

A Research Report at University of Aveiro while 2009-2014

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1. Introduction

After retirement of Gunma University, I got a special and rare happy chance as a reseacher of the University of Aveiro while June 2009 - May 2014. In order to express my deep thanks to the University and the stuff of the Department of Mathematics, I would like to recall my research activity at the University whoes contents are the general applications of reproducing kernels.

2. Investigated problems

2.1. Aveiro Discretization Method in Mathematics: A New Discretization Principle

We were able to obtain general and global results combining analysis and computers; functional analysis method (theory of reproducing kernels) and discretization, and the results were published in the book under the above titled in

L. P. Castro, H. Fujiwara, M.M. Rodrigues, S. Saitoh and V.K. Tuan,
Aveiro Discretization Method in Mathematics: A New Discretization Principle, MATHEMATICS WITHOUT BOUNDARIES : SURVEYS IN PURE

MATHEMATICS, Edited by Panos Pardalos and Themistocles M. Rassias (2014)(Springer), 37-92.

Further applications and numerical experiments were given in the paper:

L. P. Castro, H. Fujiwara, T. Qian and S. Saitoh, How to catch smoothing properties and analyticity of functions by computers?, MATHEMATICS WITHOUT BOUNDARIES: SURVEYS IN INTERDISIPINARY RESEARCH, Edited by Panos Pardalos and Themistocles M. Rassias (2014) (Springer), 101-116.

L. P. Castro, H. Fujiwara, M.M. Rodrigues, S. Saitoh and V.K. Tuan, Reproducing Kernels and Discretization, Current Trends in Analysis and its Applications/Proceedings of the 9th ISAAC Congress, Krakow 2013, Edited by Vladimir Mityushev and Michael Ruzhansky, Springer (2015), 553-559.

We found a very general discretization method by applying the theory of reproducing kernels and we made numerical experiments. We think our method will become the next generation method solving general analytical problems by using computers. In particular, we will be able to solve very general linear PDEs satisfying very general boundary conditions and initial values - independently the boundary and domains. Furthermore, we will be able to give clearly an ultimate sampling theory and ultimate realizations of general reproducing kernel Hilbert spaces. We developed the general theory in a self contained manner with some related history and many concrete examples.

2.2 Announcement 142: An Aveiro Dream in Mathematics

By combining the very specialized research result of Professor M. M. Rodrigues and the Aveiro discretization method using the fundamental theory of linear mappings, we found the basic relations among linear operators, eigenfunctions, linear initial value problems, integral transforms and reproducing kernels.

Roughly speaking, when we know some eigenfunctions of a linear operator, we can consider the related partial differential equation and we can solve the associated initial value problem; in this method, we shall consider

the reproducing kernel forms and related integral transforms (linear mappings), and we can discuss the existence problem and construction problem of the initial value problem, and furthermore, we can consider the complete property of the solutions by using the theory of reproducing kernels. From this general method, we find that we can consider many and many integral transforms and reproducing kernels in concrete forms from the known eigenfunctions. We know a great tradition on concrete forms in Russia; many definite integrals, many eigenfunctions, many analytical solutions in differential and integral equations. Our theory will give a great impact on these topics.

Such definite and concrete results expected may be looked as *An Aveiro Dream in Mathematics*.

Our output results in the Aveiro Dream in Mathematics may be stated as follows:

- 1) Many concrete reproducing kernels may be calculated and the related reproducing kernel Hilbert spaces should be realized with concrete norms.
- 2) Eigenfunctions and the related initial value problems in partial differential and integral equations should be examined with their properties of the solutions.
- 3) Many new integral transforms and their properties; that is, isometric identities and inversion formulas should be established.
- 4) For the associated t kernels and the related small reproducing kernels appeared in the general theory, we can consider the similar problems above.

From the great references by Russian mathematicians containing the special function theory, we may consider expected new materials as the Aveiro Dream in Mathematics. We believe such materials in mathematics are definite values and fundamentals in mathematics.

The basic references are given by:

L. P. Castro, M. M. Rodorigues and S. Saitoh, Initial value problems in linear integral operators equations, Topics in Mathematical Analysis and Applications, Edited by Laszlo Toth and Themistocles M. Rassias, Springer (2014), 175-188.

L. P. Castro, M. M. Rodorigues and S. Saitoh, A fundamental theorem on initial value problems by using the theory of reproducing kernels, *Complex Anal. Oper. Theory* **9**(2015), 87-98.

M. M. Rodorigues and S. Saitoh, Whittaker differential equations associated to the initial heat problem, *Current Trends in Analysis and its Applications/Proceedings of the 9th ISAAC Congress, Krakow 2013*, Edited by Vladimir Mittyushev and Michael Ruzhansky, Springer (2015), 523-530.

2.3 Explicit representations of implicit functions

As in the Kramer formula in the matrix theory, we derived the explicit representations of simultaneous nonlinear equations and as their applications, we gave the explicit representations of implicit functions that are ensured by the fundamental implicit function theorem in calculus. We use the singular integrals and the Green-Stokes theorem as the method. The materials were published in

L. P. Castro, K. Murata, S. Saitoh and M. Yamada, Explicit integral representations of implicit functions. *Carpathian J. Math.* **29** (2013), no. 2, 141-148.

M. Yamada, S. Saitoh, Explicit and direct representations of the solutions of nonlinear simultaneous equations. *Progress in analysis and its applications*, 372-378, World Sci. Publ., Hackensack, NJ, 2010.

2.4 Introduction of general fractional functions

For arbitrary non-identically zero functions f , we introduced some natural fractional functions f_1 having f as denominators and we considered their representations f_1 by appropriate numerator functions within the reproducing kernel Hilbert spaces framework. That is, in the work we would like to introduce very general fractional functions (e.g., having the possibility of admitting zeros in their denominators) by means of the theory of reproducing kernels and the Tikhonov regularization. The results may be applied to solve the convolution equations, basically, because we meet to solve the product type equations. The materials were published in

L. P. Castro and S. Saitoh, Fractional functions and their representations. *Complex Anal. Oper. Theory* 7 (2013), no. 4, 1049-1063.

2.5 Discrete differential equations

Computers can deal with only a finite number of data and so, for the use of computers to analytical problems, we gave the concept of discrete differential equations and gave approximate solutions for some general linear ordinary, partial differential equations and singular integral equations with variable coefficients. The materials were published in

L. P. Castro and S. Saitoh, Optimal and approximate solutions of singular integral equations by means of reproducing kernels. *Complex Anal. Oper. Theory* 7 (2013), no. 6, 1839-1851.

L. P. Castro, H. Fujiwara, M. M. Rodrigues and S. Saitoh, A new discretization method by means of reproducing kernels. *Interactions between real and complex analysis*, 185-223, Sci. Technics Publ. House, Hanoi, 2012.

L. P. Castro, H. Itou and S. Saitoh, Numerical solutions of linear singular integral equations by means of Tikhonov regularization and reproducing kernels. *Houston J. Math.* 38(2012), no. 4, 1261-1276.

L. P. Castro, S. Saitoh, Y. Sawano and S. Anabela, Discrete linear differential equations. *Analysis (Munich)* 32 (2012), no. 3, 181-198.

L. P. Castro, S. Saitoh, Y. Sawano and A. M. Simoes, General inhomogeneous discrete linear partial differential equations with constant coefficients on the whole spaces. *Complex Anal. Oper. Theory* 6 (2012), no. 1, 307-324.

L. P. Castro, Q. Chen and S. Saitoh, Source inversion of heat conduction from a finite number of observation data. *Appl. Anal.* 89 (2010), no. 6, 801-813.

L. P. Castro, E. M. Rojas and S. Saitoh, Inversion from different kinds of information and real inversion formulas of the Laplace transform from a finite number of data. *Math. Eng. Sci. Aerosp. MESA* 1, No. 2, 181-190 (2010).

2.6 Convolution inequalities and convolution equations

Various convolution operators may be appeared in analysis containing singular integral equations. By using the theory of reproducing kernels, we derived fundamental estimates and solutions by using the Tikhonov regularization. The results were published in

L. P. Castro, S. Saitoh and T. N. Minh, Convolutions, integral transforms and integral equations by means of the theory of reproducing kernels. *Opuscula Math.* 32 (2012), no. 4, 633-646.

L. P. Castro and S. Saitoh, New convolutions and norm inequalities. *Math. Inequal. Appl.* 15 (2012), no. 3, 707-716.

3. International Conferences:

2009:

S. Saitoh, Explicit and direct representations of the solutions of non-linear simultaneous equations, ISAAC, 2009/7/16 Imperial College London.

2010:

S. Saitoh, Constructions of the approximate solutions of singular integral equations by using the Tikhonov regularization and the theory of reproducing kernels, ICNPAA 2010 World Congress: 8th International Conference on Mathematical Problems in Engineering, Aerospace and Sciences, 2010. July. 1, Sao Jose dos Campos (Brazil).

S. Saitoh, Fundamental error estimates inequalities for the Tikhonov regularization using reproducing kernels, 2010. September. 22, Hajduszoboszlo (Hungary).

2011:

S. Saitoh, Applications of the theory of reproducing kernels to convolutions and integral equations, IWOTA Sevilla 2011, 2011.7.8. Universidad de Sevilla (Spain).

2012:

S. Saitoh, Theory of reproducing kernels and its general applications, ICML 2012 Workshop on RKHS and kernel methods, June 26-July 1, 2012, Edinburgh Univ. Scotland, UK.

S. Saitoh, Approximate solutions of bounded linear operator equations by the Tikhonov regularization using reproducing kernels, ICNPAA 2012, July 10-14, 2012, Vienna Univ., Austria.

S. Saitoh, Inversion of linear systems by a finite number of data, ICNPAA 2012, July 10-14, 2012, Vienna Univ., Austria.

S. Saitoh, Bounded linear operator equations and a new discretization method by using the reproducing kernel theory, The 20th International Conference on finite and infinite dimensional Complex Analysis, July 30-August 3, 2012, Hanoi Univ., Vietnam.

S. Saitoh, Approximate solutions of general linear integral equations by a finite number of data, Conference of Applied Analysis and Mathematical Biology, August 8-9, Delaware Univ. USA.

S. Saitoh, A new discretization principle in analysis, International Conference on Sciences and Applications, 2012. December 26-31, Abu Dhabi Univ. UAE.

2013:

S. Saitoh, Reproducing kernels and discretization, ISAAC 9th Congress, Krakov, 2013, August 5-9. Pedagogical University, Poland.

S. Saitoh, Theory of reproducing kernels and general applications, International Workshop on Learning Theory, September 13-16, 2013, University of Shaoxing, P.R. China.

4. Other Conferences or Seminars:

2009:

L.P. Castro, S. Saitoh, Y. Sawano and A.M. Simoes, Discretization by the theory of reproducing kernels, Recent developments of numerical analysis and numerical computation algorithms, 2009, December 16th. RIMS Research Center, University of Kyoto (Kyoto).

S. Saitoh, Operator equations with continuous parameter inverse problems, The 18th seminar on function spaces 2009, 2009. December 24, Hokkaido, University (Hokkaido).

2010:

S. Saitoh, Analytical and numerical solutions of linear integral equations for bounded operators by using the theory of reproducing kernels, Advanced School on Integral Equations, April 7, IST. (Lisbon).

L. P. Castro, H. Fujiwara, S. Saitoh, Y. Sawano, A. Yamada, and M. Yamada, Fundamental error estimate inequalities for the Tikhonov regularization using reproducing kernels, The 19th seminar on function spaces 2010, 2010, December 24, Hokkaido University (Hokkaido).

S. Saitoh, Heat conduction from a finite number of initial heat data, First Annual Workshop of Functional Analysis and Applications Group, CIDMA 2010. May. 5, University of Aveiro (Portugal).

2011:

S. Saitoh, Applications of reproducing kernels to fractional functions and convolution inequalities, Second Annual Workshop of Functional Analysis and Applications Group, CIDMA 2011. October 29, 2011, University of Aveiro (Portugal).

L. P. Castro and S. Saitoh, Applications of reproducing kernels to fractional functions and convolution inequalities, The 20th seminar on function spaces 2011, 2011. December 24, Hokkaido University (Hokkaido).

2013

S. Saitoh, Representations of the solutions of some general Tikhonov functional equations, The 4th Annual Workshop of Functional Analysis and Applications Group, University of Aveiro, June 8, 2013.

L. P. Castro, H. Fujiwara, M.M. Rodrigues, S. Saitoh and V.K. Tuan, Aveiro Discretization Method in Mathematics: A New Discretization Principle, The 22th Function Spaces Seminar, Tokyo Science University, December 22-24, 2013. Tokyo, Japan.

2014

S. Saitoh, Representations of solutions of general Tikhonov functional equations: the mysterious power of the Tikhonov regularization (joint work

with L. P. Castro and A. Yamada), Conformal Mappings and Value Distribution Theory: - Research Meeting, Housei Univ. January 10-11, 2014, Tokyo, Japan.

5. Organization of Conferences:

2009. July, 13-18: The 7th International ISAAC Congress at the Imperial College London, UK: The scientific committee member and the session: Reproducing kernel and related topics, organizer.

2011. August 22-27: The 8th International Moscow ISAAC Congress at Moscow: The scientific committee member and the session, Integral transforms and reproducing kernels, organizer.

2012. August 8-9: Conference on Applied Analysis & Mathematical Biology, Scientific Committee, University of Delaware, USA.

2013. August 5-9: The 9th International Krakov ISAAC Congress: The scientific committee member and the session, Integral transforms and reproducing kernels, organizer.

6. PhD thesis Jury :

I had also the privilege to read their PhD thesis and take part in the corresponding jury:

Edixon Manuel Rojas: A study of singular integral operators with shift.

Alberto Manuel Tavares Simoes: Problemas do tipo de Sommerfeld com Condições de Fronteira de ordem superior.

Anabela de Sousa e Silva: Regularity of Wiener-Hopf plus Hankel operators.

7. Publications:

MR3119107 L. P.; Haque, M. R.; Murshed, M. M.; Saitoh, S.; Tuan, N. M. Quadratic Fourier transforms. *Ann. Funct. Anal.* 5 (2014), no. 1, 10-23.

MR3061914 Fujiwara, H.; Rodrigues, M. M.; Saitoh, S.; Tuan, V. K. A new discretization principle in analysis. *Int. J. Math. Comput.* 22 (2014), no. 1, 75-88.

MR3137545 Castro, L. P.; Murata, K.; Saitoh, S.; Yamada, M. Explicit integral representations of implicit functions. *Carpathian J. Math.* 29 (2013), no. 2, 141-148.

MR3129896 Castro, L. P.; Saitoh, S. Optimal and approximate solutions of singular integral equations by means of reproducing kernels. *Complex Anal. Oper. Theory* 7 (2013), no. 6, 1839-1851.

MR3079842 Castro, L. P.; Saitoh, S. Fractional functions and their representations. *Complex Anal. Oper. Theory* 7 (2013), no. 4, 1049-1063.

MR3135824 Castro, L. P.; Fujiwara, H.; Rodrigues, M. M.; Saitoh, S. A new discretization method by means of reproducing kernels. *Interactions between real and complex analysis*, 185-223, Sci. Technics Publ. House, Hanoi, 2012.

MR3019034 Castro, L. P.; Itou, H.; Saitoh, S. Numerical solutions of linear singular integral equations by means of Tikhonov regularization and reproducing kernels. *Houston J. Math.* 38(2012), no. 4, 1261-1276.

MR3001773 Castro, Luis P.; Saitoh, Saburo; Tuan, Nguyen Minh Convolutions, integral transforms and integral equations by means of the theory of reproducing kernels. *Opuscula Math.*32 (2012), no. 4, 633-646.

MR2959029 Castro, Luis P.; Saitoh, Saburo; Sawano, Yoshihiro; Silva, Anabela S. Discrete linear differential equations. *Analysis (Munich)* 32 (2012), no. 3, 181-198.

MR2962465 Castro, L. P.; Saitoh, S. New convolutions and norm inequalities. *Math. Inequal. Appl.* 15 (2012), no. 3, 707-716.

MR2886621 Castro, L. P.; Saitoh, S.; Sawano, Y.; Simes, A. M. General inhomogeneous discrete linear partial differential equations with constant coefficients on the whole spaces. *Complex Anal. Oper. Theory* 6 (2012), no. 1, 307-324.

MR2876751 Castro, L. P.; Saitoh, S. Natural outputs and global inputs of linear systems with a finite number of input data. *Appl. Anal.* 91 (2012), no. 2, 225-236.

MR2776778 Butzer, P. L.; Ferreira, P. J. S. G.; Higgins, J. R.; Saitoh, S.; Schmeisser, G.; Stens, R. L. Interpolation and sampling: E. T. Whittaker, K. Ogura and their followers. *J. Fourier Anal. Appl.* 17 (2011), no. 2, 320-354.

MR2766944 Yamada, M.; Saitoh, S. Explicit and direct representations of the solutions of nonlinear simultaneous equations. *Progress in analysis and its applications*, 372-378, World Sci. Publ., Hackensack, NJ, 2010.

MR2759461 Saitoh, Saburo Theory of reproducing kernels: applications to approximate solutions of bounded linear operator equations on Hilbert spaces [translation of MR2427178]. *Selected papers on analysis and differential equations*, 107-134, Amer. Math. Soc. Transl. Ser. 2, 230, Amer. Math. Soc., Providence, RI, 2010.

MR2760496 Takahasi, Sin-Ei; Rassias, John M.; Saitoh, Saburo; Takahashi, Yasuji Refined generalizations of the triangle inequality on Banach spaces. *Math. Inequal. Appl.* 13 (2010), no. 4, 733-741.

MR2666541 Castro, L. P.; Chen, Q.; Saitoh, S. Source inversion of heat conduction from a finite number of observation data. *Appl. Anal.* 89 (2010), no. 6, 801-813.

MR2662019 Sawano, Yoshihiro; Yamada, Masato; Saitoh, Saburo Singular integral inequalities and natural regularizations. *Math. Inequal. Appl.* 13 (2010), no. 2, 289-303.

MR2581660 Yamada, M.; Saitoh, S. Practical inversion formulas for linear physical systems. *Further progress in analysis*, 584-589, World Sci. Publ., Hackensack, NJ, 2009.

MR2581659 Fujiwara, H.; Matsuura, T.; Saitoh, S.; Sawano, Y. Numerical real inversion of the Laplace transform by using a high-accuracy numerical method. *Further progress in analysis*, 574-583, World Sci. Publ., Hackensack, NJ, 2009.

MR2581963 Uchida, Keitaroh; Kumahara, Keisaku; Saitoh, Saburo Normal solutions of linear ordinary differential equations of the second order. *Int. J. Appl. Math.* 22 (2009), no. 6, 981-996.

MR2536026 Yamada, Masato; Saitoh, Saburo Numerical solutions of two non-linear simultaneous equations. *Appl. Anal.* 88 (2009), no. 2, 151-160.

8. Others:

In Aveiro, I was able to have a very happy birthday of 70th and I had very honorable words:

Libertas Mathematica (new series) Volume 34(2014), No. 2, 1-3

To Professor Saburou Saitoh on the occasion of his 70th birthday:

Professor Saburou Saitoh celebrated his 70th birthday at the Center for Research and Development in Mathematics and Applications (CIDMA), hosted by the University of Aveiro, Portugal, where for the last five years, as a Researcher within CIDMA (supported by the Portuguese Foundation for Science and Technology - FCT), he had a significant far beyond mathematics. Professor Saburou Saitoh was born at Tochigi Prefecture, Japan on March 4th, 1944. He completed the undergraduate courses at Gunma University and the postgraduate courses of Master and Ph.D. at Tokyo Institute of Technology. He got academic positions at Shibaura Institute of Technology (1971-1976) and Gunma University (1976-2009). He was appointed as Emeritus Professor of Gunma University in 2009. After that, he got a five years Researcher Position at CIDMA, University of Aveiro (2009-2014). The Ph.D. thesis of Professor Saitoh had the title *The Bergman norm and the Szegő norm*, and these topics held a substantial influence on his future research. At that time, he was already exchanging ideas with colleagues from all over the world. Namely, he visited the United States of America for research in the University of California, Stanford University, University of Pittsburgh and University of Delaware, in the period 1986-1987, supported by the Japanese Government. He has been participating in the ISAAC congresses since the very first congress at the University of Delaware, in 1997, and all this time he has been organizing sessions related with reproducing kernels. Associated with this, he published two volumes of the Proceedings from Kluwer Academic Publishers with the related leading mathematicians. Moreover, he was the Vice-President of ISAAC for six years. Professor Saitoh is a very special mathematician that allows his own research to be driven by his great personality. His concern about integrating mathematics in the spirit and motivations of life and the human being is an example for the younger ones. On the top of his present concerns, we can find the search for the purpose of our life and the interpretation of what mathematics is. This is performed by Professor Saitoh in a rather wide spectrum, where the relation between mathematics and the global laws of the universe are constantly on his mind. Within this scope, the general emails sent by Professor Saitoh are well-known to some of us, and not so well understood by a few others. The point is that Professor Saitoh is always trying to think above the human nature, and this leads to the case that more often than not, when he is writing about one specific topic, he is in fact already considering a somehow future

possibility, of a more global nature, than that of the original special problem. Not rarely, Professor Saitoh is even trying to implement or generalize very general rules, even in the mathematics field, such as the Pythagoras theorem. His research group is not so large and its main theme is concentrated in some restricted nature on the theory of reproducing kernels. This, however, does not make it impossible for Professor Saitoh to develop applications in several different fields of mathematics by using the theory of reproducing kernels. Indeed, his dedication to reproducing kernels is without doubt very deep. Professor Saitoh's main rule about research on mathematics is that it should be fundamental, beautiful, and produce a significant impact on human beings. This spirit had led Professor Saitoh to several fundamental results on the theory of linear transforms, Pythagorean theorems, several very general norm inequalities, representations of non-linear simultaneous equations and implicit functions, different types of applications of the Tikhonov regularization (including a typical main result on a numerical and real inversion formula of the Laplace transform, with the coauthors Professors Hiroshi Fujiwara and Tutomu Matsuu). The last five years that Professor Saitoh spent at University of Aveiro were very fruitful in his research: he had the opportunity to introduce his great ideas and methods on reproduction kernels to the research group in Aveiro and he also kept himself open to the research interests of the Aveiro group members. This turned out to generate some relevant development in the areas of Integral Equations, Differential Equations and Operator Theory. Moreover, during this period, he had the chance to realize one of his plans outside the research on mathematics: Together with his son, he published an essay book on the universal problems beyond mathematics. On mathematics, besides other subjects, he published the so-called *Aveiro discretization method in mathematics*, with the colleagues L.P. Castro, H. Fujiwara, M.M. Rodrigues and Vu Kim Tuan. This is basically a very general discretization method, by applying the theory of reproducing kernels, which allows significant numerical experiments. In particular, with this method, it is possible to solve very general linear PDEs satisfying global boundary conditions and initial values (somehow independently of the type of boundary and domains). Furthermore, Professor Saitoh was able to clearly give an ultimate sampling theory and realizations of general reproducing kernel Hilbert spaces. In Professor Saitoh's papers of the past five years one can find this general theory in a self-contained manner, with some related history and many concrete examples. As an example, we can point out a developed method which, roughly speaking, when To Professor

Saburo Saitoh we know some eigenfunctions of a linear operator, we can consider the related partial differential equation and solve an associated initial value problem. In this method, we shall consider the reproducing kernel forms and related integral transforms (linear mappings), it being therefore possible to discuss the corresponding existence and construction problems of the initial value problem. Furthermore, it is possible to consider the complete properties of the corresponding solutions by using the theory of reproducing kernels. From this general method, we are capable of analysing in detail many integral transforms and reproducing kernels in concrete forms from the known eigenfunctions. Professor Saitoh published, with his collaborators, over 150 papers and 7 books (indexed in the new zbMATH interface). Moreover, together with Professor Yoshihiro Sawano, he is planning to publish in Springer a fundamental monograph entitled *Theory of Reproducing Kernels and Applications*. As a tribute to his involvement in the life and activities of CIDMA, for his research activities, and also for his attitude concerning the Romanian school of mathematics and the journal *Libertas Mathematica*, it is our honour to dedicate to Professor Saitoh this issue of the journal.

Vasile Staicu

Editor in Chief of LM(n.s.) ;

Luis F. P. Castro

Director of CIDMA

A Tribute to the 70th Birthday of Prof Saburo Saitoh,

by Tsutomu Matsuura,

Current Trends in Analysis and its Applications/Proceedings of the 9th ISAAC Congress, Krakow 2013, Edited by Vladimir Mityushev and Michael Ruzhansky, Springer (2015), 3-4.

The essay for mathematics, human beings, and social problems was published in Japanese: No.81, May 2012(pdf 432kb) (www.jams.or.jp/kaiho/kaiho-81.pdf).

The essay for mathematics, human beings, and social problems was published in a book: S. Saito and Y. Saito, *Yoakemae - Yocchan no Omoi* (Predawn - Thoughts of Yocchan) (in Japanese). Bungeisya, Tokyo (2010).

While all summer vacations in Aveiro, I concentrated to write the book:

Saburo Saitoh and Yoshihiro Sawano: Theory of Reproducing Kernels and Applications, Springer (2016).

This book provides a large extension of the general theory of reproducing kernels published by N. Aronszajn on 1950, with many concrete applications:

Presents a unified theory of reproducing kernels which is fundamental, beautiful and widely applicable in mathematics.

Deals with the new discretizations and the Tikhonov regularization for practical constructions of the solutions by computers, in analysis.

Introduces global up-to-date, topics of general interest from the general theory of N. Aronszajn.

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