数理解析研究所講究録
第 1930 巻 2015 年 88-92

Kyoto Workshop 2012: Research Community of Computer Algebra System, July 4-6, 2012,RIMS, Japan

The Pedagogical Implementation of GeoGebra as Technology Based Mathematics Learning

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Abstract

Mathematical proof, the art to understand the mathematics process, or the way each argument, definition and theorem are used is about to switch off its light among many students. In order to keep mathematical proof alive in the heart of students, we conducted a survey to find out what does mathematical proof really mean for college students.

Keywords: *Mathematics, proof, college students, Mathematical proof.*

I. Introduction

No one can doubt that the world we are living now compare to the world that our ancestors lived hundred and thousands years ago becomes the world that Technology is playing an important role in everyday aspects of our lifestyle. As an example, (1) telephone and facsimile is more used than airmail as a fast way to pass the information, (2) we can find the place or location we want to go easily using GPS, (3) in real time we can exchange ideas with friends using chat mail or we can from house transfer money to bank without to going to the bank via internet money transfer system and so forth to mention only a few. All those systems would not be possible without the support of mathematics. Mathematics is the most important field that all sciences need to prosper. For example the application of statistics in agronomy and meteorology. In agronomy experimentation, to compare a number of fertilizers, it was thought necessary to devote only a single plot to each treatment and determine yields in order to arrive at valid conclusions concerning the relative values of the treatments. However, the agronomist found soon that the yields of a series of plots treated alike differed greatly among themselves even when soil conditions appeared uniform and experimental conditions were carefully designed to reduce errors in harvesting. For this reason it becomes necessary to find some means for determining whether differences in yields were due to differences in treatment or to uncontrollable factors that also contribute to the variability of the yields. Statistical methods were applied, and their value in the scientific investigation of agronomic practices was soon proved.
Next, the modern science of meteorology is to a great degree dependent on statistical methods for its existence. For example, the methods that give weather forecasting the accuracy it has today have been developed using modern sample survey techniques. The mathematics education needs some kind of support. Thus, Technology. In this study the pedagogical implementation of geoGebra as technology based mathematics learning is used. Because geogebra is a great dynamic mathematics software for teaching and learning mathematics as it covers all of the basics of dynamic mathematics software. It offers interactive geometry, algebra, tables, graphics and calculus including statistics tools in one environment. It is a great support indeed for linking mathematics concept.

But there is one question that we should make it clear is that:

Is geogebra easy to use? Response: Yes and No

- Yes, for mathematics instructors or teachers because they know the mathematics and can therefore easily understand the ideas and logic behind the tools.

- Yes, for students who have been instructed or under training on how to use the tools and understand the mathematics and logic behind it. They can use it in solving problems and for investigating mathematical relationships.

- No, for students, especially those who have not learned the basic of graphing, equations, and geometric relationships, the use of geogebra is limited to manipulating ready-made geogebra applets. Since, geogebra applets are easy to use but most of the time if they do not know the mathematics behind the construction or can't construct it by themselves, then the learning of the mathematics may be superficial. And it is what we should avoid.

II. What GeoGebra is Capable of

There are uncountable numbers of mathematics learning based software on the market. Each of this mathematics software has their strong points as well as weak points and the evaluation of each of this software dependent on the users. What makes geogebra different from these software is the flexibility of its user interface that can be adapted to the need of any students' level. Starting from elementary pupil to university graduated students. The geogebra user interface consists of graphics view and algebra view, where graphs constructions can be done using points, vectors, segments, lines and conic sections as well as function while changing them dynamically afterwards. On the top of that, equations and coordinates can be entered directly as well as finding the derivatives and integrals of functions. It offers also commands such as root or vertex respectively. One of the most features of the geogebra is that it provides geometry tools with the mouse to create geometric constructions and the graphical representation of all objects is displayed in the graphics view respectively.

III. Basic Implementation of GeoGebra in Mathematics Class

I have been teaching mathematics for more than ten years now starting from algebra, geometry, probability statistics, Laplace transforms, Fourier series and discrete mathematics to mention a few. During these ten years of teaching experience in mathematics, I always advised my students not to use software for studying mathematics, as it will make them become lazy and will slow their ability of thinking. But during the 15th Asian Technology Conference in Mathematics that was hosted by the University of Malaya, Kuala Lumpur, Malaysia in 2012 that I first heard about geogebra. Since then I started to learn and now it becomes one of my tools of teaching mathematics to my students. Below are some examples of problems that my students were asked to solve using geogebra.

- **Problem-1:**
  Consider a connected graph G. The distance between vertices and vertex v in G, written G (u, v) is the length of the shortest path between u and v. The diameter of G, written dim (G) is the maximum distance between any two points in G.

- ***Question-1:***
  Draw the graph G and find the distance between u and v as well as the diameter of the graph G using geogebra.

- **Answer-1:**
  To solve this problem the class was divided into two groups and each group was asked to propose different solution to solve the problem and their answers if correct should be the same. To this instruction two options were given by the students.

i. **Option-1:**
First group proposed the use of grid, where seven points (A, B, C, D, E, F, and G) on the graphics view were placed. All seven points have the same distance from one another and all connected except points D and G. From the coordinates of these points on the algebra view (dependent objects) the graph distance between vertices B and vertex F , written d (B, F) is calculated and found to be 2.That is to say d(B, F) = 2. Then the diameter of the graph, written diam (G) is also calculated using the same coordinates which is 3. That is to say diam (G) =3 respectively. Fig.1 and Fig.2 are the outputs of the d(B,F) and diam (G).

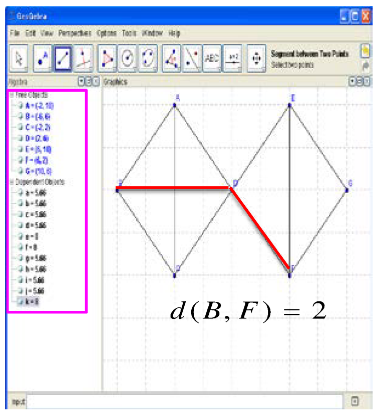

Fig.1 Proposed distance output for group1.

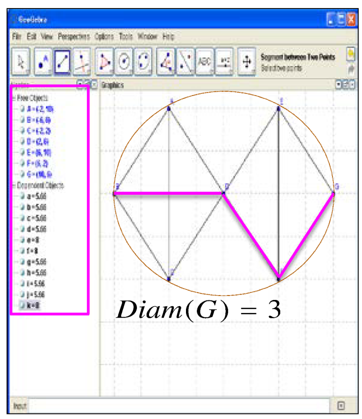

Fig.2 Proposed diameter output for group1.

ii. Option-2:

Second group proposed the use of circles, where seven circles (A, B, C, D, E, F, and G) on the graphics view were placed. All seven circles have the same distance from one another and two as radius. As for the grpoup1 all seven circles are connected except circles D and G. Next from the equations of the circles on the algebra view (dependent objects) the distance between vertices B and vertex E written d (B, E) is calculated and has found to be 2. Hence d (B, E)=2. The graph diameter is also calculated using the circle equations displayed on the algebra view which was found to be 3. That is to say diam (G)=3 Fig.3 and Fig.4 are the outputs of the d(B,F) and diam (G) respectively

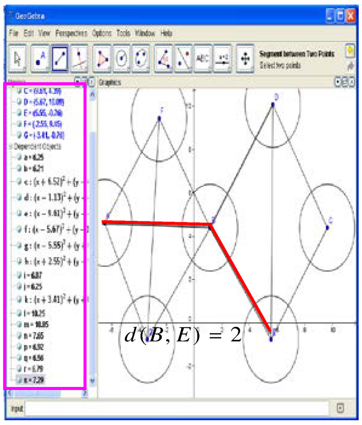

Fig.3 Proposed distance output for group2.

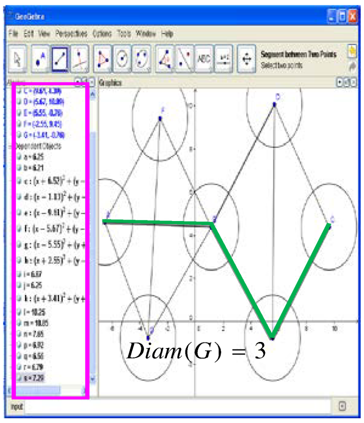

Fig.4 Proposed diameter output for group2.

- **Problem-2:**
  The following function. $f : x \to \frac{x^3}{(x-1)^2}$

Has as:

$D =]-\infty, 1[\cup]1, +\infty[$ as domain

While its sign and variation table is

x	-∞		0		1		3		+∞
f'(x)		+	0	+	‖	-	0	+	
f(x)	-∞	↗	0	↗	+∞ ‖ +∞	↘	$\frac{24}{4}$	↗	+∞

- ***Question-2:***
  Using the above information draw the graph of this function using both geogebra and usually graph draw method. Here again the class is divided into two groups and each group should report the time they spent to draw the graph. The reason of doing this kind of exercise is that some students no familiar in using software for mathematics learning are still hesitating to use geogebra. To let them know that using geogebra they can save a lot of time

- **Answer-2:**

The following is the graph drawn by students of gropu1 using geogebra following with the time spent for the both of the groups. For space constrains the graph drawn by group 2 is not reported.

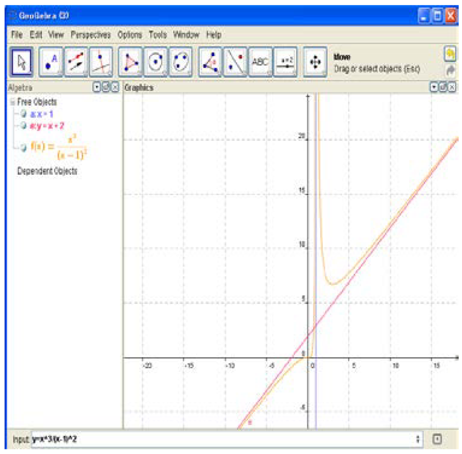

Fig.5 Graph drawn by group1 using geogebra

- **Time spent by group-1:** 90 seconds
- **Time spent by group-2:** 20 minutes

- **Problem-3:**
  A graph is said to be complete if every vertex in G is connected to every other vertex in G. Thus a complete graph must be connected. The complete graph with n vertices is denoted by K_n.

- ***Question-3:***
  Draw K_5 and K_6 using geogebra. Which one of the two graphs is Eulerian graph and tell why?

- **Answer-3:**
  To draw the graph of K_5 and K_6 all students used the grid and placed on it five pints (A, B, C, D, E) for graph K_5 and six points (A, B, C, D, E, F) for graph K6. Then connected all vertices are connected to each other. Below are the output

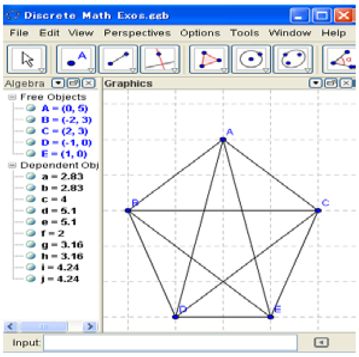

Fig.6 Graph K_5

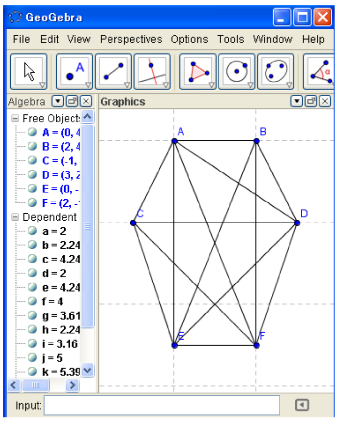

Fig.7 Graph K_6

K5is the Eulerian graph as each vertex has even degree. That is to say:

deg(A) = deg(B) = deg(c) = deg(D) = deg(E) = 4

While K_6 is not due to its odd vertex degree.

- **Problem-4:**
  From the following two signs and variation tables using Geogebra

Sign and variation table of function1

x	$-\infty$	1	$+\infty$
f'(x)		+	
f(x)	$-\infty$ ↗	0	↗ $+\infty$

Sign and variation table of function2

x	$-\infty$	0	$+\infty$
f'(x)		+	
f(x)	$-\infty$ ↗	1	↗ $+\infty$

- ***Question-4:***
  Find the appropriate functions and draw their graphs.

- **Answer-4:**
  The two functions that were found by students are as follows:

i. Function1: $g(x) = \ln x$

ii. Function2: $f(x) = e^x$

And their graphs are plotted in Fig.8

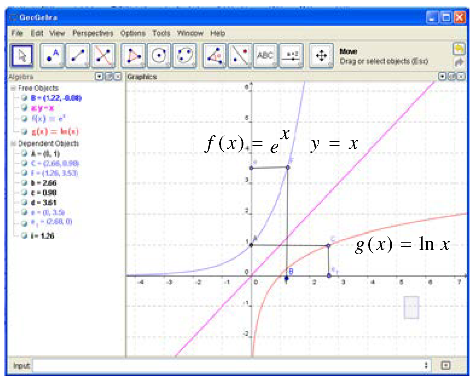

Fig.8 shows the graphs of the two functions above.

Next students changed the graphs's size from the object's properties and added some colors to make the graphs beautiful (Fig.9).

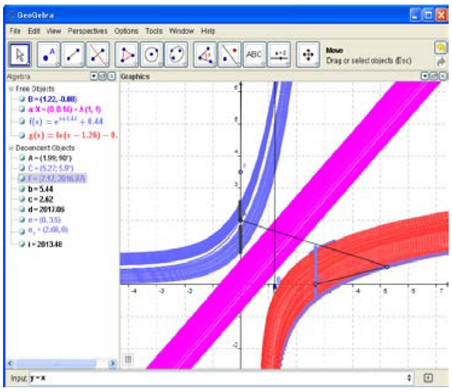

Fig.9 dynamic change of functions from Geogebra its properties window

- **Problem-5:**
  Evaluate and demonstrate the Riemann sun for

$$f(x) = 2x^3 + 8x^2 + 4x - 2$$

Taking the sample points to be under the curve where

$$a = -3,$$

$$b = -1,$$

$$n = 2$$

And upper the curve where

$$a = 1,$$

$$b = 3,$$

$$n = 4$$

- ***Question-5:***
  Sketch a graph of the function and the Riemann rectangle and use geogebra to determine the areas.

- **Answer-3:**
  The lower sum of the function f(x) is between -3 and -1 with 8 rectangles. The upper sum of the function f is between 1 and 3 with 8 rectangles as well. On the other hand, the Area for LowerSum:8.06 and Area for Upper Sum: 135.25 respectively.

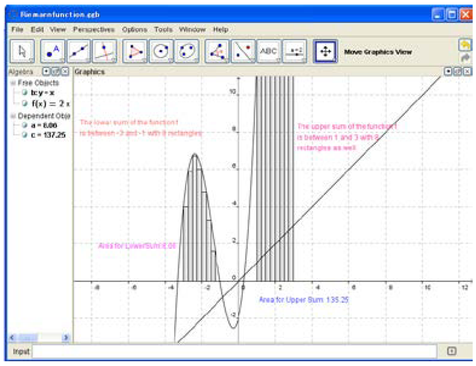

Fig.10 Riemann rectangle and its areas.

IV Conclusion

The outcomes of this discussion are as follows: The Dynamism of Geogebra can be used to custom training to use mathematical concepts and methods in addressing everyday situations or to solve practical problems. Training motivation to study mathematics as an area relevant to social and professional life. Expressing the geometric representation of the concepts related to geometry, function and so on. Development of teaching strategies based on specific skills in school curriculum.

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