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Strategic Capacity Investment under Time-Inconsistent Preferences[1]

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1. Introduction

In this paper, we extend "the strategic capacity investment" model developed by Huisman and Kort [2013] to the case that the firm has "time inconsistent preferences" developed by Grenadier and Wang [2007].

As an application of our model, we will show some macroeconomic implications that the firm might overinvest and the price of the product would be decline as a result. These results just explain the recent serious deflation after the bubble economy (such as Japan and Euro area etc.). In particular, we focus our analysis on changes of the interest rate (discount rate) and we consider the firm who takes the discount rate as a time (in)-consistent manner. In the analysis, we compare the time consistent firm and time inconsistent firm respectively.

2. Base model: Strategic Capacity Investment

X_t follows a geometric Brownian motion:

$$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dW_t, \tag{1}$$

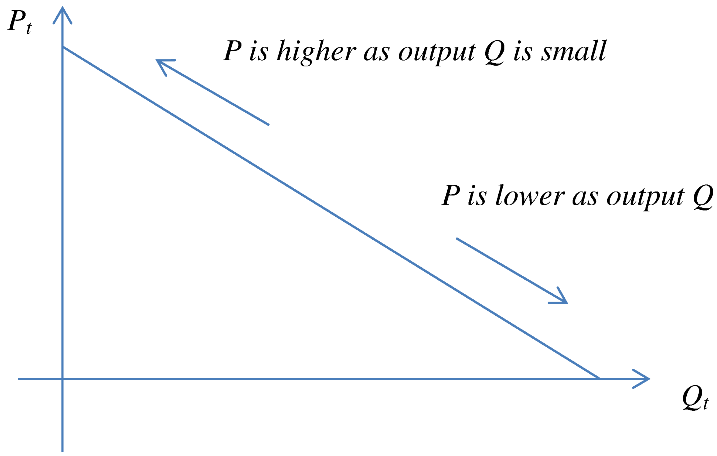
P_t is the price of the firm's output.

$$P_t = X_t \cdot (1 - \eta \cdot Q_t), \tag{2}$$

where Q_t is the firm's productive capacity (=the total quantity of market output) and $\eta > 0$ is constant (price elasticity).

[1] This paper is an abbreviated version. All proofs, remarks and some results are omitted due to the page restriction.
[2] Views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan.

The quantity of product Q is larger, the price P becomes lower. The profit of the investment depends on the trade-off between P and Q, therefore, there must be the optimal Q (as noted later, the profit is quadratic function for Q).



We consider the firm's optimal investment problem:

$$V(X_t) = \max_{\tau,Q} \mathbf{E}_t\left(\int_\tau^\infty e^{-r\cdot(s-t)}\cdot\pi_s\,ds - e^{-r\cdot(\tau-t)}I\right)$$
$$= \max_{\tau,Q} \mathbf{E}_t\left(\int_\tau^\infty e^{-r\cdot(s-t)}\cdot Q\cdot X_s(1-\eta Q)\cdot ds - e^{-r\cdot(\tau-t)}\cdot\delta Q\right) \quad (3)$$

,where $\pi_t = QP_t$ is the firm's profit and $I = \delta Q$ is investment cost.

First, we consider the case of $\tau = t$ to derive "the value match term" (the investment value if the firm had invested) as follows. The investment value V is quadratic to Q, and it is easy to derive the optimal value V and optimal Q.

$$V(X_t) = \max_{Q} \mathbf{E}_t\left(\int_t^\infty e^{-r\cdot(s-t)}\cdot Q\cdot X_s(1-\eta Q)\cdot ds - \delta Q\right)$$
$$= \max_{Q}\left(\frac{Q\cdot X_t\cdot(1-\eta Q)}{r-\mu} - \delta Q\right)$$

→ Quadratic form

$$= \max_{Q} \left(\frac{-\eta \cdot X_t \cdot \left(Q - \frac{X_t - \delta(r-\mu)}{2\eta \cdot X_t}\right)^2 + \frac{\left(X_t - \delta(r-\mu)\right)^2}{4\eta \cdot X_t}}{r-\mu} \right)$$

→ optimal Q is

$$Q^*(X_t) = \frac{X_t - \delta(r-\mu)}{2\eta \cdot X_t}$$

→ value match V is

$$V_0(X_t) = \frac{\left(X_t - \delta(r-\mu)\right)^2}{4\eta \cdot X_t(r-\mu)}$$

Next, we use the standard smooth pasting technique, and we could derive the optimal solution of the base model.

$$V(X) = \begin{cases} \dfrac{\left(X^* - \delta(r-\mu)\right)^2}{4\eta \cdot (r-\mu)X^*}\left(\dfrac{X}{X^*}\right)^{\alpha} & if\ X < X^* \\ \left(X - \delta(r-\mu)\right)^2 / \left(4X\eta(r-\mu)\right) & if\ X \geq X^* \end{cases} \tag{4}$$

$$X^* = \tfrac{\alpha+1}{\alpha-1}\delta(r-\mu) \tag{5}$$

$$Q^*(X) = \tfrac{1}{2\eta}\left(1 - \tfrac{\delta(r-\mu)}{X}\right) \tag{6}$$

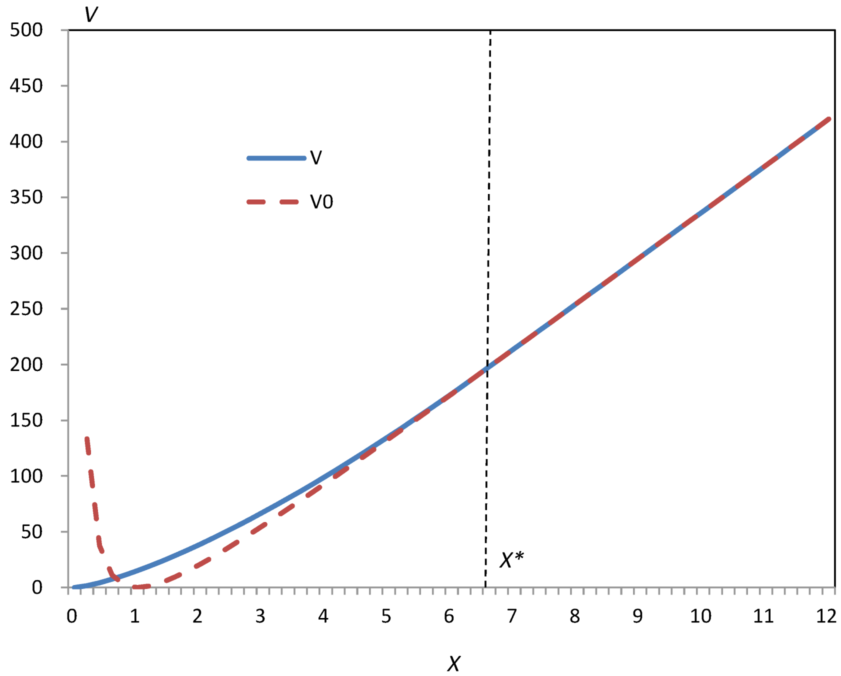
where

$$\alpha = \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}} \tag{7}$$

We show the first numerical example as follow:

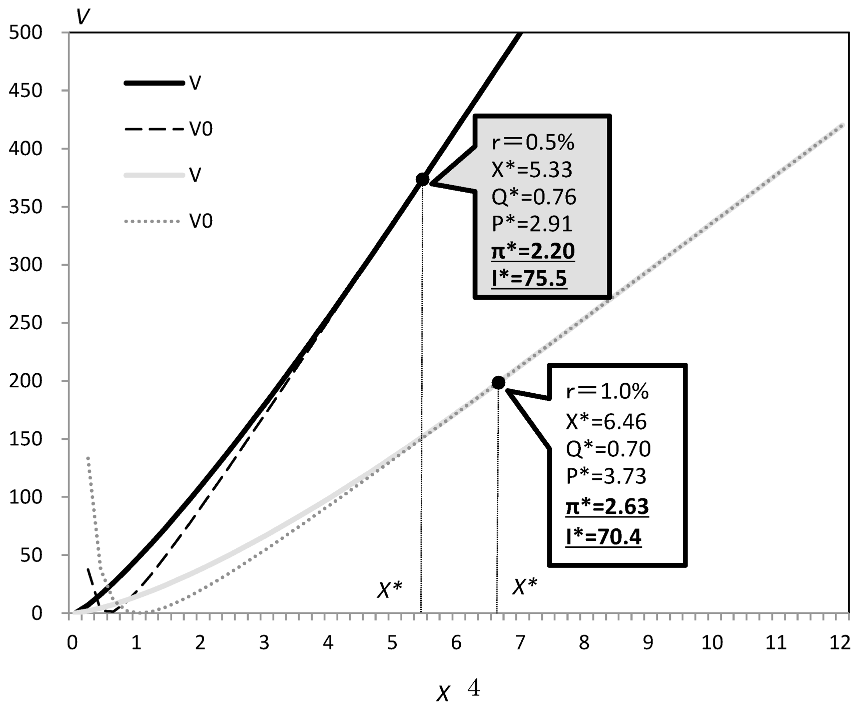
μ=0%, σ=20%, r=1.0%,δ=100,η=0.6,r=1.0%

→ X^*=6.48, Q^*=0.70, P^*=3.73, π^*=2.63, I^*=70.4

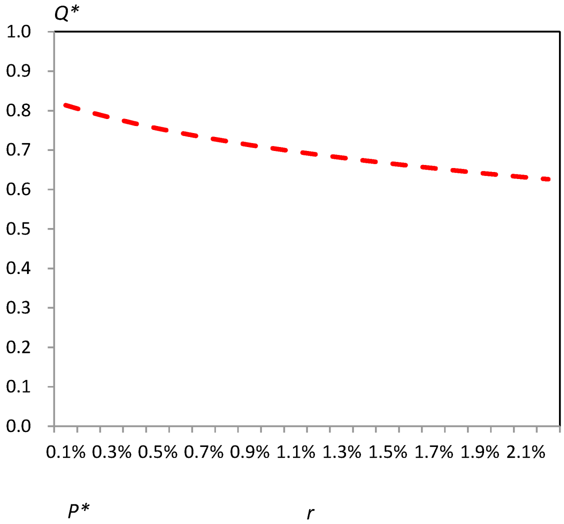


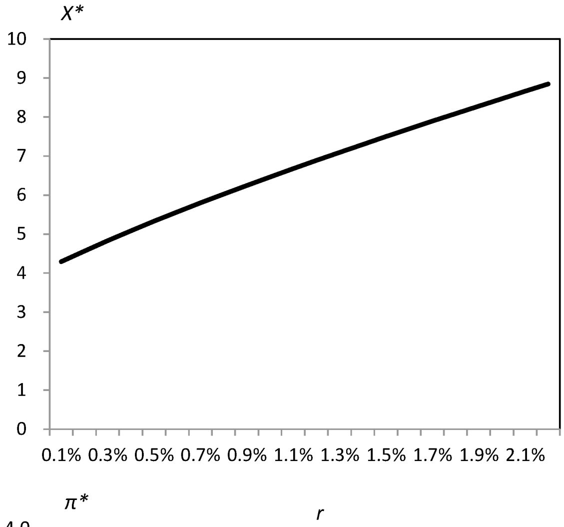
Next, we show the comparative statics as to the risk free rate r. We find that the firm invests earlier (X* is smaller), larger quantity Q*, the price of product becomes decline (P^* is lower) as risk free rate is lower. We also find that the investment cost I^* becomes larger and profit π^*becomes lower.

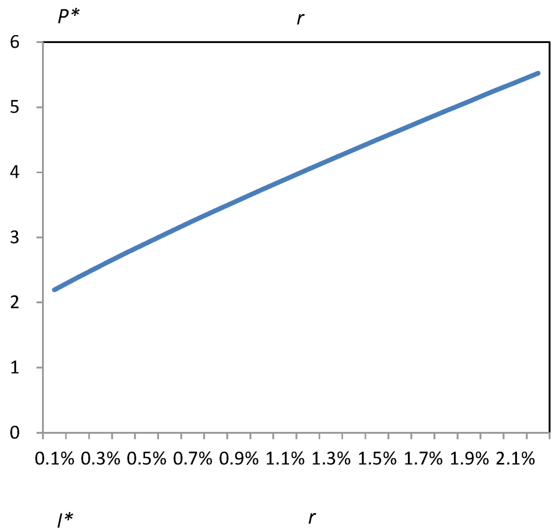
μ=0%, σ=20%, r=1.0%,δ=100,η=0.6, r=1.0%→0.5%⇒ X^*, Q^*, P^*, π^*, I^*=?

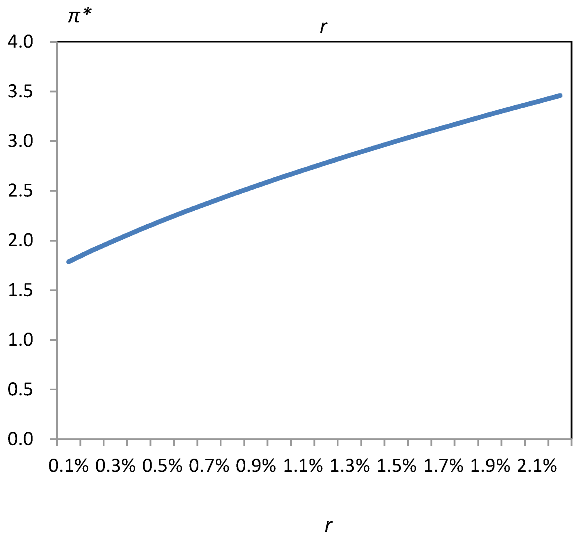


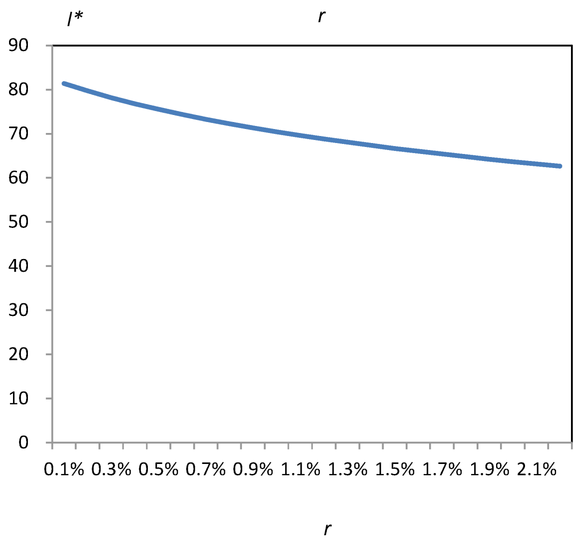
We show the comparative statics as to the risk free rate r more in detail.



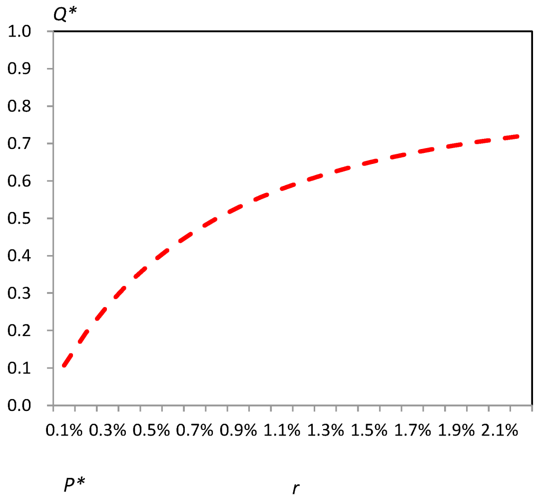


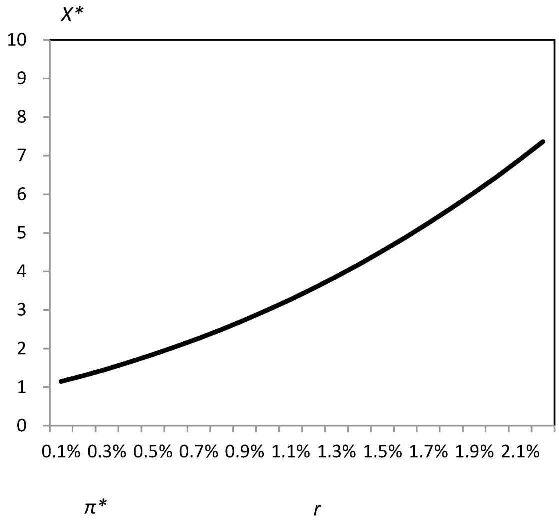


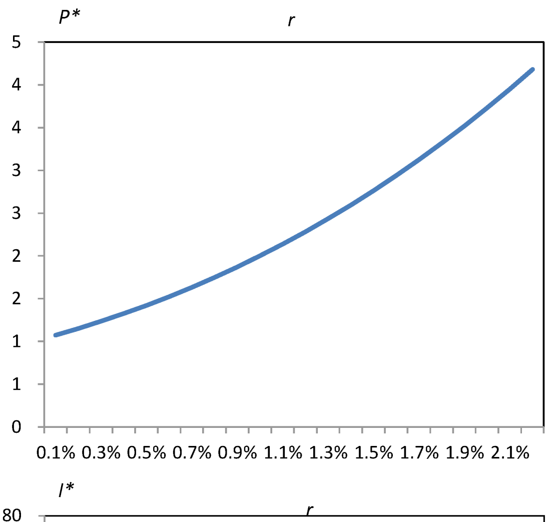


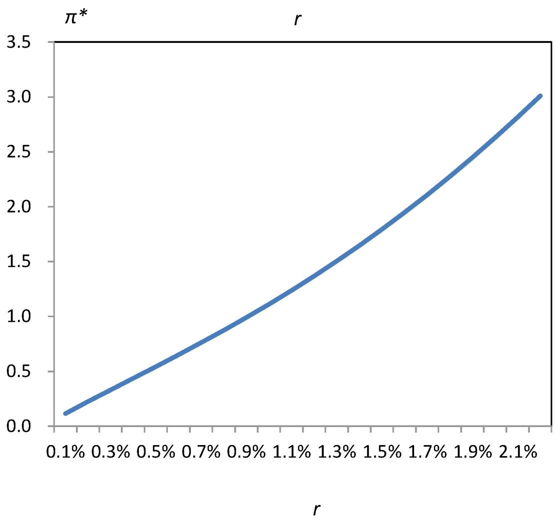


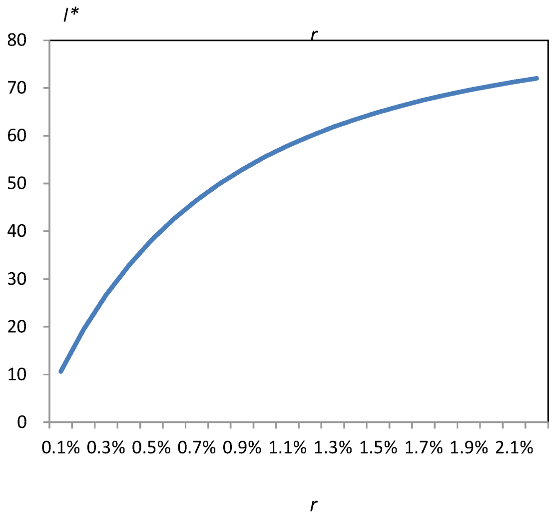
We also show the comparative statics as to volatility σ.











3. Main model : time-inconsistent preferences case

We consider the firm's optimal investment decision with time-inconsistent preferences:

$$V(X_t) = \max_{\tau,Q} \mathbf{E}_t \left[\begin{array}{l} e^{-r_1\cdot(\tau-\tau_1)-r_0\cdot(\tau_1-t)} \left(\int_{\tau}^{\infty} e^{-r_1\cdot(s-\tau)} \cdot \pi_s \, ds - I \right) 1_{\{\tau>\tau_1\}} \\ + e^{-r_0\cdot(\tau-t)} \left(\int_{\tau}^{\infty} e^{-r_0\cdot(s-\tau)} \cdot \pi_s \, ds - I \right) 1_{\{\tau\le\tau_1\}} \end{array} \right] \tag{8}$$

,where $\pi_t = QP_t$ is the firm's profit and $I = \delta Q$ is investment cost.

In addition, we also consider the firm's optimal investment decision with time-consistent preferences.

$$V(X_t) = \max_{\tau,Q} \mathbf{E}_t \left[\begin{array}{l} e^{-r_1\cdot(\tau-\tau_1)-r_0\cdot(\tau_1-t)} \left(\int_{\tau}^{\infty} e^{-r_1\cdot(s-\tau)} \cdot \pi_s \, ds - I \right) 1_{\{\tau>\tau_1\}} \\ +e^{-r_0\cdot(\tau-t)} \left(\begin{array}{c} \int_{\tau}^{\tau_1} e^{-r_0\cdot(s-\tau)} \cdot \pi_s \, ds \\ + \int_{\tau_1}^{\infty} e^{-r_1\cdot(s-\tau)} \cdot \pi_s \, ds - I \end{array} \right) 1_{\{\tau\le\tau_1\}} \end{array} \right] \tag{9}$$

,where $\pi_t = QP_t$ is the firm's profit and $I = \delta Q$ is investment cost.

First, we solve the time-inconsistent preferences cases as follows:

$$V(X_t) = \begin{cases} \dfrac{\lambda}{\lambda - (r_1 - r_0)} V_1(X) + \left(V_0({X_0}^*) - \dfrac{\lambda}{\lambda - (r_1 - r_0)} V_1({X_0}^*) \right) \left(\dfrac{X}{{X_0}^*} \right)^{\alpha_0} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad if\ X < {X_0}^* \\ V_0(X_t) = \dfrac{\left(X_t - \delta(r_0 - \mu)\right)^2}{4\eta \cdot X_t (r_0 - \mu)} \quad if\ X \geq {X_0}^* \end{cases} \tag{10}$$

$$V_1(X_t) = \begin{cases} \dfrac{\left({X_1}^* - \delta(r_1 - \mu)\right)^2}{4\eta \cdot (r_1 - \mu){X_1}^*} \left(\dfrac{X}{{X_1}^*} \right)^{\alpha_1} & if\ X < {X_1}^* \\ \left(X - \delta(r_1 - \mu)\right)^2 / \left(4X\eta(r_1 - \mu)\right) & if\ X \geq {X_1}^* \end{cases} \tag{11}$$

$${X_1}^* = \frac{(\alpha_1 + 1)}{(\alpha_1 - 1)} \delta(r_1 - \mu) \tag{12}$$

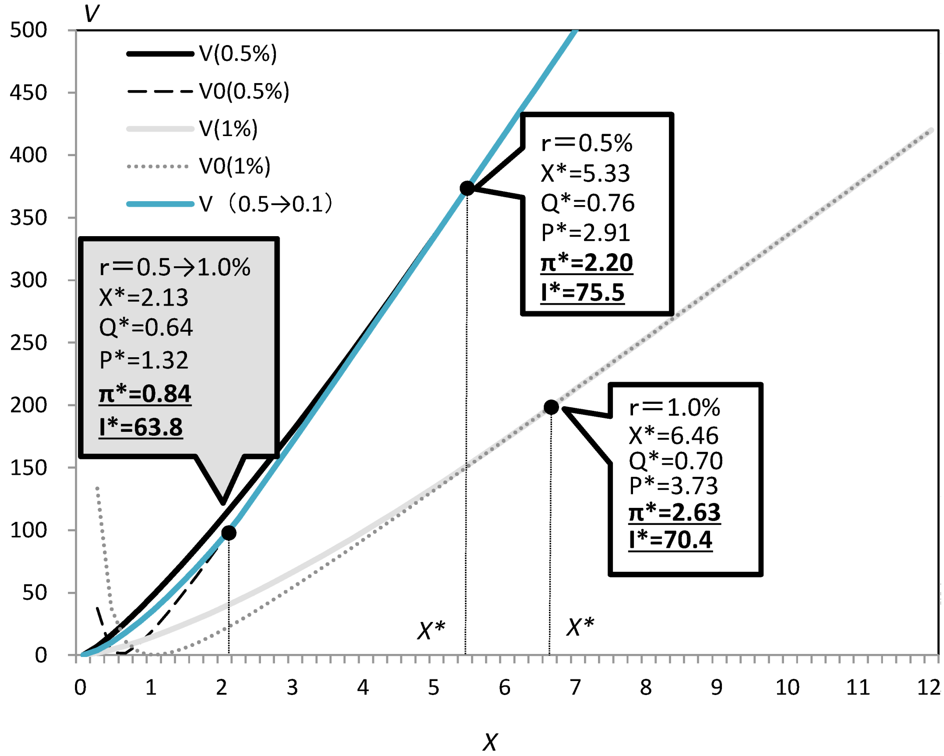
$$\frac{\lambda}{\lambda - (r_1 - r_0)} \left(\alpha_0 V_1({X_0}^*) - {X_0}^* V_1'({X_0}^*) \right) = \alpha_0 V_0({X_0}^*) - {X_0}^* V_0'({X_0}^*) \tag{13}$$

$$V_k'(X^*) = \frac{\left(X^* - \delta(r_k - \mu)\right)\left(X^* + \delta(r_k - \mu)\right)}{4\eta(r_k - \mu)(X^*)^2}$$

$$V_k\ (X^*) = \frac{\left(X^* - \delta(r_k - \mu)\right)}{4\eta(r_k - \mu)X^*} \tag{14}$$

$${Q_0}^*(X_t) = \frac{X_t - \delta(r_0 - \mu)}{2\eta \cdot X_t} \tag{15}$$

We show the numerical example as follows. We consider the case that risk free rate r will change form 0.5% to 1.0% and the other parameters are the same to the example of chapter2.



It is surprising that the firm invest at extremely earlier timing (X^* is extremely smaller) than the case of the standard real option model (constant interest rate 0.5%). Although the firm invests a little lower quantity Q^* but the price of the product P^* is extremely lower than the case of the standard real option model (constant interest rate 0.5%). Therefore, the profit of the firm's investment is extremely lower than that the standard real option model. These results suggest that the existence of myopic (time inconsistent) firm leads the serious deflation economy.

4. Main mode : time-consistent preferences case

Next, we consider the firm's optimal investment decision with time-consistent preferences.

$$V(X_t) = \max_{\tau,Q} \mathbf{E}_t \left[\begin{array}{l} e^{-r_1\cdot(\tau-\tau_1)-r_0\cdot(\tau_1-t)} \left(\int_{\tau}^{\infty} e^{-r_1\cdot(s-\tau)} \cdot \pi_s \, ds - I \right) 1_{\{\tau>\tau_1\}} \\ +e^{-r_0\cdot(\tau-t)} \left(\begin{array}{c} \int_{\tau}^{\tau_1} e^{-r_0\cdot(s-\tau)} \cdot \pi_s \, ds \\ + \int_{\tau_1}^{\infty} e^{-r_1\cdot(s-\tau)} \cdot \pi_s \, ds - I \end{array} \right) 1_{\{\tau\le\tau_1\}} \end{array} \right] \tag{16}$$

where $\pi_t = QP_t$ is the firm's profit and $I = \delta Q$ is investment cost.

We solve the time-consistent preferences cases as follow. The "difference" from the inconsistent cases is "value match" term W.

$$V(X_t) = \begin{cases} \dfrac{\lambda}{\lambda-(r_1-r_0)} V_1(X) + \left(W(X_0{}^*) - \dfrac{\lambda}{\lambda-(r_1-r_0)} V_1(X_0{}^*) \right) \left(\dfrac{X}{X_0{}^*} \right)^{\alpha_0} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad if\ X < X_0{}^* \\ W(X_t) = \dfrac{\left(X_t - \delta(r_\lambda - \mu)\right)^2}{4\eta(r_\lambda - \mu)} \quad if\ X \ge X_0{}^* \\ r_\lambda = \mu + \dfrac{(r_0+\lambda)-\mu}{(r_1+\lambda)-\mu}(r_1-\mu) \end{cases} \tag{17}$$

$$V_1(X_t) = \begin{cases} \dfrac{\left(X_1{}^* - \delta(r_1-\mu)\right)^2}{4\eta\cdot(r_1-\mu)X_1{}^*} \left(\dfrac{X}{X_1{}^*}\right)^{\alpha_1} & if\ X < X_1{}^* \\ \left(X - \delta(r_1-\mu)\right)^2 / \left(4X\eta(r_1-\mu)\right) & if\ X \ge X_1{}^* \end{cases} \tag{18}$$

$$X_1{}^* = \frac{(\alpha_1+1)}{(\alpha_1-1)} \delta(r_1-\mu) \tag{19}$$

$$\frac{\lambda}{\lambda-(r_1-r_0)} \left(\alpha_0 V_1(X_0{}^*) - X_0{}^* V_1'(X_0{}^*)\right) = \alpha_0 W(X_0{}^*) - X_0{}^* W'(X_0{}^*) \tag{20}$$

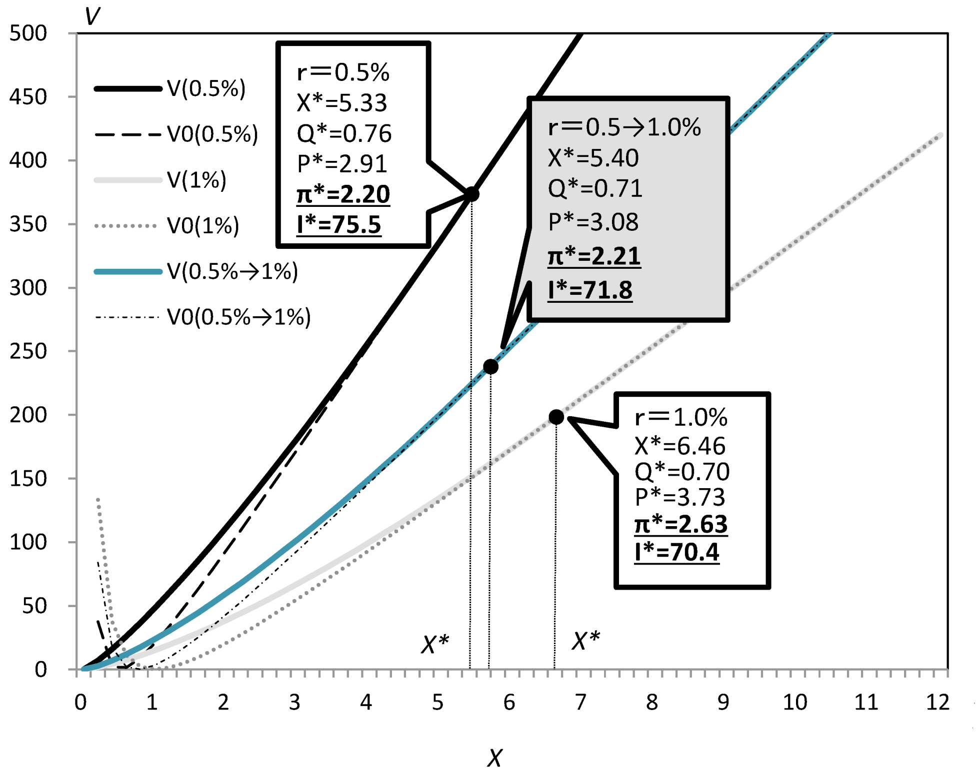
$$V_k'(X^*) = \frac{\big(X^* - \delta(r_k - \mu)\big)\big(X^* + \delta(r_k - \mu)\big)}{4\eta(r_k - \mu)(X^*)^2}$$

$$W'(X^*) = \frac{\big(X^* - \delta(r_\lambda - \mu)\big)\big(X^* + \delta(r_\lambda - \mu)\big)}{4\eta(r_\lambda - \mu)(X^*)^2} \tag{21}$$

$$V_k\ (X^*) = \frac{\big(X^* - \delta(r_k - \mu)\big)}{4\eta(r_k - \mu)X^*}$$

$$Q_0{}^*(X_t) = \frac{X_t - \delta(r_\lambda - \mu)}{2\eta \cdot X_t} \tag{22}$$

We show the numerical example. We consider the case that risk free rate r will change form 0.5% to 1.0% and the other parameters are the same to the example of chapter 2 and 3.



In contrast to the case of time inconsistent firm, the time consistent firm invest later (X^* is larger) than the standard the case of r =0.5%. Although the firm invests a little lower quantity Q^* but the price of the product P^* is higher than the case of the standard real option model (constant interest rate 0.5%). Therefore, the profit of the firm's investment is higher than that the standard real option model. These results suggest that the existence of rational (time consistent) firm does not lead the serious deflation economy.

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