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Valuation of a Game Swaption under the Generalized Ho–Lee Model

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1 Introduction

The game swaption proposed in this paper is a game version of usual interest-rate swaption. A usual swaption provides only one of the two parties (fixed rate payer and variable or floating rate payer) with the right to enter a swap at a predetermined future time. By contrast, a game swaption, which we propose in this paper, provides the both parties with the right that they can choose an exercise time to enter a swap from a set of prespecified multiple exercise opportunities. Game swaptions are classified into two types by the timing to enter into the swap, namely game spot swaptions and game forward swaptions. A game spot swaption allows us to enter the swap at the next coupon time just after the exercise time, while a game forward swaption entitles us to enter the swap at a predetermined fixed calendar time regardless of the exercise time.

In this paper, we evaluate the former type of game swaptions under the generalized Ho–Lee model. The generalized Ho–Lee model is an arbitrage–free binomial lattice interest rate model, which was proposed by Ho and Lee [2]. It generalizes the well–known original Ho–Lee model [1] so that the volatility structure is dependent on time and state. Using the generalized Ho–Lee mode as a term structure model of interest rate, we propose an evaluation method of the arbitrage–free price of a game spot swaption via a stochastic game approach and illustrate its effectiveness by some numerical results.

2 The generalized Ho–Lee model

The generalized Ho–Lee model is an arbitrage–free binomial lattice interest rate model. Let N^* be a finite time horizon and (n, i) be a node on the binomial lattice where n $(0 \leq n \leq N^*)$ denotes a time and i $(0 \leq i \leq n)$ a state. $P(n, i; T)$ represents the zero–coupon bond price at node (n, i) with a remaining maturity of T $(0 \leq T \leq N^*)$ period, which pays 1 at the end of the T–th period. We have $P(n, i; 0) = 1$ for any n and i according to a definition of discount bond. Moreover, the zero–coupon bond prices for any remaining maturity T at the initial time, $P(0,0;T)$, can be observed in the market, and these set determines the discount function at the initial time.

To represent the uncertainty of interest rates on the binomial lattice, we introduce the binomial volatilities $\delta(n,i;T)$ $(0 \le i \le n)$. $\delta(n,i;T)$ is the proportional decrease in the one–period bond value from i to $i+1$ at time n, and $\delta(n,i;T)=1$ implies that there is no risk. The binomial volatilities $\delta(n,i;T)$ is defined by

$$\delta(n,i;T) = \frac{P(n+1,i+1;T)}{P(n+1,i;T)}, \ 0 \le n \le N^*, \ 0 \le i \le n. \tag{1}$$

As the binomial volatilities are bigger, the uncertainty of interest rates also increases more. Let $\sigma(n)$ be the term structure of volatilities of interest rates. Ho and Lee [2] assumed that the function $\sigma(n)$ is given by

$$\sigma(n) = (\sigma_0 - \sigma_\infty + \alpha_0 n)\exp(-\alpha_\infty n) + \alpha_1 n + \sigma_\infty, \tag{2}$$

where σ_0 is the short–rate volatility over the first period, $\alpha_1 n + \sigma_\infty$ is approximately the short–rate forward volatility at sufficiently large time n, $\alpha_0 + \alpha_1$ and α_∞ are the short–term and long–term slopes of the term structure of volatilities, respectively.

Then, the one–period binomial volatility $\delta(n,i,1)$ is defied by

$$\delta(n,i;1) = \exp(-2\sigma(n)\min(R(n,i;1)\Delta t^{3/2})), \ 0 \le n \le N^*, \ 0 \le i \le n, \tag{3}$$

where $R(n,i;1)$ denotes the one–period yield, R the threshold rate, and Δt the time interval.

The generalized Ho–Lee model is an arbitrage–free term structure model of interest rates. Therefore, the bond prices for all different maturities at each node (n,i) are modeled to satisfy the risk–neutral valuation formula

$$P(n,i;T) = \frac{1}{2}P(n,i;1)\left\{P(n+1,i;T-1) + P(n+1,i+1;T-1)\right\} \tag{4}$$

under the risk–neutral probability $\mathbb{Q}$, where the transition probability has the same $1/2$ for the up–state and down–state movements at a next period. Then, the arbitrage–free condition for the generalized Ho–Lee model is given by

$$\delta(n,i;T) = \delta(n,i;1)\delta(n+1,i;T-1)\left(\frac{1+\delta(n+1,i+1;T-1)}{1+\delta(n+1,i;T-1)}\right). \tag{5}$$

By using straightforwardly Equations (1) and (4), we can confirm Equation (5) to be an arbitrage–free condition. Thereby, as long as the T–period binomial volatility is defined by (5), the generalized Ho–Lee model is no arbitrage. Then, the one–period bond prices at node (n,i) for the generalized Ho–Lee model are given by

$$P(n,i;1) = \frac{P(0,0;n+1)}{P(0,0;n)}\prod_{k=1}^{n}\left(\frac{1+\delta(k-1,0;n-k)}{1+\delta(k-1,0;n-k+1)}\right)\prod_{j=0}^{i-1}\delta(n-1,j;1). \tag{6}$$

Similarly, the T–period bond prices at node (n,i) are given by

$$P(n,i;T) = \frac{P(0,0;n+T)}{P(0,0;n)}\prod_{k=1}^{n}\left(\frac{1+\delta(k-1,0;n-k)}{1+\delta(k-1,0;n-k+T)}\right)\prod_{j=0}^{i-1}\delta(n-1,j;T). \tag{7}$$

Next, we explain the algorithm to derive the one–period bond prices based on the above argument. The computations of the generalized Ho–Lee model are decomposed into the following five steps:

Step 1: derive one–period bond price at node $(n, 0)$

$$P(n,0;1) = \frac{P(0,0;n+1)}{P(0,0;n)} \prod_{k=1}^{n} \left(\frac{1+\delta(k-1,0;n-k)}{1+\delta(k-1,0;n-k+1)} \right);$$

Step 2: derive one–period bond price at node (n, i)

$$P(n,i;1) = P(n,0;1) \prod_{j=0}^{i-1} \delta(n-1,j;1), \quad i = 1,\dots,n;$$

Step 3: derive one–period yields by one–period bond price

$$R(n,i;1) = -\frac{\log P(n,i;1)}{\Delta t}, \quad i = 0,\dots,n;$$

Step 4: derive one–period binomial volatilities

$$\delta(n,i;1) = \exp(-2\sigma(n)\min(R(n,i;1,R)\Delta t^{3/2})), \quad i = 0,\dots,n;$$

Step 5: derive T–period binomial volatilities

$$\delta(n,i;T) = \delta(n,i;1)\delta(n+1,i;T-1)\left(\frac{1+\delta(n+1,i+1;T-1)}{1+\delta(n+1,i;T-1)} \right), \quad i = 0,\dots,n.$$

3 Valuation of game spot swaption

Game swaptions can be classified into two types with respect to the timing of entering into the swap. A game spot swaption allows us to enter the swap at the next coupon time just after an exercise, while a game forward swaption allows us to enter the swap at a predetermined calender time regardless of the exercise time. In this paper, we consider the game spot swaption.

A swap is an agreement to exchange a fixed rate and a variable rate (or floating rate) for a common notional principal over a prespecified period. We usually use LIBOR (London Interbank Offered Rate) as the variable rate. A swaption is an option on a swap. A payer swaption gives the holder the right to enter a particular swap agreement as the fixed rate payer. On the other hand, a receiver swaption gives the holder the right to enter a particular swap agreement as the fixed rate receiver. The holder of an European swaption is allowed to enter the swap only on the expiration time. In contrast, the holder of an American swaption is allowed to enter the swap on any time that falls within a range of two time instants. A Bermudan swaption, which we consider in this paper, allows its holder to enter the swap on multiple prespecified times.

In this paper, we consider the game swaption which is an extension of Bermudan swaption. The game swaption entitles both of the fixed rate side and variable rate side to enter into the swap at multiple prespecified times. The time sequence of coupon payment is

$$0 \le N \le M_0 < M_1 < \dots < M_L \le N^*, \tag{8}$$

where N is an agreement time of the swap, $M_0, M_1, \dots, M_L$ are the L coupon payment times, and N^* is a finite time horizon. Now, we suppose $N = M_0$ for a game spot swaption. The time period is

$$M_{h+1} - M_h = \kappa\Delta t \quad h = 0,\dots,L-1. \tag{9}$$

For the following discussions, we let $\kappa = 1$.

Let $S(N,i)$ be a spot swap rate at the agreement time N. The spot swap rate, $S(N,i)$, specifies the fixed rate that makes the value of the interest rate swap equal zero at the agreement time N and is given by

$$S(N,i) = \frac{1 - P(N,i;L)}{\Delta t \sum_{l=1}^{L} P(N,i;l)}. \tag{10}$$

Next, we define the exercise rate for a game swaption. If the fixed rate side exercises at an exercisable time, she will pay the fixed rate K_F over the future period of swap. If the variable rate side exercises, the fixed rate side has to pay the fixed rate K_V over the future period. If the both sides simultaneously exercise, the fixed rate side has to pay the fixed rate K_B. Without loss of generality, we suppose

$$K_V \leq K_B \leq K_F. \tag{11}$$

Moreover, the set of admissible multiple exercise times for the fixed rate side and the variable rate side (N_F, N_V) and the prespecified exercise time intervals are given by

$$N_F := \{N_{i_1}, N_{i_2}, \ldots, N_{i_f}\} \subset \{N_b, N_{b+1}, \ldots, N_d\};$$
$$N_V := \{N_{j_1}, N_{j_2}, \ldots, N_{j_v}\} \subset \{N_c, N_{c+1}, \ldots, N_e\},$$

respectively. We assume that $\max\{N_{i_f}, N_{j_v}\} = N_m$.

Now, we apply a two–person and zero–sum stopping game to the valuation of the game swaption. The players of the game are the fixed rate payer side and the variable rate payer side. We call them fixed rate player and variable rate player, respectively. At the exercisable node (n,i) $(n \in \{N_{i_1}, N_{i_2}, \ldots, N_{i_f}\} \cup \{N_{j_1}, N_{j_2}, \ldots, N_{j_v}\})$, the fixed rate player chooses a pure strategy x and the variable rate player chooses a pure strategy y from the set of pure strategies $S := \{\text{Exercise } (E), \text{Not exercise } (N)\}$. Suppose that the pure strategy profile (pair) (x,y) is selected at node (n,i), and let the value of the game swaption at node (n,i) as $\mathcal{A}(x,y;n,i)$.

Definition 3.1 When the game spot swaption is exercised at node (n,i), the payoff value of the game spot swaption at the admissible exercise node (n,i) is given by

$$\mathcal{A}(x,y;n,i) = \begin{cases} \Delta t[S(n,i) - K_F] \sum_{l=1}^{L} P(n,i;l) & \text{if the fixed rate player exercises;} \\ \Delta t[S(n,i) - K_V] \sum_{l=1}^{L} P(n,i;l) & \text{if the variable rate player exercises;} \\ \Delta t[S(n,i) - K_B] \sum_{l=1}^{L} P(n,i;l) & \text{if the both players exercise.} \end{cases}$$

□

If the both players do not exercise at an admissible exercise time, the stochastic game moves to the following node

$$(n+1, I_{n+1}) = \begin{cases} (n+1, i+1) & \text{w.p. } 1/2; \\ (n+1, i) & \text{w.p. } 1/2, \end{cases} \tag{12}$$

at a next time. Here, I_{n+1} is a random state of interest rates at time $n+1$.

Then, the both players face a two–person and zero–sum stage game whose payoff is dependent on a state of interest rates and their strategy at every exercisable nodes (n,i) $(n \in \{N_{i_1}, N_{i_2}, \ldots, N_{i_f}\} \cup \{N_{j_1}, N_{j_2}, \ldots, N_{j_v}\})$. In the stochastic game, the fixed rate player chooses a strategy to maximize her payoff, while the variable rate player choose a strategy to minimize his payoff.

Given a two–person and zero–sum game specified by a payoff matrix $A \in \mathbb{R}^{m \times n}$ $(m, n \in \mathbb{N})$, we define the value of the game as follows:

$$\mathrm{val}[A] := \min_{q \in \Delta^n} \max_{p \in \Delta^m} \boldsymbol{p}^T A \boldsymbol{q} = \max_{p \in \Delta^m} \min_{q \in \Delta^n} \boldsymbol{p}^T A \boldsymbol{q}, \tag{13}$$

where the second equality is due to the *von Neumann Minimax Theorem*, $\boldsymbol{p}$ is an m–dimensional vector representing a mixed strategy for the row player, and $\boldsymbol{q}$ is an n–dimensional vector representing a mixed strategy for the column player.

Let $V_s(n,i)$ be the value of the game spot swaption at node (n,i). Then, we can derive it by solving the following equation backwardly in time:

Step 0 (Initial condition) for $n = N_m$,

$$V_s(n,i) = \mathrm{val}\begin{bmatrix} \Delta t[S(n,i) - K_B]\sum_{l=1}^{L} P(n,i;l) & \Delta t[S(n,i) - K_F]\sum_{l=1}^{L} P(n,i;l) \\ \Delta t[S(n,i) - K_V]\sum_{l=1}^{L} P(n,i;l) & 0 \end{bmatrix}, \; i = 0, \ldots, n;$$

Step 1–1 (Both players can exercise) for $n \in (N_F \cap N_V) \backslash \{N_m\}$,

$$V_s(n,i) = \mathrm{val}\begin{bmatrix} \Delta t[S(n,i) - K_B]\sum_{l=1}^{L} P(n,i;l) & \Delta t[S(n,i) - K_F]\sum_{l=1}^{L} P(n,i;l) \\ \Delta t[S(n,i) - K_V]\sum_{l=1}^{L} P(n,i;l) & P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V(n+1, I_{n+1})|(n,i)] \end{bmatrix}, \; i = 0, \ldots, n;$$

Step 1–2 (Fixed rate player can exercise) for $n \in (N_F \backslash N_V) \backslash \{N_m\}$,

$$V_s(n,i) = \max\left\{ \Delta t[S(n,i) - K_F]\sum_{l=1}^{L} P(n,i;l), \; P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1, I_{n+1})|(n,i)] \right\}, \; i = 0, \ldots, n;$$

Step 1–3 (Variable rate player can exercise) for $n \in (N_V \backslash N_F) \backslash \{N_m\}$,

$$V_s(n,i) = \min\left\{ \Delta t[S(n,i) - K_V]\sum_{l=1}^{L} P(n,i;l), \; P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1, I_{n+1})|(n,i)] \right\}, \; i = 0, \ldots, n;$$

Step 1–4 (Both player cannot exercise) for $n \notin (N_F \cup N_V) \backslash \{N_m\}$

$$V_s(n,i) = P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1, I_{n+1})|(n,i)], \; i = 0, \ldots, n,$$

where $P(n,i;1)$ is the one–period discount factor at node (n,i) based on the generalized Ho–Lee model, $\mathbb{Q}$ is the risk–neutral probability measure, $\mathbb{E}^{\mathbb{Q}}[\cdot|(n,i)]$ is the conditional expectation, and I_{n+1} is the random state of interest rate at $n+1$. In the initial condition, $n = N_m$, we have $P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1, I_{n+1})|(n,i)] = 0$ according to the maturity of the game swaption.

At the node (n, i) where both players can exercise, we need to solve the following two–person and zero–sum game:

$$\begin{bmatrix} \Delta t[S(n,i) - K_B] \sum_{l=1}^{L} P(n,i;l) & \Delta t[S(n,i) - K_F] \sum_{l=1}^{L} P(n,i;l) \\ \Delta t[S(n,i) - K_V] \sum_{l=1}^{L} P(n,i;l) & P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1, I_{n+1})|(n,i)] \end{bmatrix}, \tag{14}$$

where the fixed rate side chooses the row as a maximizer and the variable rate side chooses the column as a minimizer. In general, a saddle point equilibrium of two–person and zero–sum game is known to exist in mixed strategies including pure strategies. However, the following theorem shows that the above game has the saddle point in pure strategies.

Theorem 3.1 Suppose $K_V < K_F$ at an exercisable node (n, i) $(n \in \{N_{i_1}, N_{i_2}, \dots, N_{i_f}\} \cup \{N_{j_1}, N_{j_2}, \dots, N_{j_v}\})$. Then the stage–matrix games played at the node have a saddle point in pure strategies:

$$\max_{x \in S} \min_{y \in S} \mathcal{A}(x, y; n, i) = \min_{y \in S} \max_{x \in S} \mathcal{A}(x, y; n, i) \tag{15}$$

where x and y are pure strategies of the fixed rate side player and the variable rate side player, respectively. Furthermore, if we denote E and N the pure strategies 'Exercise' and 'Not Exercise,' respectively, then the equilibrium–strategy profile (x, y) is as follows:

$$\begin{cases} (E,N) & \text{if } P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1,I_{n+1})|(n,i)] \le \Delta t[S(n,i)-K_F]\sum_{l=1}^{L} P(n,i;l) < \Delta t[S(n,i)-K_V]\sum_{l=1}^{L} P(n,i;l); \\ (N,N) & \text{if } \Delta t[S(n,i)-K_F]\sum_{l=1}^{L} P(n,i;l) < P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1,I_{n+1})|(n,i)] < \Delta t[S(n,i)-K_V]\sum_{l=1}^{L} P(n,i;l); \\ (N,E) & \text{if } \Delta t[S(n,i)-K_F]\sum_{l=1}^{L} P(n,i;l) < \Delta t[S(n,i)-K_V]\sum_{l=1}^{L} P(n,i;l) \le P(n,i;1)\mathbb{E}^{\mathbb{Q}}[V_s(n+1,I_{n+1})|(n,i)]. \end{cases}$$

□

4 Numerical examples

In this section, we show a pricing of a game spot swaption by numerical examples. The parameters in the generalized Ho–Lee model are set as follows: $\Delta t = 0.25$ (3 months), $R = 0.3$, and a flat yield curve of 5%. The game spot swaption has 5–year maturity, the protection period is 1–year. The both players can choose strategies at prescribed exercise times after the protection period. If either or both player exercise the right, they enter the swap at next coupon time.

Firstly, Table 1 shows the American–type game spot swaption. We assume that the both players can exercise at any time for $n = 4$ to 20 and the exercise rates are $K_F = 5.3$, $K_B = 5.0$, and $K_V = 4.7$. In the table, the horizontal axis and the vertical axis corresponds to the time step and the state of interest rates, respectively. The upper surrounded area is the exercise regions of the fixed rate player, while the lower surrounded area is the exercise regions of the variable rate player. When the swaption prices are positive, it has an advantage over the fixed rate player. In contrast, when the swaption prices are negative, it has an advantage over the variable rate player.

Secondly, the Bermudan–type game spot swaption is indicated in Table 2. It has properties that both players can exercise at any $n \in \{4, 8, 12, 16, 20\}$, only a fixed rate player can also exercise at any $n \in \{6, 10, 14, 18\}$. Similar to the first example, we suppose $K_F = 5.3$, $K_B = 5.0$, and $K_V = 4.7$.

																				0.12173
																			0.11393	0.10619
																		0.10625	0.09863	0.09113
																	0.09869	0.09119	0.08381	0.07655
																0.09125	0.08387	0.07661	0.06947	0.06245
															0.08394	0.07667	0.06952	0.06250	0.05560	0.04881
														0.07674	0.06959	0.06256	0.05565	0.04886	0.04219	0.03564
													0.06967	0.06263	0.05572	0.04892	0.04225	0.03569	0.02925	0.02293
												0.06271	0.05579	0.04899	0.04231	0.03575	0.02930	0.02297	0.01676	0.01067
											0.05587	0.04906	0.04238	0.03581	0.02936	0.02303	0.01681	0.01086	0.00526	0
										0.04915	0.04246	0.03588	0.02943	0.02309	0.01705	0.01157	0.00664	0.00260	0	0
									0.04254	0.03596	0.02950	0.02316	0.01739	0.01218	0.00763	0.00391	0.00128	0	0	0
								0.03605	0.02959	0.02325	0.01768	0.01270	0.00836	0.00477	0.00203	0.00020	-0.00088	-0.00179	-0.00362	-0.00733
							0.02968	0.02342	0.01793	0.01311	0.00892	0.00538	0.00253	0.00036	-0.00129	-0.00280	-0.00479	-0.00790	-0.01236	-0.01732
						0.02358	0.01817	0.01345	0.00935	0.00583	0.00291	0.00051	-0.00150	-0.00340	-0.00560	-0.00853	-0.01246	-0.01729	-0.02216	-0.02693
					0.01840	0.01374	0.00989	0.00619	0.00319	0.00063	-0.00162	-0.00379	-0.00617	-0.00907	-0.01275	-0.01725	-0.02212	-0.02689	-0.03157	-0.03615
				0.01399	0.00997	0.00647	0.00342	0.00075	-0.00168	-0.00404	-0.00656	-0.00948	-0.01300	-0.01723	-0.02208	-0.02685	-0.03153	-0.03612	-0.04061	-0.04501
			0.01021	0.00670	0.00361	0.00085	-0.00171	-0.00421	-0.00684	-0.00980	-0.01326	-0.01733	-0.02203	-0.02681	-0.03149	-0.03608	-0.04057	-0.04498	-0.04929	-0.05351
		0.00690	0.00377	0.00093	-0.00172	-0.00432	-0.00704	-0.01004	-0.01346	-0.01742	-0.02197	-0.02676	-0.03144	-0.03604	-0.04053	-0.04494	-0.04925	-0.05348	-0.05761	-0.06166
	0.00391	0.00102	-0.00171	-0.00440	-0.00718	-0.01021	-0.01362	-0.01750	-0.02191	-0.02670	-0.03139	-0.03599	-0.04049	-0.04490	-0.04922	-0.05344	-0.05758	-0.06163	-0.06560	-0.06948
0.00109	-0.00169	-0.00444	-0.00728	-0.01034	-0.01372	-0.01754	-0.02184	-0.02663	-0.03133	-0.03593	-0.04044	-0.04485	-0.04917	-0.05340	-0.05755	-0.06160	-0.06557	-0.06946	-0.07326	-0.07698
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Table 1 The American–type game spot swaption prices and the exercise area

5 Concluding remarks

This paper proposed game swaptions as game versions of a Bermudan swaption. Furthermore, we evaluated the no–arbitrage price of a game spot swaption under the generalized Ho–Lee model as the term structure model of interest rate. This game swaption is also considered to be a generalization of the payer swaption and receiver swaption .

Since, in this paper, we treated only the game spot swaption, we will consider the game forward swaption in the next research. Further, mathematical analysis of the structure of the exercise areas of both players in both game swaptions is also remained as our future research.

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													0.06850	0.06263	0.05483	0.04892	0.04162	0.03569	0.02885	0.02293
												0.06271	0.05490	0.04899	0.04168	0.03575	0.02890	0.02297	0.01656	0.01067
											0.05498	0.04906	0.04174	0.03581	0.02895	0.02303	0.01663	0.01076	0.00526	0
										0.04915	0.04182	0.03588	0.02902	0.02309	0.01700	0.01145	0.00659	0.00260	0	0
									0.04190	0.03596	0.02909	0.02316	0.01736	0.01211	0.00757	0.00389	0.00128	0	0	0
								0.03605	0.02917	0.02324	0.01766	0.01266	0.00831	0.00473	0.00201	0.00020	-0.00088	-0.00179	-0.00362	-0.00733
							0.02925	0.02332	0.01791	0.01309	0.00888	0.00534	0.00251	0.00036	-0.00128	-0.00278	-0.00475	-0.00782	-0.01219	-0.01732
						0.02342	0.01812	0.01343	0.00932	0.00581	0.00289	0.00051	-0.00148	-0.00336	-0.00552	-0.00638	-0.01221	-0.01686	-0.02189	-0.02693
					0.01830	0.01370	0.00967	0.00617	0.00318	0.00064	-0.00160	-0.00375	-0.00611	-0.00900	-0.01268	-0.01725	-0.02135	-0.02629	-0.03123	-0.03615
				0.01393	0.00995	0.00646	0.00342	0.00075	-0.00166	-0.00400	-0.00649	-0.00939	-0.01289	-0.01706	-0.02182	-0.02685	-0.03052	-0.03537	-0.04020	-0.04501
			0.01017	0.00669	0.00361	0.00085	-0.00169	-0.00417	-0.00677	-0.00970	-0.01312	-0.01715	-0.02142	-0.02622	-0.03115	-0.03608	-0.03936	-0.04410	-0.04882	-0.05351
		0.00688	0.00377	0.00095	-0.00169	-0.00428	-0.00697	-0.00993	-0.01332	-0.01724	-0.02172	-0.02676	-0.03045	-0.03530	-0.04013	-0.04494	-0.04786	-0.05249	-0.05709	-0.06166
	0.00391	0.00103	-0.00168	-0.00434	-0.00709	-0.01008	-0.01342	-0.01720	-0.02145	-0.02612	-0.03106	-0.03599	-0.03929	-0.04403	-0.04875	-0.05344	-0.05605	-0.06056	-0.06504	-0.06948
0.00111	-0.00165	-0.00438	-0.00720	-0.01022	-0.01357	-0.01737	-0.02169	-0.02663	-0.03036	-0.03521	-0.04004	-0.04485	-0.04780	-0.05243	-0.05703	-0.06160	-0.06391	-0.06831	-0.07267	-0.07698
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Table 2 The Bermudan–type game spot swaption prices and the exercise area

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