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The Effects of Reversible Investment in the Presence of Business Cycle[1]

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1 Introduction

A framework of real options has been commonplace in corporate finance. The last thirty years, following the monumental works of Brennan and Schwartz (1985) and McDonald and Siegel (1986), have witnessed extensive debate and research on the investment decision of a firm based on the techniques of option pricing. Many attempts have been made in the framework of real options to broaden our horizons of understanding various issues in the real world. For instance, a number of recent works emphasize the effect of business cycle on a firm's investment and default decisions. Also, the problem of underinvestment and overinvestment induced by conflicts of interests has always been an essential issue to be investigated. Most research, however, has examined those issues separately, and little is known about the interaction between them and the joint determination of capital structure and investment decisions taking business cycle and debt maturity into account.

In the present paper, we propose a model that gives us a comprehensive understanding of the essential issues that need to be integrated; the optimal capital structure, investment triggers, and default boundary are jointly determined, taking business cycle, debt maturity, and the volatility of growth opportunities into account. The firm optimally switches between two diffusion regimes paying switching costs, and the cash flow generated from the firm depends on the state of the economy, which switches via Markov chain. The optimal switching of diffusion regimes can be read as the firm's investment provided that the coefficients of one regime dominate those of the other. Furthermore, the model integrates the investment's reversibility by allowing negative costs incurred when the firm switches from the regime with higher drift and volatility to that with lower drift and volatility. It is natural that the triggers of investment, disinvestment, and default depend on the state of the economy, and those events can occur not only by hitting the triggers but also by exogenous change of the state of the economy. After illustrating the theoretical framework, we present comparative statics regarding several parameters to articulate the effects

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of business cycle, the volatility of growth opportunities, and the debt maturity structure. The results show that their effects are intimately linked with each other, which clarifies the necessity of the integrated framework for these issues.

In terms of the influence of debt structure, the optimal leverage ratio increases in the maturity of debt consistently. Its effect on the investment timing, however, is sharply distinguished by the risks that the growth opportunities involve. When the firm can raise the expected growth rate without any incremental risks, the level of investment triggers is bimodal with respect to debt maturity, peaking at each end, and is more sensitive when the investment is irreversible. This result is consistent with Diamond and He (2014), and can be construed in the context of Myers (1977); the well-known debt overhang problem. The author proposed to issue short-term debt as a possible solution to mitigate the problem. Since the maturity of growth opportunities is infinity in our model in the sense that the firm can switch diffusion regimes any time, shortening debt maturity corresponds to any debt with finite maturity, and thus, the investment triggers get lower as the debt maturity shortens to the moderate level. But the investment triggers rebound as the maturity becomes very short, and this is because the optimal leverage ratio decreases significantly, leading the triggers to the level of those of an all-equity firm. The disparity in the sensitivity depending on the reversibility of the investment is attributed to the fact that the levered firm's incentive to invest is stronger when it is reversible so that the triggers increase less even though the maturity gets longer.

The result is markedly different if the investment entails not only higher drift but also higher volatility; the level of the investment triggers is unimodal with respect to the debt maturity, and is more sensitive when the investment is reversible. This result can be read in the context of Jensen and Meckling (1976). Equityholders usually have strong incentive to raise volatility at the expense of debtholders. However, the portion of risks that equityholders should bear increases as the debt maturity shortens to the moderate level, and thus the investment triggers rise. When the debt maturity becomes very short, however, the triggers decrease again for the same reason they increase in the previous case. The disparity of the sensitivity depending on the investment reversibility is also ascribed to the fact that the levered firm's incentive to invest is stronger when it is reversible.

In terms of the effects of macroeconomic condition, we present comparative statics regarding the persistence of the business cycle, and the results also differ significantly depending on the volatility of growth opportunities and the debt maturity. The level of investment triggers gets lower consistently as recession shortens because of the increase in expected cash flow. Still, more comprehensive understanding is needed regarding its effect on the optimal leverage ratio and yield spreads. When the investment only raises the expected growth rate, the leverage ratio increases as recession shortens because no further risks are taken. The effect is more significant for the firm with short-maturity debt, which can be read with the perspective of the negative sensitivity of the investment triggers with respect to the debt maturity. There is also difference in the effects on yield spreads. When the debt is issued with long maturity, the yield spreads are more affected by the increase in expected cash flow than the increase in leverage ratio, which is not significant for the debt with long maturity, and thus the spreads decrease.

However, the results are greatly changed if both expected growth rate and volatility increase by the investment. The optimal leverage ratio does not increase significantly, and it even sharply decreases when the debt maturity is long enough and investment is reversible. This result can be interpreted with the perspective of the positive sensitivity of investment triggers with respect to the debt maturity. As recession shortens, the timing of investment is pushed earlier by the increase in expected cash flow, which implies that the firm is more likely to be more volatile. This issue is more significant for the firms with longer debt maturity and reversible investment opportunities because the incentive of overinvestment is stronger for them, and thus they lower the leverage ratio. The yield spreads of the debt with long maturity are more affected by the expected cash flow, and thus they decrease as recession shortens. But, the debt with short maturity is more likely to be affected by the increase in leverage ratio, and so the yield spreads tend to increase as recession shortens.

The remainder of this paper is organized as follows: The setup of the theoretical framework is presented in Section 2.1, and the benchmark model which includes business cycle but does not involve the investment opportunities is briefly introduced in Section 2.2. The main model that incorporates both exogenous shocks from macroeconomic condition and the investment opportunities is investigated in Section 2.3. The parameters adopted for the comparative statics are introduced in Section 3.1, and the effects of the debt maturity and business cycle are analyzed in Section 3.2 and Section 3.3, respectively. The conclusion is given in Section 4.

2 The model and solutions

2.1 Setup

Suppose that a firm's asset value follows one-dimensional geometric Brownian motion, and that there exist two diffusion regimes in which the drift and diffusion coefficients differ. Then, the dynamics of the asset value in each regime $i \in \{\text{H,L}\}$ can be described as follows:

$$dX_t = \mu_i X_t dt + \sigma_i X_t dW_t, \quad X_0 = x, \tag{2.1}$$

where $(W_t)_{t\geq 0}$ is a standard Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual conditions. We postulate that the coefficients of regime H dominate those of regime L (i.e. $\mu_{\text{H}} \geq \mu_{\text{L}}$ and $\sigma_{\text{H}} \geq \sigma_{\text{L}}$). All agents are assumed to be risk neutral, and risk-free rate is given as a constant $r > \mu_i$ for $i \in \{\text{H,L}\}$ to ensure that the firm value is finite.

Following Vath and Pham (2007), we suppose that the equity holders can switch the diffusion regime, which involves switching costs. From the dominance of coefficients of diffusion regimes, we can regard switching from regime L to H as an investment in production facilities, which usually incurs positive costs.[2] Likewise, switching from regime H to L can be understood as a switch of business field that has fewer expected returns but is less volatile, which usually involves

[2]If it is nonpositive, the firm will invest instantly, which is a trivial case.

a liquidation of a portion of facilities with negative costs.[3]

In developing the analysis of the firm's investment and default decision, the effect of the business cycle on them should not be ignored. Hence, we introduce the state of the economy denoted by $(\epsilon_t)_{t\geq 0}$. It is independent of $(W_t)_{t\geq 0}$, and its transition probability follows a Poisson law such that it is a two-state Markov chain switching between states B and R, which refer to boom and recession, respectively. Denoting the rate of leaving the state k by $\lambda_k > 0$, there is a probability $\lambda_k \Delta t$ that the economy leaves the state k in an infinitesimal time Δt.

Given the usual interpretation of business cycle, it is reasonable to suppose that the cash flow, the investment costs, and the recovery rate depend on the business cycle. We suppose that the firm with diffusion regime i in the state k generates cash flow at the rate of $\delta_{ik} X_t$ where $\delta_{ik} := \delta_i \delta_k$ for $i \in \{\text{H,L}\}$ and $k \in \{\text{B,R}\}$, and it is natural to assume that $\delta_{\text{H}} \geq \delta_{\text{L}}$ and $\delta_{\text{B}} \geq \delta_{\text{R}}$. In terms of the switching costs, switching from regime i to j in the state k incurs a constant cost ψ_{ik}. The switching costs can be negative, and $\psi_{ik} + \psi_{jk} > 0$ must hold to prevent any redundant switching.

Meanwhile, a firm can use debt financing, and the capital structure is determined by the trade-off between tax shields and bankruptcy costs. A constant tax rate is denoted by θ, and a fraction γ_k of asset value is lost when default occurs in the state $k \in \{\text{B,R}\}$. It is also natural to presume $\gamma_{\text{B}} \leq \gamma_{\text{R}}$. Note that γ_k needs to be interpreted in a broad sense; it incorporates the huge losses accompanied with the default, such as the depreciation of the asset value, the damage of reputation, and the deterioration of credit availability in a simple form. Following Leland (1998) and Hackbarth, Miao, and Morellec (2006), we adopt stationary debt structure with finite maturity. Namely, the firm issues debt with principal p, pays a coupon c constantly, and rolls over a fraction m of the total debt. The average maturity of the debt is $1/m$ if the bankruptcy is neglected, and the debt structure is completely characterized by a tuple (c, m, p). As usual, it is assumed that the debt is issued at par and the coupon is chosen to maximize the firm value.

2.2 The benchmark model

Before entering into the main analysis, it might be useful for us to briefly introduce the case that does not involve the optimal switching of diffusion regimes as a benchmark model. That is, the cash flow generated by the firm depends on the business cycle, but the firm does not have the option of investing in production facilities. This corresponds to the case with $\psi_{\text{L}k} = \infty$ and $\psi_{\text{H}k} \geq 0$ for $k \in \{\text{B,R}\}$, and coincides with Hackbarth, Miao, and Morellec (2006).

An unlevered firm's value with diffusion regime i and initial state k is as follows:

$$\bar{U}_{ik}(x) := \mathbb{E}\Big[\int_0^\infty e^{-rt}(1-\theta)\delta_{ik} X_t dt \Big| \epsilon_0 = k\Big], \quad i \in \{\text{H,L}\}, k \in \{\text{B,R}\}, \tag{2.2}$$

where X_t is the solution of (2.1). Since the state of the economy $(\epsilon_t)_{t\geq 0}$ switches via a two-state Markov chain, the firm value should satisfy the following system of ordinary differential

[3]It is obvious that the equity holders will never switch from regime H to L if it incurs positive costs, which corresponds to the irreversible investment.

equations (ODEs) for $x \in (0,\infty)$ and $i \in$ {H,L}:

$$(r+\lambda_{\mathrm{B}})\bar{U}_{i\mathrm{B}}(x) = \mathcal{L}_i\bar{U}_{i\mathrm{B}}(x) + (1-\theta)\delta_{i\mathrm{B}}x + \lambda_{\mathrm{B}}\bar{U}_{i\mathrm{R}}(x), \tag{2.3}$$

$$(r+\lambda_{\mathrm{R}})\bar{U}_{i\mathrm{R}}(x) = \mathcal{L}_i\bar{U}_{i\mathrm{R}}(x) + (1-\theta)\delta_{i\mathrm{R}}x + \lambda_{\mathrm{R}}\bar{U}_{i\mathrm{B}}(x), \tag{2.4}$$

where $\mathcal{L}_i$ is the generator of the diffusion process in regime $i \in$ {H,L}. It is trivial to derive the solutions of (2.3) and (2.4), and they can be found in the original paper, Jeon and Nishihara (2014a).

The firm can use debt financing for the sake of tax shields, but it involves the bankruptcy costs, and the default boundary is endogenously determined to maximize the interests of equity-holders (e.g. Leland (1994), Leland and Toft (1996)). Taking cyclical cash flow into account, it is reasonable to conjecture that the default boundary in recession is above that in boom, which is verified from a numerical example in the following sections. Denoting the firm value and the default boundary with diffusion regime i in the state k by $\bar{V}_{ik}$ and $\bar{d}_{ik}$, respectively, the following system of ODEs should be satisfied for $x \in (\bar{d}_{i\mathrm{R}},\infty)$ and $i \in$ {H,L}:

$$(r+\lambda_{\mathrm{B}})\bar{V}_{i\mathrm{B}} = \mathcal{L}_i\bar{V}_{i\mathrm{B}}(x) + (1-\theta)\delta_{i\mathrm{B}}x + \theta c + \lambda_{\mathrm{B}}\bar{V}_{i\mathrm{R}}(x), \tag{2.5}$$

$$(r+\lambda_{\mathrm{R}})\bar{V}_{i\mathrm{R}} = \mathcal{L}_i\bar{V}_{i\mathrm{R}}(x) + (1-\theta)\delta_{i\mathrm{R}}x + \theta c + \lambda_{\mathrm{R}}\bar{V}_{i\mathrm{B}}(x). \tag{2.6}$$

For $x \in (d_{i\mathrm{B}}, d_{i\mathrm{R}}]$ and $i \in$ {H,L}, the firm in boom will default when the state switches to recession by exogenous shocks, and the following ODE should hold:

$$(r+\lambda_{\mathrm{B}})\bar{V}_{i\mathrm{B}}(x) = \mathcal{L}_i\bar{V}_{i\mathrm{B}}(x) + (1-\theta)\delta_{i\mathrm{B}}x + \theta c + \lambda_{\mathrm{B}}(1-\gamma_{\mathrm{R}})\bar{U}_{i\mathrm{R}}(x). \tag{2.7}$$

By the same argument, the following system of ODEs should hold for the debt value for $i \in$ {H,L}:

$$(r+m+\lambda_{\mathrm{B}})\bar{D}_{i\mathrm{B}}(x) = \mathcal{L}_i\bar{D}_{i\mathrm{B}}(x) + c + mp + \lambda_{\mathrm{B}}\bar{D}_{i\mathrm{R}}(x), \quad x \in (d_{i\mathrm{R}},\infty), \tag{2.8}$$

$$(r+m+\lambda_{\mathrm{R}})\bar{D}_{i\mathrm{R}}(x) = \mathcal{L}_i\bar{D}_{i\mathrm{R}}(x) + c + mp + \lambda_{\mathrm{R}}\bar{D}_{i\mathrm{B}}(x), \quad x \in (d_{i\mathrm{R}},\infty), \tag{2.9}$$

$$(r+m+\lambda_{\mathrm{B}})\bar{D}_{i\mathrm{B}}(x) = \mathcal{L}_i\bar{D}_{i\mathrm{B}}(x) + c + mp + \lambda_{\mathrm{B}}(1-\gamma_{\mathrm{R}})\bar{U}_{i\mathrm{R}}(x), \quad x \in (d_{i\mathrm{B}}, d_{i\mathrm{R}}]. \tag{2.10}$$

The solutions of these equations can be found in the original paper, and we omit the derivation for the same reason. Equity value is the firm value less the debt value in each regime and state (i.e. $\bar{E}_{ik}(x) = \bar{V}_{ik}(x) - \bar{D}_{ik}(x)$ for $i \in$ {H,L} and $k \in$ {B,R}). The default boundary and the coefficients of option values are simultaneously determined by the smooth-fit condition of equity value and debt value at each trigger.

2.3 The main model

Having outlined the effects of the business cycle on the firm value, we shall now proceed to analyze the firm that has options to invest and disinvest. That is, now we suppose that the firm not only is affected by business cycle but also has options to switch the diffusion regime, which incurs switching costs.[4] As noted earlier, equityholders have no incentive to switch from regime H to L if it involves nonnegative costs, which corresponds to the case of irreversible investment,

[4]See Pham (2009) for the preliminaries of optimal switching of diffusion regimes.

and for a moment we assume that the investment is reversible (i.e. $\psi_{\mathrm{H}k} < 0$ for $k \in \{\mathrm{B,R}\}$). Before developing this argument regarding a levered firm, we shall briefly introduce the case of the all-equity firm.

An increasing sequence of stopping times $(\tau_n)_{n\geq 1}$ with $\tau_n \in \mathcal{T}$ and $\tau_n \to \infty$ represents the decision when to switch, and $(\iota_n)_{n\geq 1}$ with $\iota_n \in \{\mathrm{H,L}\}$ denotes the regime at τ_n until τ_{n+1}. Also, we define $I_t^i := \sum_{n\geq 0} \iota_n \mathbf{1}_{[\tau_n,\tau_{n+1})}(t)$ with $I_{0^-}^i = i$ to trace the regime value that began from initial regime i. The process $(\epsilon_t)_{t\geq 0}$ with $\epsilon_0 = m$ is denoted by ϵ_t^m. Given these notations, the value of an unlevered firm can be written as follows:

$$U_{ik}(x) := \sup_{\tau_n \in \mathcal{T}} \mathbb{E}\Big[\int_0^\infty e^{-rt}(1-\theta)\delta_{I_t^i \epsilon_t^k} X_t^{x,i} dt - \sum_{n=1}^{\infty} e^{-r\tau_n}\psi_{\iota_{n-1}\epsilon_{\tau_n}^k}\Big], \tag{2.11}$$

where $X_t^{x,i}$ is the solution of the controlled diffusion process

$$dX_t = \mu(X_t, I_t^i)dt + \sigma(X_t, I_t^i)dW_t, \qquad t \geq 0, \quad X_0 = x. \tag{2.12}$$

It is natural to guess that the triggers of investment and disinvestment depend on the state of the economy, denoted by $s_{\mathrm{L}k}^U$ and $s_{\mathrm{H}k}^U$ for $k \in \{\mathrm{B,R}\}$, respectively. For a moment, we suppose that only cash flow depends on the business cycle. That is, for the sake of simplicity, we assume switching costs and recovery rate to be time-invariant. Given the cyclicality of cash flow, it is obvious that $s_{i\mathrm{B}}^U \leq s_{i\mathrm{R}}^U$ holds for $i \in \{\mathrm{H,L}\}$. In other words, the investment is advanced and disinvestment is delayed in boom in comparison with recession. Then, the value of all-equity firm with diffusion regime L should satisfy the following system of ODEs for $x \in (0, s_{\mathrm{LB}}^U)$:

$$(r+\lambda_{\mathrm{B}})U_{\mathrm{LB}}(x) = \mathcal{L}_{\mathrm{L}}U_{\mathrm{LB}}(x) + (1-\theta)\delta_{\mathrm{LB}}x + \lambda_{\mathrm{B}}U_{\mathrm{LR}}(x), \tag{2.13}$$

$$(r+\lambda_{\mathrm{R}})U_{\mathrm{LR}}(x) = \mathcal{L}_{\mathrm{L}}U_{\mathrm{LR}}(x) + (1-\theta)\delta_{\mathrm{LR}}x + \lambda_{\mathrm{R}}U_{\mathrm{LB}}(x). \tag{2.14}$$

If we define $A := U_{\mathrm{LB}} - U_{\mathrm{LR}}$ and $B := \lambda_{\mathrm{R}}U_{\mathrm{LB}} + \lambda_{\mathrm{B}}U_{\mathrm{LR}}$, they can be written as follows:

$$(r+\lambda_{\mathrm{B}}+\lambda_{\mathrm{R}})A(x) = \mathcal{L}_{\mathrm{L}}A(x) + (1-\theta)(\delta_{\mathrm{LB}} - \delta_{\mathrm{LR}})x, \tag{2.15}$$

$$rB(x) = \mathcal{L}_{\mathrm{L}}B(x) + (1-\theta)(\lambda_{\mathrm{R}}\delta_{\mathrm{LB}} + \lambda_{\mathrm{B}}\delta_{\mathrm{LR}})x, \tag{2.16}$$

and it is straightforward to calculate the solutions, which can be found in the original paper.[5] Given these solutions, the firm value of regime L in $x \in (0, s_{\mathrm{LB}}^U)$ can be written as follows:

$$U_{\mathrm{LB}} = (\lambda_{\mathrm{B}}A + B)/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), \qquad U_{\mathrm{LR}} = (B - \lambda_{\mathrm{R}}A)/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}). \tag{2.17}$$

By the same argument, the firm of regime H for $x \in (s_{\mathrm{HR}}^U, \infty)$ can be obtained as follows:

$$U_{\mathrm{HB}} = (\lambda_{\mathrm{B}}C + D)/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), \qquad U_{\mathrm{HR}} = (D - \lambda_{\mathrm{R}}C)/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), \tag{2.18}$$

where C and D are given in the original paper.

For $x \in [s_{\mathrm{LB}}^U, s_{\mathrm{LR}}^U)$, the firm of regime L in recession will switch to regime H when the state of the economy changes to boom, and thus the following ODE should hold:

$$(r+\lambda_{\mathrm{R}})U_{\mathrm{LR}}(x) = \mathcal{L}_{\mathrm{L}}U_{\mathrm{LR}}(x) + (1-\theta)\delta_{\mathrm{LR}}x + \lambda_{\mathrm{R}}(U_{\mathrm{HB}}(x) - \psi_{\mathrm{LB}}) \tag{2.19}$$

[5] The characteristic roots of ODEs are summarized in the original paper.

A general solution of $(r + \lambda_{\mathrm{R}})U_{\mathrm{LR}}(x) = \mathcal{L}_{\mathrm{L}} U_{\mathrm{LR}}(x)$ is $E_1 x^{\alpha_{\mathrm{R}}^+} + E_2 x^{\alpha_{\mathrm{R}}^-}$, and if we find a particular solution of (2.19), the general solution of (2.19) will be the sum of them. Having the solution of U_{HB} in (2.18), we can guess that the particular solution is of the form $E_3 x^{\beta_{\mathrm{BR}}^-} + E_4 x^{\beta^-} + E_5 x + E_6$, and a tedious algebra gives us the general solution as follows:

$$U_{\mathrm{LR}}(x) = E_1 x^{\alpha_{\mathrm{R}}^+} + E_2 x^{\alpha_{\mathrm{R}}^-} + E_3 x^{\beta_{\mathrm{BR}}^-} + E_4 x^{\beta^-} + E_5 x + E_6 \tag{2.20}$$

where the specific form of the functions E_3, E_4, E_5, and E_6 are given in the original paper.

Meanwhile, the firm of regime H with the asset value of $x \in (s_{\mathrm{HB}}^U, s_{\mathrm{HR}}^U]$ will switch to regime L, that is, liquidate a portion of its facilities, if the state of the economy changes to recession. The same argument allows us to obtain the firm value as follows:

$$U_{\mathrm{HB}}(x) = F_1 x^{\beta_{\mathrm{B}}^+} + F_2 x^{\beta_{\mathrm{B}}^-} + F_3 x^{\alpha_{\mathrm{BR}}^+} + F_4 x^{\alpha^+} + F_5 x + F_6, \tag{2.21}$$

where the specific form of the functions F_3, F_4, F_5, and F_6 can be found in original paper.

In summary, the value of all-equity firm can be represented as follows:

$$U_{\mathrm{LB}}(x) = \begin{cases} (\lambda_{\mathrm{B}} A(x) + B(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (0, s_{\mathrm{LB}}^U), \\ U_{\mathrm{HB}}(x) - \psi_{\mathrm{LB}}, & x \in [s_{\mathrm{LB}}^U, \infty), \end{cases}$$

$$U_{\mathrm{LR}}(x) = \begin{cases} (B(x) - \lambda_{\mathrm{R}} A(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (0, s_{\mathrm{LB}}^U), \\ E_1 x^{\alpha_{\mathrm{R}}^+} + E_2 x^{\alpha_{\mathrm{R}}^-} + E_3 x + E_4, & x \in [s_{\mathrm{LB}}^U, s_{\mathrm{LR}}^U), \\ U_{\mathrm{HR}}(x) - \psi_{\mathrm{LR}}, & x \in [s_{\mathrm{LR}}^U, \infty), \end{cases}$$

$$U_{\mathrm{HB}}(x) = \begin{cases} U_{\mathrm{LB}}(x) - \psi_{\mathrm{HB}}, & x \in (0, s_{\mathrm{HB}}^U], \\ F_1 x^{\beta_{\mathrm{B}}^+} + F_2 x^{\beta_{\mathrm{B}}^-} + F_3 x^{\alpha_{\mathrm{BR}}^+} + F_4 x^{\alpha^+} + F_5 x + F_6, & x \in (s_{\mathrm{HB}}^U, s_{\mathrm{HR}}^U], \\ (\lambda_{\mathrm{B}} C(x) + D(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (s_{\mathrm{HR}}^U, \infty), \end{cases}$$

$$U_{\mathrm{HR}}(x) = \begin{cases} U_{\mathrm{LR}}(x) - \psi_{\mathrm{HR}}, & x \in (0, s_{\mathrm{HR}}^U], \\ (D(x) - \lambda_{\mathrm{R}} C(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (s_{\mathrm{HR}}^U, \infty). \end{cases}$$

The constant coefficients and the triggers are simultaneously determined by smooth-fit conditions at each trigger. Even if we allow cyclicality to switching costs and recovery rate, the solution can be obtained by the same argument, except that the inequality regarding the level of switching triggers might be changed. For brevity, the detailed illustration regarding those cases is omitted here.

Having discussed the case of an all-equity firm so far, we shall now proceed to analyzing a levered firm. Provided that the investment is reversible (i.e. $\psi_{\mathrm{H}k} < 0$ for $k \in \{\mathrm{B,R}\}$), the default occurs in regime L only. The rationale behind this result is as follows: if disinvestment involves negative costs, equityholders of the firm in regime H will switch to regime L right before the default rather than default in regime H, unless the covenant prohibits liquidating production facilities.

Now we have triggers of investment, disinvestment, and default in each state k, denoted by $s_{\mathrm{L}k}$, $s_{\mathrm{H}k}$, and $d_{\mathrm{L}k}$, respectively, for $k \in \{\mathrm{B,R}\}$. If we only allow the cyclicality to the cash flow for the sake of simplicity, as assumed in the previous analysis, we can conjecture that all triggers are lower in boom than in recession because an upturn in the economy generates more cash flow, and this is verified by a numerical example in the following sections. It is natural that the disinvestment triggers decrease as the reversibility of investment worsens (i.e. as $\psi_{\mathrm{H}k}(<0)$ increases for $k \in \{\mathrm{B},\mathrm{R}\}$), and they can even be located below the default triggers. If this is the case, the firm would switch to regime L right before the default, but we assume for a moment that the reversibility is high enough that we can have the following inequality for the level of thresholds: $d_{\mathrm{LB}} < d_{\mathrm{LR}} < s_{\mathrm{HB}} < s_{\mathrm{HR}} < s_{\mathrm{LB}} < s_{\mathrm{LR}}$.

The technique used to calculate the firm value is similar to the case of the all-equity firm except that the firm defaults. The firm value of diffusion regimes L and H for $x \in (d_{\mathrm{LR}}, s_{\mathrm{LR}})$ and $x \in (s_{\mathrm{HB}}, \infty)$, respectively, can be obtained by the same argument used in (2.17), (2.18), (2.19), and (2.21).

For $x \in (d_{\mathrm{LB}}, d_{\mathrm{LR}}]$, the firm in boom will default if the state switches to recession, and the following ODE should hold:

$$(r+\lambda_{\mathrm{B}})V_{\mathrm{LB}}(x) = \mathcal{L}_{\mathrm{L}}V_{\mathrm{LB}}(x) + (1-\theta)\delta_{\mathrm{LB}}x + \theta c + \lambda_{\mathrm{B}}(1-\gamma_{\mathrm{R}})U_{\mathrm{LR}}(x). \tag{2.22}$$

A general solution of $(r+\lambda_{\mathrm{B}})V_{\mathrm{LB}}(x) = \mathcal{L}_{\mathrm{L}}V_{\mathrm{LB}}(x)$ is $L_1 x^{\alpha_{\mathrm{B}}^{+}} + L_2 x^{\alpha_{\mathrm{B}}^{-}}$, and if we find a particular solution of (2.22), then the general solution of (2.22) is the sum of them. Having the all-equity firm value in (2.17), we can calculate the solution as follows:

$$V_{\mathrm{LR}}(x) = L_1 x^{\alpha_{\mathrm{R}}^{+}} + L_2 x^{\alpha_{\mathrm{R}}^{-}} + L_3 x^{\beta_{\mathrm{BR}}^{-}} + L_4 x^{\beta^{--}} + L_5 x + L_6 \tag{2.23}$$

where the specific form of the functions L_3, L_4, L_5, and L_6 are given in the original paper.

In summary, the value of a levered firm can be represented as follows:

$$V_{\mathrm{LB}}(x) = \begin{cases} (1-\gamma_{\mathrm{B}})U_{\mathrm{LB}}(x), & x \in (0, d_{\mathrm{LB}}], \\ L_1 x^{\alpha_{\mathrm{B}}^{+}} + L_2 x^{\alpha_{\mathrm{B}}^{-}} + L_3 x^{\alpha_{\mathrm{BR}}^{+}} + L_4 x^{\alpha^{+}} + L_5 x + L_6, & x \in (d_{\mathrm{LB}}, d_{\mathrm{LR}}], \\ (\lambda_{\mathrm{B}} G(x) + H(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (d_{\mathrm{LR}}, s_{\mathrm{LB}}), \\ V_{\mathrm{HB}}(x) - \psi_{\mathrm{LB}}, & x \in [s_{\mathrm{LB}}, \infty) \end{cases}$$

$$V_{\mathrm{LR}}(x) = \begin{cases} (1-\gamma_{\mathrm{R}})U_{\mathrm{LR}}(x), & x \in (0, d_{\mathrm{LR}}], \\ (H(x) - \lambda_{\mathrm{R}} G(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (d_{\mathrm{LR}}, s_{\mathrm{LB}}), \\ K_1 x^{\alpha_{\mathrm{R}}^{+}} + K_2 x^{\alpha_{\mathrm{R}}^{-}} + K_3 x^{\beta_{\mathrm{BR}}^{-}} + K_4 x^{\beta^{-}} + K_5 x + K_6, & x \in [s_{\mathrm{LB}}, s_{\mathrm{LR}}), \\ V_{\mathrm{HR}}(x) - \psi_{\mathrm{LR}}, & x \in [s_{\mathrm{LR}}, \infty) \end{cases}$$

$$V_{\mathrm{HB}}(x) = \begin{cases} V_{\mathrm{LB}}(x) - \psi_{\mathrm{HB}}, & x \in (0, s_{\mathrm{HB}}], \\ M_1 x^{\beta_{\mathrm{B}}^{+}} + M_2 x^{\beta_{\mathrm{B}}^{-}} + M_3 x^{\alpha_{\mathrm{BR}}^{+}} + M_4 x^{\alpha_{\mathrm{BR}}^{-}} + M_5 x^{\alpha^{+}} + M_6 x^{\alpha^{-}} + M_7 x + M_8, & x \in (s_{\mathrm{HB}}, s_{\mathrm{HR}}], \\ (\lambda_{\mathrm{B}} I(x) + J(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (s_{\mathrm{HR}}, \infty), \end{cases}$$

$$V_{\mathrm{HR}}(x) = \begin{cases} V_{\mathrm{LR}}(x) - \psi_{\mathrm{HR}}, & x \in (0, s_{\mathrm{HR}}], \\ (J(x) - \lambda_{\mathrm{R}} I(x))/(\lambda_{\mathrm{B}} + \lambda_{\mathrm{R}}), & x \in (s_{\mathrm{HR}}, \infty). \end{cases}$$

The functional form of the coefficients above can be found in the original paper.

Likewise, the debt value in each regime and state should satisfy the following system of

ODEs:

$$
\begin{aligned}
(r+m+\lambda_{\mathrm{B}})D_{\mathrm{LB}}(x) &= \mathcal{L}_{\mathrm{L}}D_{\mathrm{LB}}(x)+c+mp+\lambda_{\mathrm{B}}D_{\mathrm{LR}}(x), && x\in(d_{\mathrm{LR}},s_{\mathrm{LB}}),\\
(r+m+\lambda_{\mathrm{R}})D_{\mathrm{LR}}(x) &= \mathcal{L}_{\mathrm{L}}D_{\mathrm{LR}}(x)+c+mp+\lambda_{\mathrm{R}}D_{\mathrm{LB}}(x), && x\in(d_{\mathrm{LR}},s_{\mathrm{LB}}),\\
(r+m+\lambda_{\mathrm{B}})D_{\mathrm{LB}}(x) &= \mathcal{L}_{\mathrm{L}}D_{\mathrm{LB}}(x)+c+mp+\lambda_{\mathrm{B}}(1-\gamma_{\mathrm{R}})U_{\mathrm{LR}}(x), && x\in(d_{\mathrm{LB}},d_{\mathrm{LR}}],\\
(r+m+\lambda_{\mathrm{R}})D_{\mathrm{LR}}(x) &= \mathcal{L}_{\mathrm{L}}D_{\mathrm{LR}}(x)+c+mp+\lambda_{\mathrm{R}}D_{\mathrm{HB}}(x), && x\in[s_{\mathrm{LB}},s_{\mathrm{LR}}),\\
(r+m+\lambda_{\mathrm{B}})D_{\mathrm{HB}}(x) &= \mathcal{L}_{\mathrm{H}}D_{\mathrm{HB}}(x)+c+mp+\lambda_{\mathrm{B}}D_{\mathrm{HR}}(x), && x\in(s_{\mathrm{HR}},\infty),\\
(r+m+\lambda_{\mathrm{R}})D_{\mathrm{HR}}(x) &= \mathcal{L}_{\mathrm{H}}D_{\mathrm{HR}}(x)+c+mp+\lambda_{\mathrm{R}}D_{\mathrm{HB}}(x), && x\in(s_{\mathrm{HR}},\infty),\\
(r+m+\lambda_{\mathrm{B}})D_{\mathrm{HB}}(x) &= \mathcal{L}_{\mathrm{H}}D_{\mathrm{HB}}(x)+c+mp+\lambda_{\mathrm{B}}D_{\mathrm{LR}}(x), && x\in(s_{\mathrm{HB}},s_{\mathrm{HR}}].
\end{aligned}
$$

The solutions can be acquired in the same manner, and they can be summarized as follows:

$$
D_{\mathrm{LB}}(x)=\begin{cases}
(1-\gamma_{\mathrm{B}})U_{\mathrm{LB}}(x), & x\in(0,d_{\mathrm{LB}}],\\
S_1x^{\alpha^+_{m\mathrm{B}}}+S_2x^{\alpha^-_{m\mathrm{B}}}+S_3x^{\alpha^+_{\mathrm{BR}}}+S_4x^{\alpha^+}+S_5x+S_6, & x\in(d_{\mathrm{LB}},d_{\mathrm{LR}}],\\
(\lambda_{\mathrm{B}}N(x)+O(x))/(\lambda_{\mathrm{B}}+\lambda_{\mathrm{R}}), & x\in(d_{\mathrm{LR}},s_{\mathrm{LB}}],\\
D_{\mathrm{HB}}(x), & x\in(s_{\mathrm{LB}},\infty)
\end{cases}
$$

$$
D_{\mathrm{LR}}(x)=\begin{cases}
(1-\gamma_{\mathrm{R}})U_{\mathrm{LR}}(x), & x\in(0,d_{\mathrm{LR}}],\\
(O(x)-\lambda_{\mathrm{R}}N(x))/(\lambda_{\mathrm{B}}+\lambda_{\mathrm{R}}), & x\in(d_{\mathrm{LR}},s_{\mathrm{LB}}],\\
R_1x^{\alpha^+_{m\mathrm{R}}}+R_2x^{\alpha^-_{m\mathrm{R}}}+R_3x^{\beta^-_{m\mathrm{BR}}}+R_4x^{\beta^-_m}+R_5, & x\in(s_{\mathrm{LB}},s_{\mathrm{LR}}],\\
D_{\mathrm{HR}}(x), & x\in(s_{\mathrm{LR}},\infty),
\end{cases}
$$

$$
D_{\mathrm{HB}}(x)=\begin{cases}
D_{\mathrm{LB}}(x), & x\in(0,s_{\mathrm{HB}}],\\
T_1x^{\beta^+_{m\mathrm{B}}}+T_2x^{\beta^-_{m\mathrm{B}}}+T_3x^{\alpha^+_{m\mathrm{BR}}}+T_4x^{\alpha^-_{m\mathrm{BR}}}+T_5x^{\alpha^+_m}+T_6x^{\alpha^-_m}+T_7, & x\in(s_{\mathrm{HB}},s_{\mathrm{HR}}],\\
(\lambda_{\mathrm{B}}P(x)+Q(x))/(\lambda_{\mathrm{B}}+\lambda_{\mathrm{R}}), & x\in(s_{\mathrm{HR}},\infty),
\end{cases}
$$

$$
D_{\mathrm{HR}}(x)=\begin{cases}
D_{\mathrm{LR}}(x), & x\in(0,s_{\mathrm{HR}}],\\
(Q(x)-\lambda_{\mathrm{R}}P(x))/(\lambda_{\mathrm{B}}+\lambda_{\mathrm{R}}), & x\in(s_{\mathrm{HR}},\infty).
\end{cases}
$$

The functional form of the coefficients above can be found in the original paper.

Equity value is the firm value less the debt value (i.e. $E_{ik}(x)=V_{ik}(x)-D_{ik}(x)$ for $i\in\{\mathrm{H,L}\}$ and $k\in\{\mathrm{B,R}\}$). The triggers and the coefficients of option values are simultaneously determined by the smooth-fit conditions of equity value and debt value at each trigger.

The solutions can be obtained by the same argument even if we allow cyclicality to switching costs and recovery rate except that the inequality regarding the level of triggers may be changed, and the illustration for those cases is omitted to avoid unnecessary duplication.

As noted earlier, the firm will not switch back to regime L if $\psi_{\mathrm{H}k}\geq 0$ for $k\in\{\mathrm{B,R}\}$, and this corresponds to the case of irreversible investment. If the investment is indeed irreversible, there are no disinvestment triggers, and the default can also occur in regime H, but the default boundaries of regime H will be lower than those of regime L. The triggers of investment and default are denoted by $\hat{s}_{\mathrm{L}k}$ and $\hat{d}_{ik}$ for each regime $i\in\{\mathrm{H,L}\}$ and state $k\in\{\mathrm{B,R}\}$. The firm value, debt value, and equity value are denoted by $\hat{V}_{ik}$, $\hat{D}_{ik}$, and $\hat{E}_{ik}$, respectively. Regarding an unlevered firm, the investment trigger and the firm value are denoted by $\hat{s}^U_{\mathrm{L}k}$ and $\hat{U}_{ik}$, respectively. They are determined following the same argument, and can be found in the original paper.

3 Comparative statics

3.1 Parameters

We adopt $r = 0.08$ as a constant risk-free rate, which is close to the historical average Treasury rates (e.g. Huang and Huang (2012)). For the diffusion regimes, it is assumed that the coefficients of regime H dominate those of regime L, and we adopt $\mu_{\mathrm{L}} = 0.2$ and $\mu_{\mathrm{H}} = 0.3$ for the drift coeffients. Regarding the diffusion coefficients, we fix $\sigma_{\mathrm{L}} = 0.2$ and analyze with respect to different volatilities of regime H to clarify the individual features of growth opportunities, namely, $\sigma_{\mathrm{H}} = 0.2$ and $\sigma_{\mathrm{H}} = 0.25$. Compared with the former in which only the expected growth rate grows, in the latter case, it is accompanied by the increase of risks.

For simplicity, we postulate that only cash flow differs depending on business cycle. That is, switching costs and recovery rate are assumed to be time-invariant.[6] Since it is natural that more cash flow is generated in boom than in recession, we presume that $\delta_{\mathrm{B}} = 1.5$ and $\delta_{\mathrm{R}} = 1$, having $\delta_{\mathrm{H}} = \delta_{\mathrm{L}} = 0.06$ fixed as Huang and Huang (2012) did.[7] We adopt $\gamma_{\mathrm{B}} = \gamma_{\mathrm{R}} = 0.4869$ for a fraction of loss at default to reflect that the average recovery rate is 51.31% in Huang and Huang (2012).

In terms of switching costs, we shall limit ourselves to investment with high reversibility, namely $\psi_{\mathrm{LB}} = \psi_{\mathrm{LR}} = 15$ and $\psi_{\mathrm{HB}} = \psi_{\mathrm{HR}} = -14$, so that we can verify the inequality regarding the level of thresholds assumed in the previous section (i.e. $d_{\mathrm{LB}} < d_{\mathrm{LR}} < s_{\mathrm{HB}} < s_{\mathrm{HR}} < s_{\mathrm{LB}} < s_{\mathrm{LR}}$). If the reversibility of investment worsens, the disinvestment triggers can be located below the default boundaries, which implies that the firm will disinvest right before the default. This is the extreme case of the agency problem,[8] which is not the main issue of the present paper, and we shall focus on other aspects of the problem.

Following Hackbarth, Miao, and Morellec (2006), we adopt $\lambda_{\mathrm{B}} = 0.1$ and $\lambda_{\mathrm{R}} = 0.15$ for the persistence of the business cycle, and $m = 0.2$ for the rate of rollover (i.e. 5 years of average debt maturity). For the tax rate, we use $\theta = 0.35$ as numerous works have done (e.g. Leland (1994), Leland and Toft (1996)). Initial asset value is assumed to be $x_0 = 100$.

3.2 The debt maturity

In this subsection, we let m (the rate of rollover) vary and examine the impact of debt maturity on the optimal leverage and investment decisions of the firm. As noted earlier, the debt is issued at par and the coupon is chosen to maximize the firm value.

First, we assume that the firm can raise the expected growth rate via growth opportunities without any increase of volatility (i.e. $\sigma_{\mathrm{H}} = \sigma_{\mathrm{L}} = 0.2$).

---

[6] Hackbarth, Miao, and Morellec (2006) also assumed that the recovery rate is irrelevant to business cycle. Refer to Elliott, Miao, and Yu (2009), Du and MacKay (2010), and Jeon and Nishihara (2014c) for the impact of time-varying switching costs on the investment decisions of a firm.

[7] They assumed that the payout ratio is fixed at 6% regardless of the credit ratings.

[8] See Jeon and Nishihara (2014b) for the detailed discussion regarding this problem.

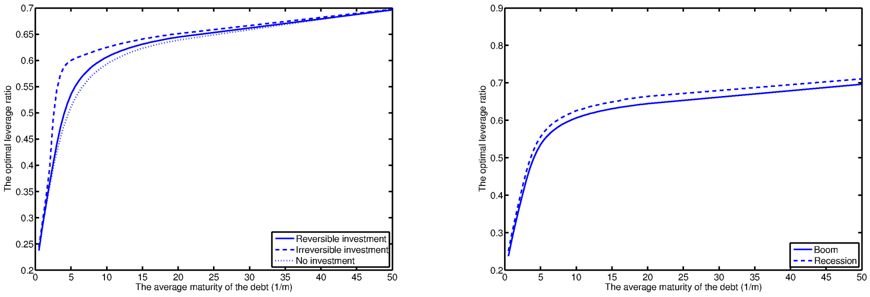
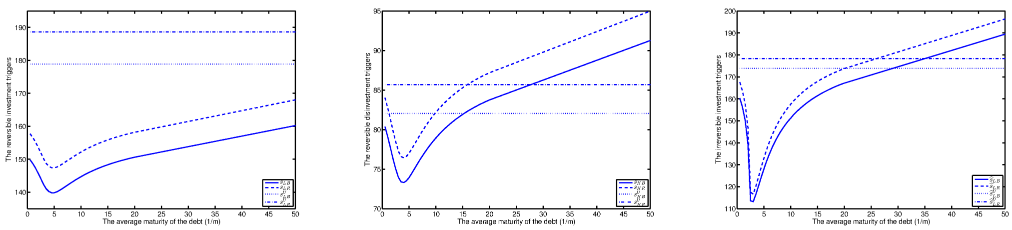


Figure 1: The optimal leverage ratio w.r.t. debt maturity provided $\sigma_{\mathrm{H}} = \sigma_{\mathrm{L}} = 0.2$

Panel (a) of Figure 1 presents the firm's optimal leverage for different investment reversibility provided that the firm is of regime L and there is an upturn in the economy. We can see that the optimal leverage increases in the debt maturity, which is in line with Leland and Toft (1996), Leland (1998), and Hackbarth, Miao, and Morellec (2006).[9] The reason for this result is delineated in Hackbarth, Miao, and Morellec (2006) as follows: a reduction in the maturity of the debt contract implies an increase in the debt service, which increases the default probability, and thus the optimal response of the firm is to lower the leverage ratio. We can also notice that the leverage ratio of the firm with reversible investment is lower than that of the firm with irreversible investment. Panel (b) of Figure 1 shows the countercyclical leverage ratio, which is consistent with Hackbarth, Miao, and Morellec (2006) and Chen and Manso (2010).



(a) Reversible / Investment (b) Reversible / Disinvestment (c) Irreversible / Investment

Figure 2: The level of the triggers w.r.t. debt maturity provided $\sigma_{\mathrm{H}} = \sigma_{\mathrm{L}} = 0.2$

Figure 2 describes the investment triggers of the firm with reversible and irreversible investment opportunities. We can see that the triggers are bimodal with respect to the debt maturity, peaking at each end, and more sensitive when the investment is irreversible. This result is consistent with Diamond and He (2014), which elucidates the effects of debt maturity, and can be construed in the context of Myers (1977): the underinvestment problem with risky debt financing. Myers (1977) says that if debt matures after the expiration of the investment option, risky debt financing may induce the incentive of underinvestment because a portion of equityholders' benefits accrue to the debtholders in the form of reduction of the default probability; to mitigate this problem, the author suggested shortening debt maturity.

[9] In Brennan and Schwartz (1978), the optimal leverage decreases as the maturity gets longer, but the default boundary is exogenous in their model.

In our model, the maturity of the options is infinity in the sense that the firm can invest in production facilities any time, and so shortening of debt maturity corresponds to any debt with finite maturity. We can see in Figure 2 that the investment triggers get lower as the average debt maturity shortens from infinity to the moderate level. As the maturity becomes very short, however, they rebound; and this result is intimately linked with Figure 1. As the debt maturity shortens, the optimal leverage decreases significantly (from 70% to 25% in the range of our analysis), and thus the level of investment triggers converges to that of an unlevered firm as the debt maturity becomes very short. It seems that this explication does not fit very well if only the reversible investment triggers are concerned. But, the tendency for the triggers to converge to those of an all-equity firm is crystal clear in the disinvestment triggers and irreversible investment triggers. The disparity of the degree of convergence and sensitivity is attributed to the fact that the incentive of investment of a levered firm is stronger when it is reversible so that the triggers increase less even though the maturity gets longer.

Now we shall proceed to the case in which the investment is accompanied by not only the increase of expected growth rate but also the increase of volatility (i.e. $\sigma_{\mathrm{H}} = 0.25$ and $\sigma_{\mathrm{L}} = 0.2$).

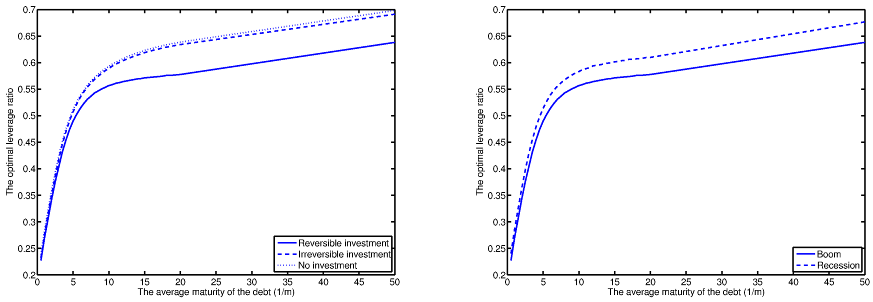


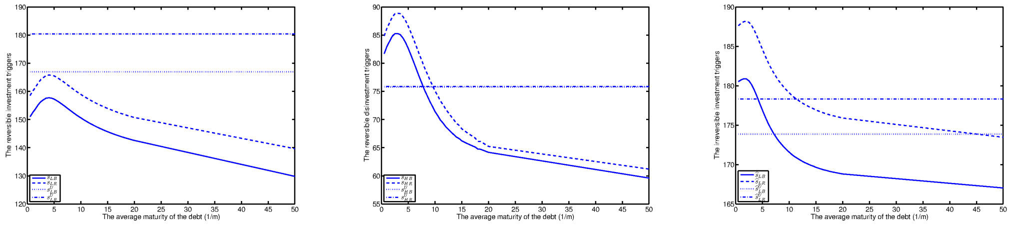
Figure 3: The optimal leverage ratio w.r.t. debt maturity provided $\sigma_{\mathrm{H}} = 0.25 > \sigma_{\mathrm{L}} = 0.2$

We can see from Panel (a) of Figure 3 that the optimal leverage ratio also increases in debt maturity regardless of the increase of risks via growth opportunities. Furthermore, the negative correlation between the growth opportunities and leverage ratio is more significant; the leverage ratio of the firm with reversible investment is lower than that with irreversible investment and even that with no real options.[10] Note that the gap between them decreases as the maturity of debt shortens. The countercyclicality of leverage ratio is also shown in Panel (b) of Figure 3, and the gap between the optimal leverage ratio in each state widens as the volatility of growth opportunity increases.

We also have similar results regarding the investment reversibility and the capital structure. The leverage ratio with reversible investment is lower than that with the irreversible one, even lower than the case with no investment. Yet, the rationale behind this result is different from that in Figure 1. The investment raises not only the expected growth rate but also the volatility, and this makes the equity holders with reversible investment opportunity advance the timing of

[10] It has been examined in numerous works that there is negative correlation between leverage ratio and growth opportunities (e.g. Mauer and Sarkar (2005) and Childs, Mauer, and Ott (2005)).

investment to earlier than those with irreversible one. This implies that the firm is more likely to be volatile, leaving less room for raising the leverage ratio.[11]



(a) Reversible / Investment (b) Reversible / Disinvestment (c) Irreversible / Investment

Figure 4: The level of the triggers w.r.t. debt maturity provided $\sigma_{\mathrm{H}} = 0.25 > \sigma_{\mathrm{L}} = 0.2$

Meanwhile, Figure 4 shows that the investment triggers are unimodal with respect to the debt maturity and more sensitive when the investment is reversible, contrasting sharply with the result from Figure 2. This result is directly linked to the overinvestment problem (e.g. Jensen and Meckling (1976)). When the maturity of both the real options and the debt is infinity, equityholders have a strong incentive to raise volatility, because their expected profits increase at the expense of debtholders. Yet, the portion of risks that equityholders should bear increases as the debt maturity shortens, and thus the investment triggers increase. When the maturity becomes very short, however, the triggers decrease again for the same reason they increase in Figure 2; namely, the optimal leverage ratio decreases significantly, and thus the triggers tend to converge to the level of those of an all-equity firm. The disparity of the convergence and the sensitivity is also ascribed to the fact that the incentive of the investment of a levered firm is stronger when it is reversible. This sharp contrast in the effects of growth opportunities on investment timing is a novel result; this result is not presented in Diamond and He (2014) because only the drift is controlled via the investment in their model.

3.3 The persistence of business cycle

In this subsection, we examine how the business cycle affects a firm's decisions to invest and default. Having in the previous subsection discussed the significant effects of debt maturity and the volatility of growth opportunities on a firm's the optimal leverage ratio and investment decision, we find it natural to conjecture that there exist a disparity in the effects of the business cycle which depends on them. Hence, we present the comparative statics of λ_k with respect to the different investment projects and debt maturity; namely, for $\sigma_{\mathrm{H}} = 0.2$ and $\sigma_{\mathrm{H}} = 0.25$, and for $m = 0.2$ (a bond with average maturity of 5 years) and $m = 0$ (a consol bond), respectively.[12]

[11] This rationale can also be confirmed by Figure 4 which presents the level of investment triggers, that is, the incentive to invest. We can see that the gap between the leverage ratios of the two cases widens as the average maturity of the debt lengthens so that the difference between the investment timing of them increases.

[12] They are chosen to represent short and long maturity. When the maturity becomes very short (e.g. 6 months), the yield spreads and the leverage ratio are very low, unsuitable for analyzing the effects, and so we adopt $m = 0.2$ (5-year maturity on average) following Hackbarth, Miao, and Morellec (2006).

Note that there is a probability $\lambda_k \Delta t$ that the economy leave the state k in an infinitesimal time Δt, and thus the state k shortens as λ_k increases for $k \in \{\mathrm{B},\mathrm{R}\}$. The same argument can be obtained from the comparative statics of λ_{B} and λ_{R}, and for brevity we introduce only the latter.

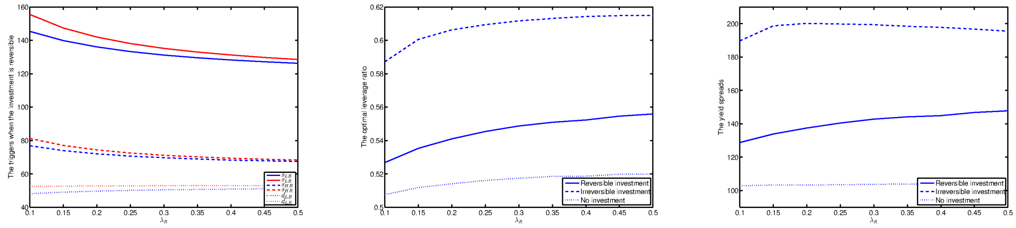


Figure 5: The various triggers, the optimal leverage ratio, and the yield spreads provided $\sigma_{\mathrm{H}} = \sigma_{\mathrm{L}} = 0.2$ and $m = 0.2$

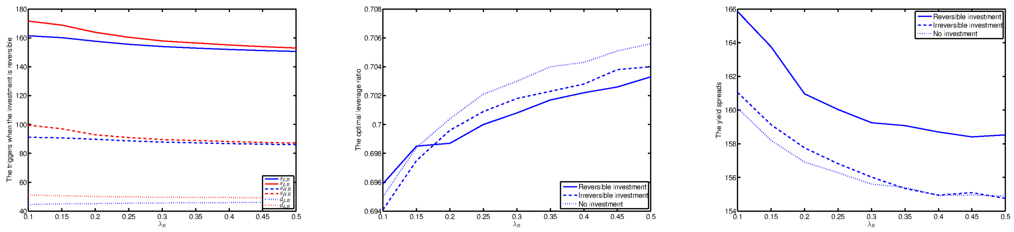


Figure 6: The various triggers, the optimal leverage ratio, and the yield spreads provided $\sigma_{\mathrm{H}} = \sigma_{\mathrm{L}} = 0.2$ and $m = 0$

First, we supose that the firm can raise the expected growth rate without increasing risks at all (i.e. $\sigma_{\mathrm{H}} = \sigma_{\mathrm{L}} = 0.2$). Figures 5 and 6 correspond to the comparative statics of λ_{R} with $m = 0.2$ and $m = 0$, respectively. It is assumed that the firm's asset value is of regime L and the economy is in an upturn. Above all, we can observe in Panel (a) of Figure 5 and 6 that the investment gets earlier as λ_{R} increases, and this is because the expected cash flow increases as the recession shortens. Also, the gap between triggers of each state diminishes as the persistence of the state shortens.

In terms of the optimal leverage ratio, we can see that it increases in λ_{R} because the expected cash flow increases without any incremental risks. It is more significant for the firm with short-maturity debt (i.e. $m = 0.2$), and this can be read in the context of what we have examined in the previous subsection: the negative sensitivity of investment triggers with respect to debt maturity (provided that it is not very short) in Figure 2. The incentive of investment is stronger when the debt is issued with short maturity, because the underinvestment problem noted in Myers (1977) is mitigated.

The effects on yield spreads also differ depending on the debt structure. When the debt is issued with long maturity, the yield spreads are more affected by the increase in expected cash flow than the leverage ratio, which does not increase significantly, and thus the yield spreads decrease. But, the influence of leverage ratio is more significant when the debt is issued with short maturity, and thus the yield spreads tend to increase as recession shortens. With irreversible investment, they sharply increase and then decrease gradually, and this result can also be construed with what we have discussed in the previous subsection; the sensitivity of the triggers is more evident when the investment is irreversible. Thus, the asset value is more likely to have higher drift when the investment is irreversible, leading to the gradual decrease in the yield spreads.

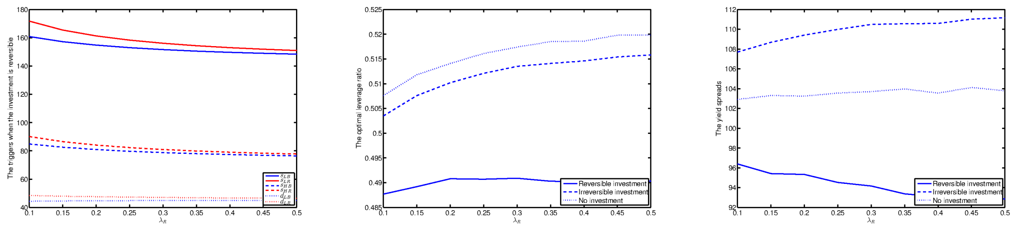
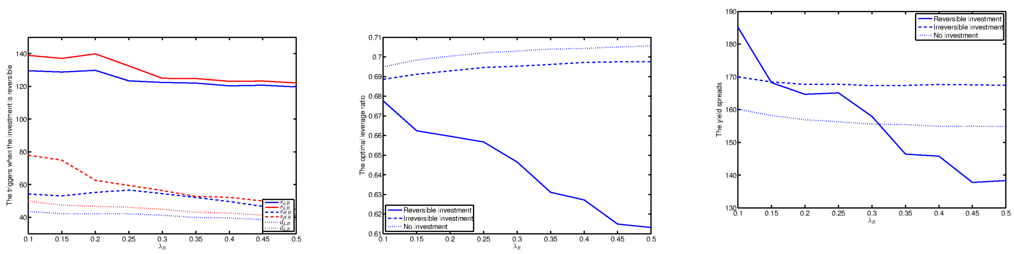


Figure 7: The various triggers, the optimal leverage ratio, and the yield spreads provided $\sigma_{\mathrm{H}} = 0.25 > \sigma_{\mathrm{L}} = 0.2$ and $m = 0.2$



(a) The yield spreads

Figure 8: The various triggers, the optimal leverage ratio, and the yield spreads provided $\sigma_{\mathrm{H}} = 0.25 > \sigma_{\mathrm{L}} = 0.2$ and $m = 0$

Now we shall proceed to the case in which the higher expected growth rate entails higher volatility (i.e. $\sigma_{\mathrm{H}} = 0.25$ and $\sigma_{\mathrm{L}} = 0.2$). Figures 7 and 8 correspond to the comparative statics of λ_{R} with $m = 0.2$ and $m = 0$, respectively. As observed earlier, the investment timing gets advanced and the gap between triggers of each state decreases as λ_{R} increases, but the other issues make a sharp distiction from the former analysis.

The optimal leverage ratio tends to increase in λ_{R} for most cases because of the increase in the expected cash flow. It sharply decreases, however, for the firm with a consol bond and reversible investment opportunities, and this can also be read in the context of the analysis in the previous subsection: the negative sensitivity of investment triggers with respect to debt maturity (provided that it is not very short) in Figure 4. As the expected cash flow increases, the investment timing gets earlier; that is, the firm is more likely to be more volatile, and this issue is more significant for the firms with longer debt maturity and reversible investment opportunities because the incentive of overinvestment is stronger for them. Note that the sensitivity of investment triggers is more significant when the investment is reversible.

The effects on yield spreads also differ depending on the maturity of debt. The yield spreads decrease in λ_R provided that the debt is issued with long maturity, and they are most significant when the investment is reversible. It is obvious for the case of reversible investment, because in that case the leverage ratio sharply decreases and the expected cash flow increases. The yield spreads also decrease in other two cases (i.e. the cases with irreversible investment and no real options) because the effect of increase in expected cash flow is more significant for the debt with longer maturity and the increase of leverage is not significant. However, this is not the case when the debt is issued with short maturity. The debt value is less affected by the increase in expected cash flow, and so the yield spreads increases if the investment is irreversible or if there is no switching in diffusion regime. When the investment is reversible, the yield spreads decrease because the change in leverage ratio is less significant and the firm can even switch back to the regime with lower volatility.

4 Conclusion

We have proposed a model that incorporates the business cycle and the investment opportunities of the firm. The state of the economy, which affects the firm's cash flow, switches via Markov chain, and the firm can switch the diffusion regime of asset value paying switching costs. The triggers of investment, disinvestment, and default, which depend on the state of the economy, are determined endogenously. Both the investment and default can occur not only by hitting the triggers but also by exogenous change of the state of the economy.

The relation between investment timing and debt maturity is associated with the volatility of investment opportunities. The level of investment triggers is bimodal with respect to the debt maturity when the investment does not involve any incremental risks, peaking at each end, and this result can be read in the context of the underinvestment problem noted in Myers (1977). They are unimodal, however, with respect to the debt maturity when higher drift entails higher volatility, which can be construed with the perspective of the overinvestment problem noted in Jensen and Meckling (1976). The optimal leverage ratio is countercyclical in either case, but the gap between the leverage ratio in each state is wider when the volatility of growth opportunity is higher.

The effects of the persistence of the business cycle are also intimitely linked with the volatility of investment opportunities and the debt maturity. As the recession shortens, the leverage ratio tends to increase, which leads to higher yield spreads for the debt with short maturity; but the yield spreads tend to decrease when the debt is issued with longer maturity, because the effect of the increase in expected cash flow is more significant for them. Furthermore, the degree of the tendency depends a great deal on the reversibility of investment and the incremental risks involved.

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