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Stability of stationary solutions to hyperbolic-parabolic systems in half space and the convergence rate

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1 Introduction

The present paper surveys the results on [8] and [10], which study large-time behavior of solutions to a system of viscous conservation laws over one-dimensional half space $\mathbb{R}_+ := (0,\infty)$,

$$U_t + f(U)_x = (G(U)U_x)_x, \quad x \in \mathbb{R}_+,\ t > 0, \tag{1}$$

where $U = U(t,x)$ is an unknown m-vector valued function taking values in an open convex set $\mathcal{O}_U \subset \mathbb{R}^m$; $f(U)$ is a smooth m-vector valued function defined on $\mathcal{O}_U$; $G(U)$ is a smooth $m \times m$ matrix valued function defined on $\mathcal{O}_U$. The paper [8] shows an existence and an asymptotic stability of a stationary solution to the system (1) and the paper [10] derives a convergence rate of a time-global solution towards the stationary solution.

To study the system (1) we rewrite it to a normal form of the symmetric hyperbolic-parabolic systems under the assumption that

> [**A1**] the system (1) admit an entropy function $\eta = \eta(U)$ defined on $\mathcal{O}_U$, which satisfies following three conditions:
>
> (i) $\eta(U)$ is a smooth strictly convex scalar function, that is, the Hessian matrix $D_U^2\eta(U)$ is positive definite for $U \in \mathcal{O}_U$;
>
> (ii) there exists a smooth scalar function $q(U)$ defined on $\mathcal{O}_U$, which is called an entropy flux, such that $D_U q(U) = D_U\eta(U) D_U f(U)$ for $U \in \mathcal{O}_U$;
>
> (iii) the matrix $G(U)(D_U^2\eta(U))^{-1}$ is real symmetric and non-negative definite for $U \in \mathcal{O}_U$.

The assumption [**A1**] allows us to rewrite the system (1) to that in a symmetric form by using the entropy function. Furthermore we can transform the symmetric system to the normal form which is a coupled system of hyperbolic and parabolic equations by assuming a null condition:

> [**N**] the null space of the viscosity matrix $G(U)$ is independent of the dependent variable U.

Using the assumptions [**A1**] and [**N**], we see there exists a diffeomorphism $U \mapsto u$ from $\mathcal{O}_U$ onto $\mathcal{O}_u \subset \mathbb{R}^m$, which allows us to rewrite the system (1) to that for a new dependent variable u as

$$A^0(u)u_t + A(u)u_x = B(u)u_{xx} + g(u,u_x). \tag{2}$$

Here $A^0(u)$, $A(u)$ and $B(u)$ are real symmetric matrices of the form

$$A^0(u) = \begin{pmatrix} A_1^0(u) & 0 \\ 0 & A_2^0(u) \end{pmatrix}, \; A(u) = \begin{pmatrix} A_{11}(u) & A_{12}(u) \\ A_{21}(u) & A_{22}(u) \end{pmatrix}, \; B(u) = \begin{pmatrix} 0 & 0 \\ 0 & B_2(u) \end{pmatrix}.$$

In (2), $A^0(u)$ is real symmetric and positive definite, that is, $A_1^0(u)$ and $A_2^0(u)$ are real symmetric and positive definite; $A(u)$ is real symmetric, that is, $A_{11}(u)$ and $A_{22}(u)$ are symmetric and ${}^\top A_{12}(u) = A_{21}(u)$; $B_2(u)$ is real symmetric and positive definite; $g(u, u_x)$ is a nonlinear term

$$g(u, u_x) = \begin{pmatrix} 0 \\ g_2(u, u_x) \end{pmatrix}.$$

Since (2) is obtained by multiplying (1) by ${}^\top U_u D_U^2 \eta$, we have expressions of A^0, A, B and g as

$$A^0 = {}^\top U_u D_U^2 \eta U_u, \quad A = {}^\top U_u D_U^2 \eta f_U U_u, \quad B = {}^\top U_u D_U^2 \eta G U_u,$$
$$g = {}^\top U_u D_U^2 \eta (G U_u)_x u_x.$$

Letting $u = {}^\top(v, w)$ where $v = v(t, x) \in \mathbb{R}^{m_1}$ and $w = w(t, x) \in \mathbb{R}^{m_2}$, we deduce the system (2) to the decomposed form

$$A_1^0(u) v_t + A_{11}(u) v_x + A_{12}(u) w_x = 0, \tag{3a}$$
$$A_2^0(u) w_t + A_{21}(u) v_x + A_{22}(u) w_x = B_2(u) w_{xx} + g_2(u, u_x). \tag{3b}$$

We prescribe the initial data for (3) as

$$u(0, x) = u_0(x) = {}^\top(v_0, w_0)(x), \quad \text{i.e.,} \quad (v, w)(0, x) = (v_0, w_0)(x), \tag{4}$$

with assuming that a spatial asymptotic state of the initial data is a constant:

$$\lim_{x \to \infty} u_0(x) = u_+ = {}^\top(v_+, w_+), \quad \text{i.e.,} \quad \lim_{x \to \infty} (v_0, w_0)(x) = (v_+, w_+). \tag{5}$$

Here the spatial asymptotic state $u_+ = {}^\top(v_+, w_+)$ is chosen to satisfy the condition

[A2] The matrix $A_{11}(u_+)$ is negative definite for a certain $u_+ \in \mathcal{O}_u$.

The assumption **[A2]** implies that the characteristic speeds of the hyperbolic equations (3a) are negative around u_+. Hence boundary conditions only for the parabolic equations (3b) are necessary and sufficient for the well-posedness if we construct the solution in a small neighborhood of u_+. Thus we prescribe the boundary conditions for (3) as

$$w(t, 0) = w_b, \tag{6}$$

where $w_b \in \mathbb{R}^{m_2}$ is a constant. We also assume 0-th order compatibility condition holds. We show the existence of a solution to the problem (3)–(6) globally in time under the smallness assumption on a boundary strength $|w_b - w_+|$. Thus the condition **[A2]** yields that the characteristics of the hyperbolic system (3a) around the boundary are negative.

The hyperbolic-parabolic system is a generalization of the concrete models arising in physical models, especially in fluid dynamics. The assumption **[A2]** corresponds to the outflow problem for the model system of compressible viscous gases. This problem is studied in [3, 4, 11]. For the heat-conductive model of compressible viscous gases in $\mathbb{R}^3$, Matsumura and Nishida in [6] show the asymptotic stability of a constant state (or a stationary solution corresponding to an external potential force) and establish a technical energy method. For the system (1) in the full space $\mathbb{R}^n$, Umeda, Kawashima and Shizuta in [13] consider a sufficient condition which guarantees a dissipative structure of the system (1) and show the asymptotic stability of the constant state.

The half space problem to the hyperbolic-parabolic coupled systems is studied by Kawashima, Nishibata and Zhu in [4], where they consider outflow problems for a barotropic model of compressible and viscous gases. They show the existence and the asymptotic stability of stationary solutions. For the heat-conductive model, Kawashima, Nakamura, Nishibata and Zhu [3] prove the existence and the asymptotic stability of stationary solutions for the outflow problem, too.

Notations. For vectors $u, v \in \mathbb{R}^m$, $|u|$ and $\langle u, v\rangle$ denote standard Euclidean norm and inner product, respectively. For a matrix A, ${}^{\top}A$ denotes a transport matrix of A. For $1 \leq p \leq \infty$, $L^p(\mathbb{R}_+)$ denotes a standard Lebesgue space over $\mathbb{R}_+$ equipped with a norm $\|\cdot\|_{L^p}$. For a non-negative integer s, $H^s(\mathbb{R}_+)$ denotes an s-th order Sobolev space over $\mathbb{R}_+$ in the L^2 sense with a norm $\|\cdot\|_{H^s}$. Notice that $H^0(\mathbb{R}_+) = L^2(\mathbb{R}_+)$ and $\|\cdot\|_{H^0} = \|\cdot\|_{L^2}$. For a function $f = f(u)$, $D_u f(u)$ denotes a Fréchet derivative of f with respect to u. Especially, in the case of $u = {}^{\top}(u_1, \ldots, u_n) \in \mathbb{R}^n$ and $f(u) = {}^{\top}(f_1, \ldots, f_m)(u) \in \mathbb{R}^m$, the Fréchet derivative $D_u f = (\frac{\partial f_i}{\partial u_j})_{ij}$ is an $m \times n$ matrix. For a function $f = f(v, w)$, we sometimes abbreviate partial Fréchet derivatives $D_v f(v, w)$ and $D_w f(v, w)$ to $f_v(v, w)$ and $f_w(v, w)$, respectively. A notation $\#^-(A)$ denotes the number of negative eigenvalues of a matrix A.

2 Existence of stationary solution

The stationary wave $\tilde{U}(x)$ is defined as a smooth stationary solution to (1) which converges to a constant state $U_+ = U(u_+)$ as $x \to \infty$. Thus $\tilde{U}$ satisfies a system of ordinary differential equations equations

$$f(\tilde{U})_x = (G(\tilde{U})\tilde{U}_x)_x, \quad x \in \mathbb{R}_+. \tag{7}$$

Let $\tilde{u} = {}^{\top}(\tilde{v}, \tilde{w})$ be a stationary solution for (3). By using a diffeomorphism $U \mapsto u$, we have a relation $\tilde{u} = u(\tilde{U})$ and $\tilde{U} = U(\tilde{u})$. We assume that $\tilde{u}$ satisfies the same boundary and spatial asymptotic conditions in (6) and (5). Namely

$$\tilde{w}(0) = w_{\mathrm{b}}, \tag{8a}$$

$$\lim_{x\to\infty} \tilde{u}(x) = u_+, \quad \text{i.e.,} \quad \lim_{x\to\infty} (\tilde{v}, \tilde{w})(x) = (v_+, w_+). \tag{8b}$$

The existence of the stationary solution for the boundary value problem (7) and (8) is summarized in the following theorem of which detailed proof is stated in the paper [8]. We note that the non-degenerate stationary solution exists if the number of negative characteristics is greater than the number of hyperbolic equations (3a). The existence of the degenerate stationary solution is showed under the assumption that the matrix $D_U f(U_+)$ has a simple zero-eigenvalue.

Theorem 1. *Assume that* **[A2]** *holds and let* $\delta := |w_+ - w_{\mathrm{b}}|$.

(i) (Non-degenerate flow) *We assume that*

$$\#^-(D_U f(U_+)) > m_1 \tag{9}$$

holds. Then there exists a local stable manifold $\mathcal{M}^{\mathrm{s}} \subset \mathbb{R}^{m_2}$ *around the equilibrium* w_+ *such that if* $w_{\mathrm{b}} \in \mathcal{M}^{\mathrm{s}}$ *and* δ *is sufficiently small, then there exists a unique smooth solution* $\tilde{u}(x)$ *to* (7) *and* (8) *satisfying an exponential decay estimate*

$$|\partial_x^k(\tilde{u}(x) - u_+)| \leq C\delta e^{-cx} \quad for \quad k = 0, 1, \ldots.$$

(ii) (Degenerate flow) *We assume that* $D_U f(U_+)$ *has a simple zero-eigenvalue* $\mu(U_+) = 0$. *Moreover we assume that the characteristic field corresponding to* $\mu(U_+) = 0$ *is genuinely nonlinear, that is,*

$$D_U \mu(U_+) R(U_+) \neq 0,$$

where $\mu(U)$ *is an eigenvalue of the matrix* $D_U f(U)$ *satisfying* $\mu(U_+) = 0$ *and* $R(U)$ *be a right eigenvector of* $D_U f(U)$ *corresponding to* $\mu(U)$. *Then there exists a certain region* $\mathcal{M} \subset \mathbb{R}^{m_2}$ *such that if* $w_{\mathrm{b}} \in \mathcal{M}$ *and* δ *is sufficiently small, then there exists a unique smooth solution* $\tilde{u}(x)$ *satisfying an algebraic decay estimate*

$$|\partial_x^k(\tilde{u}(x) - u_+)| \leq C\frac{\delta^{k+1}}{(1+\delta x)^{k+1}} + C\delta e^{-cx} \quad for \quad k = 0, 1, \ldots.$$

The asymptotic stability of the stationary solution thus constructed in the above theorem are studied in section 3. The convergence rate is also studied under a stability condition In section 4, we derive the convergence rate without stability condition. In the present summary, We study the convergence rate only for the non-degenerate flow for simplicity. For the degenerate flow, readers are referred to [8] and [10].

3 Asymptotic stability and convergence rate of stationary solution with stability condition

We study the asymptotic stability of the stationary solution, of which existence is shown in Theorem 1, under a condition **[K]** guaranteeing a dissipative structure of the system. This kind of dissipative structure is firstly studied by Kawashima in [1] under a condition

[K] There exists an $m \times m$ real matrix K such that $KA^0(u_+)$ is skew-symmetric and $[KA(u_+)] + B(u_+)$ is symmetric and positive definite, where $[A] := (A + {}^{\top}\!A)/2$ is a symmetric part of a matrix A.

Shizuta and Kawashima in [12] prove the equivalence of the condition **[K]** and

[SK] Let $\lambda A^0(u_+)\phi = A(u_+)\phi$ and $B(u_+)\phi = 0$ for $\lambda \in \mathbb{R}$ and $\phi \in \mathbb{R}^m$. Then $\phi = 0$.

Kawashima proves the asymptotic stability of a constant state in full space under the stability condition **[K]**, or equivalently **[SK]**, in [1, 2, 5, 12, 13]. The main purpose of our researches in [8, 9, 10] is to generalize his ideas and methods to the half space problem for the asymptotic analysis on stationary solutions in half space. Precisely, we prove the asymptotic stability of the non-degenerate and the degenerate stationary solutions. However we only show in the present survey the asymptotic stability of the non-degenerate stationary in Theorem 1-(i) for simplicity. For the asymptotic stability of the degenerate stationary solution, please see [8] and [10].

Theorem 2. *Assume that the same assumptions as in Theorem* 1-(i) *hold. Then there exists a positive constant* ε_0 *such that if*

$$\|u_0 - \tilde{u}\|_{H^2} + \delta \le \varepsilon_0,$$

the initial boundary value problem (3), (4) *and* (6) *has a unique solution* $u(t,x)$ *globally in time satisfying*

$$u - \tilde{u} \in C([0,\infty), H^2(\mathbb{R}_+)).$$

Moreover the solution converges to the stationary solution $\tilde{u}$*:*

$$\lim_{t\to\infty} \|u(t) - \tilde{u}\|_{L^\infty} = 0.$$

The crucial point of proof of Theorem 2 is to obtain a uniform a priori estimate of a perturbation from the stationary solution

$$(\varphi, \psi)(t,x) := (v,w)(t,x) - (\tilde{v}, \tilde{w})(x).$$

We have the equation for (φ, ψ) from (3) as

$$A_1^0(u)\varphi_t + A_{11}(u)\varphi_x + A_{12}(u)\psi_x = h_1, \tag{10a}$$

$$A_2^0(u)\psi_t + A_{21}(u)\varphi_x + A_{22}(u)\psi_x = B_2(u)\psi_{xx} + h_2, \tag{10b}$$

where h_1 and h_2 are remainder terms. The initial and the boundary conditions are prescribed as

$$(\varphi,\psi)(0,x) = (\varphi_0,\psi_0)(x) := (v_0,w_0)(x) - (\tilde{v},\tilde{w})(x), \tag{11}$$
$$\psi(t,0) = 0. \tag{12}$$

To summarize the a priori estimate for a solution (φ,ψ) in Sobolev space H^2, we define an energy norm $N(t)$

$$N(t) := \sup_{0\le\tau\le t} \|(\varphi,\psi)(\tau)\|_{H^2}.$$

Proposition 3. *Let $(\varphi,\psi) \in C([0,T];H^2(\mathbb{R}_+))$ be a solution to* (10)–(12) *for a certain $T>0$. Then there exists a positive constant ε_1 such that if $N(T)+\delta \le \varepsilon_1$, the solution satisfies*

$$\|(\varphi,\psi)(t)\|_{H^2}^2 + \int_0^t \left(\|\varphi_x(\tau)\|_{H^1}^2 + \|\psi_x(\tau)\|_{H^2}^2\right) d\tau \le C\|(\varphi_0,\psi_0)\|_{H^2}^2$$

for $t \in [0,T]$.

The first step in deriving the a-priori estimate is to obtain the basic L^2 estimate with using an energy form $\mathcal{E}$ defined by

$$\mathcal{E} := \eta(U) - \eta(\tilde{U}) - D_U\eta(\tilde{U})(U-\tilde{U}).$$

Note that, if $N(t)$ is sufficiently small, the energy form $\mathcal{E}$ is equivalent to $|(\varphi,\psi)|^2$ because the Hessian matrix $D_U^2\eta$ is positive. Then we derive the estimates for the higher order derivatives. To do this, we combine the energy method in half space discussed in [7] and the dissipative estimate of the hyperbolic part under the stability condition. For detailed proof, see [9].

By assuming a condition

[A3] The matrix $A(u_+)$ is negative definite for a certain $u_+ \in \mathcal{O}_u$,

which is a stronger condition than **[A2]**, we derive the convergence rate towards the stationary solution. The result is summarized in

Theorem 4. *Assume the same assumptions as in Theorem 2 and* **[A3]** *hold.*
(i) (Exponential decay.) *Let $u_0 - \tilde{u} \in H^2(\mathbb{R}_+)$ and $e^{\alpha x/2}(u_0-\tilde{u}) \in L^2(\mathbb{R}_+)$ for a certain positive constant α. Then for a constant $\beta \in (0,\alpha]$ there exists a positive constant ε_0 such that if*

$$\|u_0-\tilde{u}\|_{H^2} + \|e^{\beta x/2}(u_0-\tilde{u})\|_{L^2} + \delta \le \varepsilon_0,$$

then the initial boundary value problem (3), (4) *and* (6) *has a unique solution globally in time as*

$$u - \tilde{u} \in C([0,\infty);H^2(\mathbb{R}_+)).$$

Moreover there exists a certain constant $\nu \in (0, \beta)$ *such that the solution* u *verifies the decay estimate*

$$\|u(t) - \tilde{u}\|_{H^2} + \|e^{\beta x/2}(u(t) - \tilde{u})\|_{L^2} \le C(\|u_0 - \tilde{u}\|_{H^2} + \|e^{\beta x/2}(u_0 - \tilde{u})\|_{L^2})e^{-\nu t/2}$$

for $t > 0$.
(ii) (Algebraic decay.) *We assume* $u_0 - \tilde{u} \in H^2(\mathbb{R}_+)$ *and* $(1+x)^{\alpha/2}(u_0 - \tilde{u}) \in L^2(\mathbb{R}_+)$ *hold for a certain positive constant* α. *Then there exists a positive constant* ε_0 *such that if*

$$\|u_0 - \tilde{u}\|_{H^2} + \|(1+x)^{\alpha/2}(u_0 - \tilde{u})\|_{L^2} + \delta \le \varepsilon_0,$$

then the initial boundary value problem (3), (4) *and* (6) *has a unique solution globally in time satisfying*

$$u - \tilde{u} \in C([0,\infty); H^2(\mathbb{R}_+)).$$

Moreover the solution u *verifies the decay estimate*

$$\|u(t) - \tilde{u}\|_{H^2} \le C(\|u_0 - \tilde{u}\|_{H^2} + \|(1+x)^{\alpha/2}(u_0 - \tilde{u})\|_{L^2})(1+t)^{-\alpha/2}$$

for $t > 0$.

This theorem is proved by the weighted energy method. The detailed proof is given in [10].

4 Asymptotic stability of stationary solution without stability condition

Even though the stability condition **[SK]** does not hold, we can also derive the global existence of solution and its convergence rate towards the stationary solution under the assumption **[A3]**. This result is summarized in

Theorem 5. *Assume the same assumptions as in Theorem 2 and* [**A3**] *except* [**SK**] *hold.*
(i) (Exponential decay.) *We assume* $e^{\alpha x/2}(u_0 - \tilde{u}) \in H^2(\mathbb{R}_+)$ *holds for a certain positive constant* α. *Then, for a certain constant* $\beta \in (0, \alpha]$, *there exists a positive constant* ε_0 *such that if*

$$(\|e^{\beta x/2}(u_0 - \tilde{u})\|_{H^2} + \delta)\beta^{-1} \le \varepsilon_0,$$

then the initial boundary value problem (3), (4) *and* (6) *has a unique solution globally in time as*

$$e^{\beta x/2}(u - \tilde{u}) \in C([0,\infty); H^2(\mathbb{R}_+)).$$

Moreover there exists a certain constant $\nu \in (0, \beta)$ *such that the solution* u *verifies the decay estimate*

$$\|e^{\beta x/2}(u(t) - \tilde{u})\|_{H^2} \le C\, \|e^{\beta x/2}(u_0 - \tilde{u})\|_{H^2}\, e^{-\nu t/2}$$

for $t > 0$.

(ii) (Algebraic decay.) *We assume* $(1+\gamma x)^{\alpha/2}(u_0 - \tilde{u}) \in H^2(\mathbb{R}_+)$ *holds for a certain positive constant* γ *and a certain constant* $\alpha \geq 2$. *Then, for an arbitrary constant* $\theta \in (0, \alpha]$, *there exists a positive constant* ε_0 *such that if*

$$(\|(1+\gamma x)^{\alpha/2}(u_0 - \tilde{u})\|_{H^2} + \delta)\gamma^{-1} + \gamma \leq \varepsilon_0,$$

then the initial boundary value problem (3), (4) *and* (6) *has a unique solution globally in time as*

$$(1+\gamma x)^{\alpha/2}(u - \tilde{u}) \in C([0,\infty); H^2(\mathbb{R}_+)).$$

Moreover the solution verifies the decay estimate

$$\|u(t) - \tilde{u}\|_{H^2} \leq C \,\|(1+\gamma x)^{\alpha/2}(u_0 - \tilde{u})\|_{H^2}\, (1+t)^{-(\alpha-\theta)/2}$$

for $t > 0$.

Please see [10] for the detailed proof.

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