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Investment strategies, random shock and asymmetric information*

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1 Introduction

In this paper, we consider a firm's investment problem in the presence of asymmetric information and possibility of random shock. We examine both the optimal timing (trigger) and quantity strategies for the investment.

This paper is based on many previous studies related to the investment decision problem. The standard framework by McDonald and Siegel (1986) examines the optimal timing of investment when the investment cost is fully irreversible. Following McDonald and Siegel (1986), there are many extended models from different angles. The first extension is to incorporate the reversibility of investment. See Abel and Eberly (1999) and Wong (2010, 2011). The reversibility of investment means that the firm could sell the capital after the investment when the profitability of capital becomes low. Thus, a reversible investment implies that the firm owns an abandonment option. The main result of Wong (2011) is that higher reversibility accelerates the investment but not necessarily increases the investment quantity.

The second extension is to incorporate the asymmetric information. As we know, in most modern firms, firm owners would like to delegate the management to managers, taking advantage of managers professional skills. In this situation, it is possible to exist asymmetric information between owners and managers. For example, managers have private information that owners cannot observe. Grenadier and Wang (2005), Shibata (2009) and Cui and Shibata (2016a, 2016b) provide frameworks on examining the investment strategies under asymmetric information. The important results are that under asymmetric information, the investment timing is more delayed and the quantity is more increased than under full information.

The third extension is to incorporate the possibility of random shock. The arrival of random shock can be regarded as the occurrence of some exogenous event that affects the profit flows generated by the capital. For example, the technology improvement may increase the revenue or decrease the operational cost. Alvarez and Stenbacka (2001) present an example in which the firm faces a cost saving technology improvement at an exponentially distributed arrival time. Cui and Shibata (2015) consider an investment problem with random shock where the random shock is associated with a fixed level of revenue. One important result of Cui and Shibata (2015) is that the investment quantity is decreasing with the arrival probability of random shock.

Thus, in this study, we combine the three features: the reversibility of investment, the asymmetric information and the possibility of random shock. Here, we obtain three important

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results. First, higher reversibility accelerates the investment, but not necessarily increases the investment quantity. The investment quantity exhibits a U-shape with the degree of reversibility. Second, higher arrival probability of random shock accelerates the investment, but decreases the investment quantity. The result of investment quantity is contrary to our intuition. From our intuition, because the arrival of random shock saves the operational cost of per unit quantity, we conjecture that the firm should increase the quantity. However, we find that the investment quantity is decreasing with the arrival probability of random shock. Third, even for a reversible investment and with the possibility of random shock, the quantity is more increased and the investment timing is more delayed under asymmetric information than under full information.

The remainder of the paper is organized as follows. Section 2 describes the model setup and derives the firm value after investment. Section 3 formulates the investment problem under asymmetric information. As a benchmark, we also provide the solution to the problem under full information. Section 4 solves for the optimal strategies under asymmetric information and discusses the properties. Section 5 concludes.

2 The model

In this section, we first describe the model setup. We then derive the firm value after investment.

2.1 Setup

Consider a firm that is endowed with an option to invest in a project. To commence the project, the firm simultaneously chooses the quantity and the timing of investment. We assume that the firm owner delegates the investment decision to a manager. Throughout our analysis, we assume that both the owner and the manager are risk neutral and aim to maximize their expected pay-offs.

The investment quantity, $q > 1$, affects the random profit flows $\{q(X_t - f) : t \geq 0\}$ generated from the project, where $\{X_t : t \geq 0\}$ denotes the random revenue flows and $f > 0$ denotes the operational cost per unit time, both of which are of per unit quantity. The stochastic process, $\{X_t : t \geq 0\}$ is governed by the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}z_t, \quad X_0 = x > 0, \tag{2.1}$$

where $\mu > 0$, $\sigma > 0$ are constant parameters, and z_t is a standard Brownian motion. For convergence, we assume that $r > \mu$ where $r > 0$ is a constant interest rate. In addition, we assume that the initial value x is too small to make an immediate investment optimal.

The cost to undertake the investment is $I(q; F) := C(q) + F$. $C(q)$ denotes the cost of investment quantity with $C'(q) > 0$ and $C''(q) > 0$ for all $q > 1$. At the time of investment, $q > 1$ is endogenously chosen to maximize the owner's value. In addition, we assume the fixed set-up cost $F \in \{F_1, F_2\}$ with $F_2 > F_1 > 0$. We denote $\Delta F = F_2 - F_1 > 0$. One could interpret F_1 as "lower-fixed cost" and F_2 as "higher-fixed cost". The probabilities of drawing $F = F_i$ ($i \in \{1, 2\}$) are exogenously given, i.e., $P(F_i) = p_i \in (0, 1)$ with $\sum_{i \in \{1,2\}} p_i = 1$.

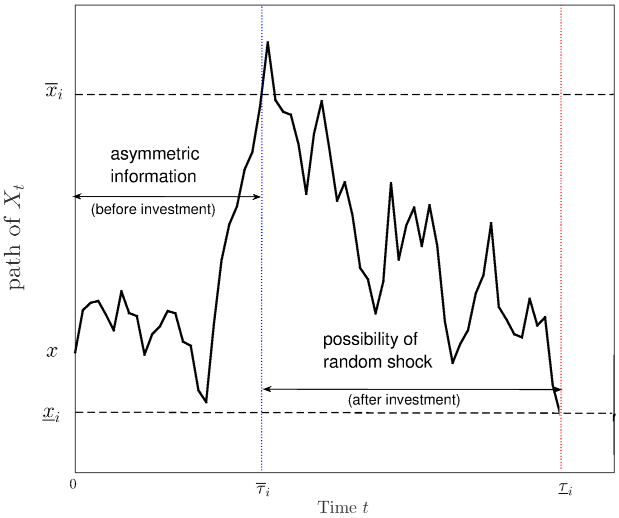

Figure 1: Scenario of model

We assume that the project's profit flows $\{q(X_t - f) : t \geq 0\}$ are observed by both the owner and the manager. However, the fixed set-up cost F is observed privately only by the manager.† Immediate after making a contract with the owner, the manager observes whether F is equal to F_1 or F_2, but the owner cannot observe the true value of F. In this situation, the manager could divert a value of $\triangle F$ to himself by reporting F_2 when he truly observes F_1. The owner suffers losses from the manager's diversion. Thus, to prevent the losses, the owner must encourage the manager to tell truth at the time of investment by providing incentives.

After the investment, if the project's profit becomes unfavorable, the firm could abandon the project. The abandonment decision, once made, is irreversible. We assume that the salvage at the time of abandonment is $sI(q;F)$, where $s \in [0,1]$ denotes the recovery rate of the initial investment cost. Thus, a higher value of s implies a higher reversibility of investment. If $s = 0$ or $s = 1$, the investment is called fully irreversible or fully reversible.

Before the abandonment and after the investment, there exists a possibility of random shock that influences the project's profit. Let τ^R denote the arrival timing of the random shock. Here, we assume that once the random shock occurs, the operational cost of per unit quantity, f decreases to 0. That is, the random profit flows after the random shock becomes $\{qX_t; t \geq \tau^R\}$. For simplicity, we model the arrival of random shock as a Poisson process with intensity λ. That is, over a small time interval Δt, the random shock occurs with a probability $\lambda \Delta t$.

We use Figure 1 to explain the scenario of the model. Let $q_i = q(F_i)$ denote the investment quantity, $\overline{x}_i = \overline{x}(F_i)$ and $\underline{x}_i = \underline{x}(F_i)$ individually denote the investment trigger and abandonment trigger for $F = F_i$ $(i \in \{1,2\})$. In addition, let $\overline{\tau}_i = \inf\{t \geq 0; X_t = \overline{x}_i\}$ and $\underline{\tau}_i = \inf\{t \geq \overline{\tau}_i; X_t = \underline{x}_i\}$ individually denote the (random) first passage time when X_t reaches $\overline{x}_i$ from below

†In the asymmetric information structure, it is quite common to assume that a portion of investment value is privately observed by one party (here, the manager) and not observed by the other party (here, the firm owner). Laffont and Martimort (2002) give an excellent overview of situations with asymmetric information.

and reaches $\underline{x}_i$ from above. The model scenario is, when X_t, starting off x, increases and arrives at $\overline{x}_i$, the firm undertakes the investment and decides q_i endogenously. Before investment, there is asymmetric information between the owner and the manager on the value of F. After investment, if X_t, starting off $\overline{x}_i$, decreases and arrives at $\underline{x}_i$, the firm exercises the abandonment. There exists a possibility of random shock that reduces the operational cost f to 0. We could recognize that a smaller $\overline{x}_i$ implies an earlier (later) investment, and a smaller $\underline{x}_i$ implies a later (earlier) abandonment.

2.2 Value after investment

In this subsection, we derive the owner's value after investment, while the abandonment option is kept alive.

Given q_i, $\overline{x}_i$ s and λ, the firm's value at time $\overline{\tau}_i$ is formulated as

$$V(q_i,\overline{x}_i;s,\lambda) = \sup_{\underline{\tau}_i} \mathbb{E}^{\overline{x}_i}\left[\int_{\overline{\tau}_i}^{\underline{\tau}_i} \mathrm{e}^{-r(t-\overline{\tau}_i)} q_i X_t \mathrm{d}t - \int_{\overline{\tau}_i}^{\tau^R\wedge\underline{\tau}_i} \mathrm{e}^{-r(t-\overline{\tau}_i)} q_i f \mathrm{d}t + \mathrm{e}^{-r(\underline{\tau}_i-\overline{\tau}_i)} sI(q_i;F_i)\right], \tag{2.2}$$

where $\mathbb{E}^{\overline{x}_i}[\cdot]$ denotes the expectation operator conditional on $\overline{x}_i$. The first term on the right-hand side of (2.2) is the present value of the revenue flows $\{qX_t : t \in [\overline{\tau}_i, \underline{\tau}_i]\}$. The second term is the present value of the operational cost f, which is stopped either due to the arrival of random shock at time τ^R, or due to the abandonment at time $\underline{\tau}_i$. The third term is the present value of salvage $sI(q_i;F_i)$ upon the abandonment.

Using the arguments in Dixit and Pindyck (1994), we write (2.2) as

$$V(q_i,\overline{x}_i;s,\lambda) = v_1 q_i \overline{x}_i - v_2 q_i f + (sI(q_i;F_i) + v_2 q_i f - v_1 q_i \underline{x}(q_i;s,\lambda))\left(\frac{\overline{x}_i}{\underline{x}(q_i;s,\lambda)}\right)^{\gamma}, \tag{2.3}$$

where $v_1 = 1/(r-\mu)$, $v_2 = 1/(r+\lambda)$ and $\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(1/2-\mu/\sigma^2)^2 + 2r/\sigma^2} < 0$. The third term on the right-hand side of (2.3) captures the value of the abandonment option, defined by $AO(q_i,\overline{x}_i;s,\lambda)$. Additionally, $\underline{x}(q_i;s,\lambda)$ is the optimal abandonment trigger given by

$$\underline{x}(q_i;s,\lambda) = \frac{\gamma}{\gamma-1}\frac{sI(q_i;F_i) + v_2 q_i f}{v_1 q_i}, \tag{2.4}$$

for any fixed q_i, s and λ. (2.4) implies an important property. That is, holding $I(q_i;F_i)$ fixed, an increase in λ (a decrease in v_2) decreases the value of $\underline{x}(q_i;s,\lambda)$.

Substituting (2.4) into (2.3), we obtain the value of the abandonment option $AO(q_i,\overline{x}_i;s,\lambda)$ as:

$$AO(q_i,\overline{x}_i;s,\lambda) = \left(\frac{v_1 q_i \overline{x}_i}{-\gamma}\right)^{\gamma}\left(\frac{1-\gamma}{sI(q_i;F_i) + v_2 q_i f}\right)^{\gamma-1} > 0. \tag{2.5}$$

(2.5) implies that holding $I(q_i;F_i)$ fixed, an increase in s or a decrease in λ (an increase in v_2) increases the value of $AO(q_i,\overline{x}_i;s,\lambda)$.

3 Investment problem

In this section, we formulate the investment problem under asymmetric information. As a benchmark, we also provide the solution to the investment problem under full information.

3.1 Asymmetric information model

In this subsection, we formulate the investment problem when the manager has private information on F.

As explained earlier, under asymmetric information, the owner must induce the manager to reveal the private information truthfully by providing incentives. Otherwise, the manager diverts value for his own interest by misreporting the value of F. In this study, we assume that the owner signs a contract with the manager at time zero. The contract commits the owner to give the manager a bonus incentive at the time of investment, to induce the manager to tell truth. There is no renegotiation after the contract is signed. Here, we describe the bonus incentive as $w_i = w(F_i)$. We make no distinguish between the manager's reported $\widetilde{F}_i$ and true F_i because at the equilibrium, the manager reports the true F_i as private information. Thus, the contract under asymmetric information is described as

$$S^{**} = (q_i, \overline{x}_i, \underline{x}_i, w_i), \quad i \in \{1,2\},$$

where the superscript "**" refers to the *asymmetric information* problem.

Then, the investment problem under asymmetric information is to maximize the owner's option value through the choice of S^{**}, i.e.,

$$\max_{q_1,q_2,\overline{x}_1,\overline{x}_2,w_1,w_2} \sum_{i\in\{1,2\}} p_i \left\{V(q_i,\overline{x}_i;s,\lambda) - I(q_i;F_i) - w_i\right\} \left(\frac{x}{\overline{x}_i}\right)^{\beta}, \tag{3.1}$$

subject to

$$w_1 \left(\frac{x}{\overline{x}_1}\right)^{\beta} \geq (w_2 + \triangle F) \left(\frac{x}{\overline{x}_2}\right)^{\beta}, \tag{3.2}$$

$$w_2 \left(\frac{x}{\overline{x}_2}\right)^{\beta} \geq (w_1 - \triangle F) \left(\frac{x}{\overline{x}_1}\right)^{\beta}, \tag{3.3}$$

$$w_i \geq 0, i \in \{1,2\}, \tag{3.4}$$

where $\beta = 1/2 - \mu/\sigma^2 + \sqrt{(1/2-\mu/\sigma^2)^2 + 2r/\sigma^2} > 1$.

Here, the objective function (3.1) is the *ex ante* owner's option value. The problem includes four previous models: Grenadier and Wang (2005), Wong (2011), and Cui and Shibata (2016a, 2016b). First, when $s = 0$, $\lambda \to +\infty$ and $q_i = 1$, the problem is the same as that in Grenadier and Wang (2005). Second, if $p_1 = 1$ and $\lambda = 0$, the problem becomes that in Wong (2011). Third, when $s = 0$ and $\lambda \to +\infty$, the problem corresponds to Cui and Shibata (2016a). Forth, when $\lambda \to +\infty$, the problem becomes that in Cui and Shibata (2016b).

We explain the four constraints (3.2) - (3.4) as follows. (3.2) and (3.3) are the *ex post* incentive-compatibility constraints for the manager who observes F_1 and F_2, respectively. Taking (3.2) as an example, the manager's value is $w_1 (x/\overline{x}_1)^{\beta}$ if he observes F_1 and tells the truth, while the manager's value is $(w_2 + \triangle F) (x/\overline{x}_2)^{\beta}$ if he observes F_1 but reports F_2. Then, if (3.2) is satisfied, the manager who observes F_1 has no incentive to tell lie. Similarly, (3.3) is imposed for the manager who observes F_2. (3.4) are the *ex post* limited-liability constraints. They are imposed to ensure that the manager could accept the contract.

3.2 Full information benchmark

In this subsection, we consider the investment problem when the owner observes the true value of F.

Under full information, there is no delegation of the investment because the manager has no informational advantage. Thus, the contract under full information is described as

$$S^* = (q_i, \overline{x}_i, \underline{x}_i), \quad i \in \{1,2\},$$

where the superscript "$*$" refers to the *full (symmetric) information* problem.

The owner's maximization problem under full information is defined as

$$\max_{q_1,q_2,\overline{x}_1,\overline{x}_2} \sum_{i\in\{1,2\}} p_1 H(x, q_1, \overline{x}_1; F_1, s, \lambda) + p_2 H(x, q_2, \overline{x}_2; F_2, s, \lambda), \tag{3.5}$$

where $x < \overline{x}_i$ for any i ($i \in \{1,2\}$) and

$$H(x, q_i, \overline{x}_i; F_i, s, \lambda) = (V(q_i, \overline{x}_i; s, \lambda) - I(q_i; F_i)) \left(\frac{x}{\overline{x}_i}\right)^{\beta}. \tag{3.6}$$

Then, we have the following result.

Proposition 1 *Suppose the investment problem under full information. For any i ($i \in \{1,2\}$), q_i^* and $\overline{x}_i^*$ are the solutions to the following system of equations:*

$$C'(q_i^*) = \frac{\beta}{\beta-1}\frac{1}{q_i^*}\left[I(q_i^*; F_i) + \frac{1}{\beta}\frac{1-\eta(q_i^*, \overline{x}_i^*; s, \lambda)}{1-s\eta(q_i^*, \overline{x}_i^*; s, \lambda)} v_2 q_i^* f\right], \tag{3.7}$$

and

$$\overline{x}_i^* = \frac{\beta}{\beta-1}\frac{1}{v_1 q_i^*}\left[I(q_i^*; F_i) + v_2 q_i^* f - \frac{\beta-\gamma}{\beta} AO(q_i^*, \overline{x}_i^*; s, \lambda)\right], \tag{3.8}$$

where $\eta(q_i^*, \overline{x}_i^*; s, \lambda) = (1-\gamma)(sI(q_i^*; F_i) + v_2 q_i^* f)^{-1} AO(q_i^*, \overline{x}_i^*; s, \lambda)$. In addition, by (2.4), we have $\underline{x}_i^* = \underline{x}(q_i^*; s, \lambda)$.

When $\lambda \to +\infty$ ($v_2 \to 0$), q_i^* becomes independent of s, the solutions become the same as in Wong (2010).

4 Model solution

In this section, we provide the solution to the asymmetric information problem. We then discuss the solution properties.

4.1 Optimal contract

Although the optimization problem under asymmetric information is subject to four inequality constraints (3.2) - (3.4), we could simplify the problem through two steps. First, (3.3) is satisfied automatically because the manager who observes F_2 has no incentive to tell lie. The manager who observes F_2 suffers a loss of ΔF if he reports F_1. Second, (3.2) is binding. This is because if holding (3.2) as a strict inequality, we can increase the owner's value by decreasing w_1. Thus, we obtain that at the optimum, w_i $(i \in \{1,2\})$ satisfy

$$w_2 = 0, \quad w_1 = \left(\frac{\overline{x}_1}{\overline{x}_2}\right)^{\beta} \Delta F. \tag{4.1}$$

As a result, the owner's maximization problem under asymmetric information is simplified as follows:[‡]

$$\max_{q_1,q_2,\overline{x}_1,\overline{x}_2} \sum_{i\in\{1,2\}} p_1 H(x, q_1, \overline{x}_1; F_1, s, \lambda) + p_2 H(x, q_2, \overline{x}_2; F_2 + \phi\Delta F, s, \lambda), \tag{4.2}$$

where $\phi = p_1/p_2 > 0$ and $x < \overline{x}_i$ for any i $(i \in \{1,2\})$. Then we have the following result.

Proposition 2 *Suppose the investment problem under asymmetric information.*
(1). For $i = 1$, the solutions are $q_1^{**} = q_1^*$, $\overline{x}_1^{**} = \overline{x}_1^*$, $\underline{x}_1^{**} = \underline{x}_1^*$, $w_1^{**} = (\overline{x}_1^*/\overline{x}_2^{**})^{\beta}\,\Delta F$.
(2). For $i = 2$, q_2^{**} and $\overline{x}_2^{**}$ are the solutions to the following system of equations:

$$C'(q_2^{**}) = \frac{\beta}{\beta-1}\frac{1}{q_2^{**}}\left[I(q_2^{**}; F_2) + \frac{\phi\Delta F}{(1 - s\eta(q_2^{**}, \overline{x}_2^{**}; s, \lambda))} + \frac{1}{\beta}\frac{1-\eta(q_2^{**}, \overline{x}_2^{**}; s, \lambda)}{1 - s\eta(q_2^{**}, \overline{x}_2^{**}; s, \lambda)} v_2 q_2^{**} f\right], \tag{4.3}$$

and

$$\overline{x}_2^{**} = \frac{\beta}{\beta-1}\frac{1}{v_1 q_2^{**}}\left[I(q_2^{**}; F_2 + \phi\Delta F) + v_2 q_2^{**} f - \frac{\beta-\gamma}{\beta} AO(q_2^{**}, \overline{x}_2^{**}; s, \lambda)\right], \tag{4.4}$$

where $\eta(q_2^{**}, \overline{x}_2^{**}; s, \lambda) = (1-\gamma)(sI(q_2^{**}; F_2) + v_2 q_2^{**} f)^{-1} AO(q_2^{**}, \overline{x}_2^{**}; s, \lambda)$. In addition, $\underline{x}_2^{**} = \underline{x}(q_2^{**}; s, \lambda)$.

In Proposition 2, there are two important remarks. First, we have $q_2^{**} \neq q_2^*$, $\overline{x}_2^{**} \neq \overline{x}_2^*$ and $\underline{x}_2^{**} \neq \underline{x}_2^*$ although $q_1^{**} = q_1^*$, $\overline{x}_1^{**} = \overline{x}_1^*$ and $\underline{x}_1^{**} = \underline{x}_1^*$. This implies that it is less costly for the owner to distort $(q_2^{**}, \overline{x}_2^{**}, \underline{x}_2^{**})$ away from $(q_2^*, \overline{x}_2^*, \underline{x}_2^*)$ than to distort $(q_1^{**}, \overline{x}_1^{**}, \underline{x}_1^{**})$ away from $(q_1^*, \overline{x}_1^*, \underline{x}_1^*)$. Second, we have $w_1^{**} > 0$ and $w_2^{**} = 0$. This is because the manager who observes F_1 has an informational rent defined by ΔF that the manager who observes F_2 doesn't have. These results are the same as in Grenadier and Wang (2005), Shibata (2009) and Cui and Shibata (2016a, 2016b).

4.2 Discussion

To see the solution properties, we consider numerical examples. In order to do to, we assume that the cost of investment quantity is[§]

$$C(q_i) = q_i^3. \tag{4.5}$$

Suppose that the basic parameters are $r = 0.05$, $\mu = 0.02$, $\sigma = 0.25$, $F_1 = 5$, $F_2 = 10$, $f = 1$, $s = 0.5$, $\lambda = 0.05$, $p_1 = 0.5$ and $x = 1$.

[‡]For simplification, we use the relation $I(q_2; F_2) + \phi\Delta F = I(q_2; F_2 + \phi\Delta F)$.

[§]We use $C(q_i) = q_i^3$ here just to show the results more obviously. The properties of results also hold with $C(q_i) = q_i^2$.

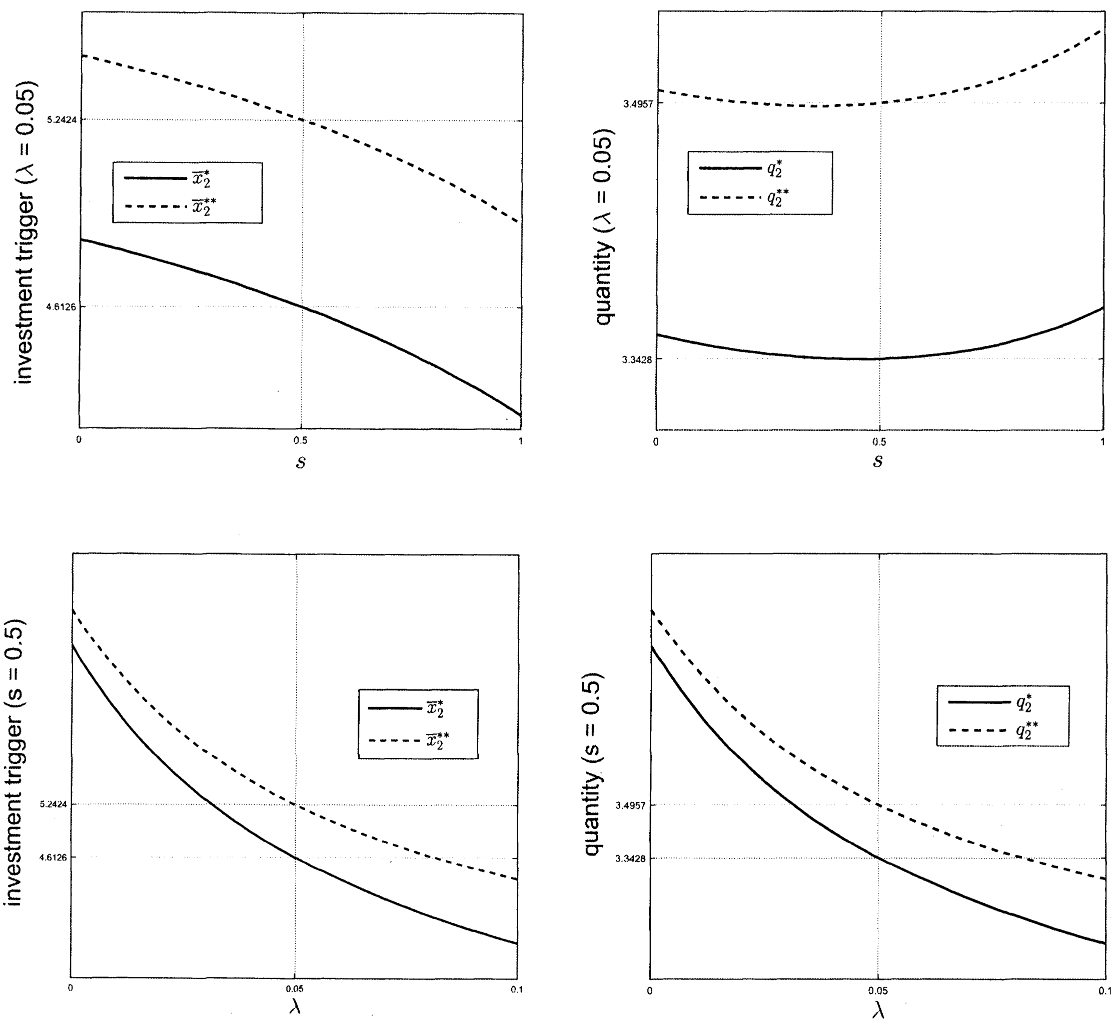

Figure 2: Optimal quantity and investment trigger

We begin by examining the effects of reversibility, i.e., s, on the optimal investment timing (trigger) and quantity strategies. We have the following remark.

Remark 1 *Higher reversibility accelerates the investment, but not necessarily increases the investment quantity.*

The upper-left panel of Figure 2 illustrates that $\overline{x}_2^{**}$ is monotonically decreasing with s. This result is the same as Wong (2010, 2011) and Cui and Shibata (2016b). That is, even under asymmetric information and with the possibility of random shock, higher reversibility accelerates the investment. The intuition is that higher reversibility increases the value of the abandonment option.

The upper-right panel shows that q_2^{**} exhibits a U-shape against s, with a minimum reached at around $s = 0.5$. This result is the same as under full information and without the possibility of random shock, i.e., Wong (2011). That is, an increase in s decreases q_2^{**} when s is relatively low, and increases q_2^{**} when s is sufficiently large.

We then consider the impact of the arrival probability of random shock, i.e., λ. We obtain the following remark.

Remark 2 *Higher arrival probability of random shock accelerates the investment, but decreases the investment quantity.*

The lower-left panel of Figure 2 illustrates that $\overline{x}_2^{**}$ is monotonically decreasing with λ. The reason is as follows. On the one hand, as shown in (2.5), an increase in λ (a decrease in v_2) decreases the value of the abandonment option. This effect decreases the investment value. On the other hand, an increase in λ reduces the value of operational cost. This effect increases the investment value. Because the latter effect dominates the former effect, a higher value of λ increases the investment value and then accelerates the investment.

The lower-right panel shows that q_2^{**} is monotonically decreasing with λ. This is an interesting result that contrary to our intuition. From our intuition, we conjecture that an increase in λ should increase q_2^{**} because the occurrence of random shock saves the operational cost of per unit quantity. However, we obtain that higher arrival probability of random shock (i.e., λ) induces the firm to undertake a smaller investment quantity q_2^{**}.

Finally, we compare the investment strategies under full and asymmetric information for any fixed value of s and λ. We have the following remark.

Remark 3 *Under asymmetric information, the investment timing is more delayed and the quantity is more increased than under full information.*

On the one hand, the upper-left panel of Figure 2 shows $\overline{x}_2^{**} > \overline{x}_2^{*}$ for any s and the lower-left panel illustrates $\overline{x}_2^{**} > \overline{x}_2^{*}$ for any λ. These results imply that even for a reversible investment and with the possibility of random shock, the investment timing is delayed under asymmetric information than under full information. On the other hand, the upper-right panel shows $q_2^{**} > q_2^{*}$ for any s and the lower-right panel demonstrates $q_2^{**} > q_2^{*}$ for any λ. These results imply that the investment quantity is increased under asymmetric information than under full information. The intuition is that the firm increases the quantity to compensate for the losses due to the delayed investment. There are tradeoffs between the efficiencies on the investment timing and quantity strategies.

5 Conclusion

In this study, we examine a firm's optimal timing and quantity strategies for a reversible investment, under which there exists asymmetric information before investment and possibility of random shock after investment. We obtain three important results. First, higher reversibility accelerates the investment, but not necessarily increases the investment quantity. Second, higher arrival probability of random shock accelerates the investment, but decreases the investment quantity. Third, under asymmetric information, the investment timing is more delayed and the quantity is more increased than under full information.

The asymmetric information model we have considered in this study assumes that the manager's bonus is a one-time payment incurred at the time of investment. That is, the manager's bonus has no influence on the value after investment. However, it should be interesting to consider a continuous bonus incentive that is related to the the profit flows and examine the optimal investment strategies. We leave this challenge for future work.

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