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Apparent parameter technique and vanishing of cohomology groups with Whitney holomorphic functions

By

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Abstract

We extend apparent parameter technique introduced in [2] to the case of Whitney holomorphic functions and give its application.

§ 1. Introduction

In the paper T. Aoki, N. Honda and S. Yamazaki [2], we have established compatibility of composition of analytic pseudodifferential operators $\mathscr{E}_X^{\mathbb{R}}$, which is defined in two ways; one comes from Leibniz's rule in the symbol theory of $\mathscr{E}_X^{\mathbb{R}}$ as in [1] and the other is given by the cohomological residue map, for example, as in [7]. It was a long-standing issue to show the compatibility of both the definitions and it has been done in [2] by employing, so called, an apparent parameter technique. This technique is based on Theorem 2.3 in Section 2 (see also Proposition 1.3 in [7]) which establishes, roughly speaking, a certain isomorphism between a local cohomology groups with coefficients in holomorphic functions and the corresponding ones on the space equipped with an apparent parameter.

The theorem is not only a crucial key in showing the compatibility but also a useful tool for several applications. In this paper, as another application of the theorem, we extend it to the case of Whitney holomorphic functions, and then, show that some difficulty observed in a Čech representation of holomorphic microfunctions of Whitney class is overcome by introducing an apparent parameter thanks to the theorem. The details and proofs of this note are given in our forthcoming paper.

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§ 2. Local cohomology groups on a vector space

Let $X = \mathbb{R}^m$ and let Z be a closed subset in X. Let us consider a continuous deformation mapping $\varphi(x,s)\colon X \times [0,1] \to X$ which satisfies the following conditions A1., A2. and A3.:

A1. $\varphi(x,1) = x$ for any $x \in X$ and $\varphi(x,s) = x$ for any $x \in Z$.
A2. $\varphi(\varphi(x,s),0) = \varphi(x,0)$ for any $s \in [0,1]$ and $x \in X$.
A3. Set

$$\rho_\varphi(x,s) := |\varphi(x,s) - \varphi(x,0)|. \tag{2.1}$$

Then $\rho_\varphi(x,s)$ is a strictly increasing function of s outside Z, i.e., if $s_1 < s_2$, we have $\rho_\varphi(x,s_1) < \rho_\varphi(x,s_2)$ for any $x \in X \setminus Z$.

Define, for short,

$$\rho_\varphi(x) := \rho_\varphi(x,1) = |x - \varphi(x,0)|, \tag{2.2}$$

and call it *the level function* of φ.

Example 2.1. Let $X = \mathbb{C}^2$ with (z_1, z_2) and $Z = \{z_1 = 0\}$. Define

$$\varphi(z,s) = (sz_1, z_2). \tag{2.3}$$

Then clearly $\varphi(z,s)$ satisfies the above conditions.

Set, for some $0 < a < \pi$ and $r > 0$,

$$\Gamma := \{\eta \in \mathbb{C};\ |\arg \eta| < a,\ 0 < |\eta| < r\} \tag{2.4}$$

and $\widehat{X} := X \times \mathbb{C}_\eta$ with coordinates (x,η). We denote by π_η the canonical projection $\widehat{X} \mapsto X$ defined by $(x,\eta) \mapsto x$. Let $G \subset X$ be a closed subset and $U \subset X$ an open subset. For $\varrho > 0$, we define the subsets $\widehat{G}$ and $\widehat{U}$ in $\widehat{X}$ as follows:

$$\begin{aligned}\widehat{G} &:= \{(\varphi(x,s),\eta) \in X \times \Gamma;\ \rho_\varphi(x) \leqslant \varrho|\eta|,\ 0 \leqslant s \leqslant 1,\ x \in G\}\\ &= \bigcup_{0 \leqslant s \leqslant 1} \{(\varphi(x,s),\eta) \in X \times \Gamma;\ \rho_\varphi(x) \leqslant \varrho|\eta|,\ x \in G\},\\ \widehat{U} &:= \{(x,\eta) \in X \times \Gamma;\ x \in U,\ \rho_\varphi(x) < \varrho|\eta|\}.\end{aligned} \tag{2.5}$$

Note that $\widehat{G} \cap \widehat{U}$ is a closed subset in $\widehat{U}$.

Example 2.2. Let $X = \mathbb{C}^2$ with (z_1, z_2) and let us consider the mapping $\varphi(z, s) := (sz_1, z_2)$ where $Z = \{z_1 = 0\}$. Let $G = \{(z_1, z_2);\ \varrho^2|z_2| \leqslant |z_1|\}$ for $\varrho > 0$. By noticing

$$\rho_\varphi(z) = |\varphi(z, 1) - \varphi(z, 0)| = |z_1|,$$

we have

$$\begin{aligned}
\widehat{G} &:= \bigcup_{0\leqslant s\leqslant 1} \{(\varphi(z, s), \eta) \in \widehat{X};\ \rho_\varphi(z) \leqslant \varrho|\eta|,\ z \in G\} \\
&= \bigcup_{0\leqslant s\leqslant 1} \{(sz_1, z_2, \eta) \in \mathbb{C}^2 \times \Gamma;\ |z_1| \leqslant \varrho|\eta|,\ \varrho^2|z_2| \leqslant |z_1|\} \\
&= \{(z_1, z_2, \eta) \in \mathbb{C}^2 \times \Gamma;\ |z_1| \leqslant \varrho|\eta|,\ \varrho|z_2| \leqslant |\eta|\} \\
&= \{(z_1, \eta) \in \mathbb{C} \times \Gamma;\ |z_1| \leqslant \varrho|\eta|\} \underset{\mathbb{C}_\eta}{\times} \{(z_2, \eta) \in \mathbb{C} \times \Gamma;\ |z_2| \leqslant \varrho^{-1}|\eta|\}.
\end{aligned}$$

Since $\pi_\eta^{-1}(G) \cap \widehat{U}$ is a closed subset in $\widehat{G} \cap \widehat{U}$ and $\widehat{U}$ is an open subset in $\pi_\eta^{-1}(U)$, the canonical morphism $\mathcal{F} \to \boldsymbol{R}\pi_{\eta *}\pi_\eta^{-1}\mathcal{F}$ induces

$$\boldsymbol{R}\Gamma_{G\cap U}(U; \mathcal{F}) \longrightarrow \boldsymbol{R}\Gamma_{\pi_\eta^{-1}(G)\cap\pi_\eta^{-1}(U)}(\pi_\eta^{-1}(U); \pi_\eta^{-1}\mathcal{F}) \longrightarrow \boldsymbol{R}\Gamma_{\widehat{G}\cap\widehat{U}}(\widehat{U}; \pi_\eta^{-1}\mathcal{F}). \tag{2.6}$$

We are now ready to state the theorem:

Theorem 2.3 (Propostion 1.3 [7]). *Let $\mathcal{F}$ be a complex of Abelian sheaves on X. Assume that U satisfies $\sup_{x\in U} \rho_\varphi(x) < \varrho r$. Then the above canonical morphism*

$$\boldsymbol{R}\Gamma_{G\cap U}(U; \mathcal{F}) \longrightarrow \Gamma_{\widehat{G}\cap\widehat{U}}(\widehat{U}; \pi_\eta^{-1}\mathcal{F}) \tag{2.7}$$

is isomorphic.

Now we extend the above theorem to the case of Whitney holomorphic functions. We consider the following situation: Let $X = \mathbb{C}^n = \mathbb{R}^{2n}$ with the complex coordinates $(x) = (x_1, \ldots, x_n)$, and let Z be a closed subanalytic subset in X. Set

$$\Gamma := \{\eta \in \mathbb{C};\ |\arg \eta| < a,\ |\eta| < r\} \subset \mathbb{C} \tag{2.8}$$

for some $0 < a < \pi$ and $r > 0$. Define $\widehat{X} := X \times \mathbb{C}_\eta$ with complex coordinates (x, η) and the canonical projection $\pi_\eta : \widehat{X} \to X$ in the same way as those at the beginning of this section. Let $\varphi(x, s)\colon X \times [0, 1] \to X$ be a continuous deformation mapping which satisfies the conditions A1., A2. and A3. introduced already and the following additional one:

A4. The φ is a C^1 subanalytic map and it satisfies

$$\operatorname{rank}_{\mathbb{R}}(d_{\mathbb{R}^{2n}\times(0,1)}\,\varphi) = 2n \quad \text{on } (X \setminus Z) \times (0, 1) \tag{2.9}$$

and

$$d_{\mathbb{R}^{2n}}\, \rho_\varphi\big|_{\wedge d_{\mathbb{R}^{2n}}\varphi=0} \neq 0 \quad \text{on } (X \setminus Z) \times (0,1). \tag{2.10}$$

Here $d_{\mathbb{R}^{2n}\times(0,1)}$ and $d_{\mathbb{R}^{2n}}$ denote the differentials with respect to the real coordinate variables $(\operatorname{Re} x,\ \operatorname{Im} x,\ s)$ and $(\operatorname{Re} x,\ \operatorname{Im} x)$ respectively, and ρ_φ is regarded as a function on $X \times (0,1)$, i.e., ρ_φ is independent of the variable s.

Let V be an open subanalytic subset and K a compact subanalytic subset. Set, for $\sigma > 0$,

$$\begin{aligned} \widehat{V} &:= \{(\varphi(x,s),\eta) \in \widehat{X};\ |\rho_\varphi(x)| \le \sigma|\eta|,\ 0 < s \le 1,\ x \in V\}, \\ \widehat{K} &:= \{(x,\eta) \in K \times \overline{\Gamma};\ |\rho_\varphi(x)| \le \sigma|\eta|\}. \end{aligned} \tag{2.11}$$

Note that $\widehat{V}$ and $\widehat{K}$ are subanalytic subsets and $\widehat{V} \cap \widehat{K}$ is an open subset in $\widehat{K}$. Set

$$\mathfrak{D}_{\widehat{X}} := \Gamma(\widehat{X},\, \mathcal{D}_{\widehat{X}}) \quad \text{and} \quad \mathfrak{M} := \mathfrak{D}_{\widehat{X}}/\mathfrak{D}_{\widehat{X}}\partial_\eta, \tag{2.12}$$

where $\mathcal{D}_{\widehat{X}}$ denotes the sheaf of linear analytic differential operators on $\widehat{X}$. In what follows, we extensively use the theory of Whitney tensor product $\bullet \overset{\mathrm{w}}{\otimes} \mathcal{O}_X$ and that of sheaves on subanalytic sites, in particular, the sheaf $\mathcal{O}^t_{X_{sa}}$ of temperate holomorphic functions and $\mathcal{O}^{\mathrm{w}}_{X_{sa}}$ of Whitney holomorphic functions on the subanalytic site X_{sa}. For these notions, refer the readers to [3], [4] and [5]. Now we can state the counterpart of the previous theorem:

Theorem 2.4. *Assume the condition* $\sup_{x\in K} \rho_\varphi(x) \le \sigma r$. *Then we have the canonical isomorphism in* $D^b(\mathbb{C})$

$$\mathrm{R}\Gamma\left(X,\ \mathbb{C}_{V\cap K} \overset{\mathrm{w}}{\otimes} \mathcal{O}_X\right) \xrightarrow{\ \sim\ } \mathrm{R}\mathrm{Hom}_{\mathfrak{D}_{\widehat{X}}}\left(\mathfrak{M},\ \mathrm{R}\Gamma\left(\widehat{X},\ \mathbb{C}_{\widehat{V}\cap\widehat{K}} \overset{\mathrm{w}}{\otimes} \mathcal{O}_{\widehat{X}}\right)\right).$$

§ 3. Application to holomorphic microfunction of Whitney class

In this section, we give an application of Theorem 2.4 to a Čech representation of holomorphic microfunctions of Whitney class.

A holomorphic microfunction in the complex domain is a counterpart of a well-known Sato's microfunction in the real domain. Let $X = \mathbb{C}^n$ with complex coordinates $(z) = (z', z'')$ and Y a closed complex submanifold with its complex codimension $d > 0$. We assume Y to be defined by $\{z' = 0\}$. We denote by $T^*_Y X$ the conormal bundle of Y. Then the sheaf $\mathscr{C}^{\mathbb{R}}_{Y|X}$ of holomorphic microfunctions on $T^*_Y X$ is defined by

$$\mathscr{C}^{\mathbb{R}}_{Y|X} := \mu_Y(\mathcal{O}_X)[d], \tag{3.1}$$

where $\mu_Y(\bullet)$ is the microlocalization functor along Y (see [7]). We can also obtain the sheaf $\mathscr{C}^{\mathbb{R},t}_{Y|X}$ of temperate holomorphic microfunctions and the sheaf $\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X}$ of holomorphic microfunctions of Whitney class by replacing the sheaf $\mathcal{O}_X$ with $\mathcal{O}^t_{X_{sa}}$ and $\mathcal{O}^{\mathrm{w}}_{X_{sa}}$ in (3.1) respectively, see [6] for details.

Let $p := (0; dz_1) \in T^*_Y X$. Then it follows from a fiber formula of microlocalization that a stalk of $\mathscr{C}^{\mathbb{R}}_{Y|X}$ at p is defined by

$$\mathscr{C}^{\mathbb{R}}_{Y|X,p} = \varinjlim_{U,G} \mathrm{H}^d_G(U; \mathcal{O}_X), \tag{3.2}$$

where U is an open neighborhood of the origin in X and G has a form $G' \times \mathbb{C}^{n-d} \subset \mathbb{C}^d \times \mathbb{C}^{n-d} = \mathbb{C}^n$ with G' being a closed conic cone in $\mathbb{C}^d$ containing the vector $dz_1 = (1, 0, \ldots, 0)$. That is, G is a closed wedge whose edge is Y.

Similarly, a stalk of $\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X}$ is

$$\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X,p} = \varinjlim_{U,W} \mathrm{H}^d\left(X; \mathbb{C}_{W\cap\overline{U}} \overset{\mathrm{w}}{\otimes} \mathcal{O}_X\right), \tag{3.3}$$

where U is a subanalytic open neighborhood of the origin in X and W has a form $W' \times \mathbb{C}^{n-d} \subset \mathbb{C}^d \times \mathbb{C}^{n-d} = \mathbb{C}^n$ with W' being a subanalytic open conic cone in $\mathbb{C}^d$ containing the vector dz_1. That is, Ω is an open wedge whose edge is Y.

One of reasons why these objects seem interesting and important in analysis is that a section of, for example, $\mathscr{C}^{\mathbb{R}}_{Y|X}$ is given by a boundary value of a holomorphic function locally defined on a cone along Y, which can be seen through Čech representation of local cohomology groups. As a matter of fact, let us consider the following simple case: Let $X = \mathbb{C}^2$ with coordinates (z_1, z_2) and $Y = \{0\}$. Define open subsets, for $\epsilon > 0$,

$$\begin{aligned} U &= \{|z| < \epsilon\}, \\ S &= \{z \in U; |\arg z_1 - \pi| < \pi/2 + \epsilon\}, \\ V &= \{z \in U;\ |z_1| < \epsilon|z_2|\}, \end{aligned} \tag{3.4}$$

where $|z| = \max\{|z_1|, |z_2|\}$. Then a pair $\{\{U, S, V\}, \{S, V\}\}$ becomes a covering of the pair $\{U, U/G\}$ in (3.2). Clearly U, S and V are Stein open subsets, and hence, the higher cohomology groups of $\mathcal{O}_X$ on these open subsets vanish, i.e.,

$$\mathrm{H}^k(U; \mathcal{O}_X) = \mathrm{H}^k(V; \mathcal{O}_X) = \mathrm{H}^k(S; \mathcal{O}_X) = 0 \quad (k \neq 0). \tag{3.5}$$

Therefore, by the theory of Čech cohomology groups, we have obtained

$$\mathscr{C}^{\mathbb{R}}_{Y|X,p} = \varinjlim_{\epsilon>0} \mathcal{O}_X(S\cap V)/(\mathcal{O}_X(S) + \mathcal{O}_X(V)). \tag{3.6}$$

As a conclusion, a holomorphic microfunction at p is represented by a boundary value of a holomorphic function defined on a cone $S \cap V$.

Now let us define, for a subanalytic open subset Ω in X,

(3.7) $OT(\Omega) := \mathrm{H}^0(\Omega;\, \mathcal{O}^t_{X_{sa}}) = \{f \in \mathcal{O}_X(\Omega);\, f$ has a temperate growth along $\partial\Omega\}$.

Since the higher cohomology groups of $\mathcal{O}^t_{X_{sa}}$ on U, S and V still vanish, i.e.,

$$\mathrm{H}^k\left(U;\, \mathcal{O}^t_{X_{sa}}\right) = \mathrm{H}^k\left(V;\, \mathcal{O}^t_{X_{sa}}\right) = \mathrm{H}^k\left(S;\, \mathcal{O}^t_{X_{sa}}\right) = 0 \quad (k \neq 0), \tag{3.8}$$

by the same argument as that for $\mathscr{C}^{\mathbb{R}}_{Y|X}$, we also have

$$\mathscr{C}^{\mathbb{R},t}_{Y|X,p} = \varinjlim_{\epsilon>0} OT(S \cap V)/(OT(S) + OT(V)). \tag{3.9}$$

Hence we can see that a temperate holomorphic microfunction at p is a holomorphic microfunction whose representative has a temperate growth along boundary of $S \cap V$.

We now expect a similar fact that $\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X}$ has the same kind of representation as that for $\mathscr{C}^{\mathbb{R}}_{Y|X}$ or $\mathscr{C}^{\mathbb{R},t}_{Y|X}$. Let us define, for a subanalytic open subset Ω,

(3.10)
$OW(\Omega) := \mathrm{H}^0(X;\, \mathbb{C}_{\overline{\Omega}} \overset{\mathrm{w}}{\otimes} \mathcal{O}_X) = \{f \in \mathcal{O}_X(\Omega);\, f$ extends to X as a C^∞ function$\}$.

Contrary to the case for either $\mathcal{O}_X$ or $\mathcal{O}^t_{X_{sa}}$, we cannot show vanishing of higher cohomology groups for Whitney holomorphic functions on the Stein open subset V. In fact, we have the following lemma.

Lemma 3.1. *We have*

$$\mathrm{H}^1(X;\, \mathbb{C}_{\overline{V}} \overset{\mathrm{w}}{\otimes} \mathcal{O}_X) \neq 0.$$

Furthermore, we also have

$$\mathrm{H}^1(V;\, \mathcal{O}^{\mathrm{w}}_{X_{sa}}) \neq 0.$$

Hence we can no longer expect a formula like

$$\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X,p} = \varinjlim_{\epsilon>0} OW(S \cap V)/(OW(S) + OW(V))$$

because $\left\{\{\overline{U}, \overline{S}, \overline{V}\},\, \{\overline{S}, \overline{V}\}\right\}$ is not a relative Leray covering of the pair $\{\overline{U},\, \overline{U}/W\}$ in (3.3) with respect to the functor $\boldsymbol{R}\Gamma(X;\, \mathbb{C}_{(\bullet)} \overset{\mathrm{w}}{\otimes} \mathcal{O}_X)$ as the above lemma shows.

To overcome difficulty mentioned above, by the aide of Theorem 2.4, we consider the representation of $\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X}$ with an apparent parameter in the following way: We consider the problem in the original geometrical situation, that is, $X = \mathbb{C}^n$, $Y = \mathbb{C}^{n-d} =$

$\{(z', z''); z' = 0\}$ and $p = (0; dz_1)$. It follows from (3.3) and Theorem 2.4 ($\widehat{V}$ and $\widehat{K}$ in the theorem correspond to $\widehat{W}_\epsilon$ and $\overline{\widehat{U}_\epsilon}$ in the equation below, respectively) that we have

$$\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X,p} = \varinjlim_{\epsilon>0} \left\{ u \in \mathrm{H}^d(\widehat{X}; \mathbb{C}_{\widehat{W}_\epsilon \cap \overline{\widehat{U}_\epsilon}} \overset{\mathrm{w}}{\otimes} \mathscr{O}_{\widehat{X}}); \partial_\eta u = 0 \right\}, \tag{3.11}$$

where

$$\widehat{U}_\epsilon := \{(z, \eta) \in X \times \Gamma; |z_1| < \epsilon|\eta|, |z| < \epsilon, |\eta| < \epsilon\} \tag{3.12}$$

and

$$\widehat{W}_\epsilon := \{(z, \eta) \in \widehat{X}; |\arg z_1| < \pi/2 - \epsilon, \epsilon|z_k| < |\eta| \, (k = 2, \dots, d)\}. \tag{3.13}$$

Set $\widehat{U} := \widehat{U}_\epsilon$ for simplicity and

$$\begin{aligned} \widehat{V}^{(1)} &:= \{z \in \widehat{U}; |\arg z_1 - \pi| < \pi/2 + \epsilon\}, \\ \widehat{V}^{(k)} &:= \{z \in \widehat{U}; |\eta| < \epsilon|z_k|\} \quad (2 \le k \le d). \end{aligned} \tag{3.14}$$

We also set, for a non-empty subset α in $\{1, 2, \dots, d\}$,

$$\overline{\widehat{V}^{(\alpha)}} := \bigcap_{k\in\alpha} \overline{\widehat{V}^{(k)}}.$$

Then, since we have

$$\widehat{W}_\epsilon \cap \overline{\widehat{U}_\epsilon} = \left(\overline{\widehat{U}} \setminus \overline{\widehat{V}^{(1)}}\right) \cap \cdots \cap \left(\overline{\widehat{U}} \setminus \overline{\widehat{V}^{(d)}}\right)$$

and since $\mathbb{C}_{\overline{\widehat{U}} \setminus \overline{\widehat{V}^{(k)}}}$ is isomorphic to the complex

$$\mathscr{L}_k : 0 \longrightarrow \mathbb{C}_{\overline{\widehat{U}}} \longrightarrow \mathbb{C}_{\overline{\widehat{V}^{(k)}}} \longrightarrow 0,$$

we have isomorphisms

$$\begin{aligned} \mathbb{C}_{\widehat{W}_\epsilon \cap \overline{\widehat{U}_\epsilon}} &\simeq \mathscr{L}_1 \underset{\mathbb{C}}{\otimes} \cdots \underset{\mathbb{C}}{\otimes} \mathscr{L}_d \\ &\simeq 0 \to \mathbb{C}_{\overline{\widehat{U}}} \to \bigoplus_{\#\alpha=1} \mathbb{C}_{\overline{\widehat{V}^{(\alpha)}}} \to \bigoplus_{\#\alpha=2} \mathbb{C}_{\overline{\widehat{V}^{(\alpha)}}} \to \cdots \to \bigoplus_{\#\alpha=d} \mathbb{C}_{\overline{\widehat{V}^{(\alpha)}}} \to 0. \end{aligned} \tag{3.15}$$

The subset $\widehat{V}^{(\alpha)}$ appearing in the above sequence is Stein, which also enjoys the following good property for $\mathscr{O}^{\mathrm{w}}_{\widehat{X}}$:

Lemma 3.2. *We have*

$$\mathrm{H}^k(\widehat{X}; \mathbb{C}_{\overline{\widehat{U}}} \overset{\mathrm{w}}{\otimes} \mathscr{O}_{\widehat{X}}) = \mathrm{H}^k(\widehat{X}; \mathbb{C}_{\overline{\widehat{V}^{(\alpha)}}} \overset{\mathrm{w}}{\otimes} \mathscr{O}_{\widehat{X}}) = 0 \qquad (k \neq 0)$$

for any non-empty subset α in $\{1, \dots, d\}$.

We put (3.15) into (3.11), and then, take its d-th cohomology group, by noticing the above lemma, we finally obtain the following representation:

Proposition 3.3. *We have*

$$\mathscr{C}^{\mathbb{R},\mathrm{w}}_{Y|X,p} = \varinjlim_{\epsilon>0} \left\{ u \in \frac{OW(\widehat{V}^{(*)})}{\bigoplus_{\#\alpha=d-1} OW(\widehat{V}^{(\alpha)})};\ \partial_\eta u = 0 \right\}. \tag{3.16}$$

Here the set $OW(\Omega)$ of Whitney holomorphic functions is defined in the same way as that in (3.10)*, that is, for a subanalytic open subset $\Omega \subset \widehat{X}$,*

$$OW(\Omega) := \mathrm{H}^0(\widehat{X};\, \mathbb{C}_{\overline{\Omega}} \overset{\mathrm{w}}{\otimes} \mathcal{O}_{\widehat{X}}) = \{f \in \mathcal{O}_{\widehat{X}}(\Omega);\ f \text{ extends to } \widehat{X} \text{ as a } C^\infty \text{ function}\}, \tag{3.17}$$

and $\widehat{V}^{(*)} := \widehat{V}^{(1)} \cap \cdots \cap \widehat{V}^{(d)}$, i.e.,

$$\widehat{V}^{(*)} = \left\{ (z,\eta) \in X \times \Gamma;\ \begin{array}{l} |\arg z_1 - \pi| < \pi/2 + \epsilon,\ |z_1| < \epsilon|\eta| \\ |\eta| < \epsilon|z_k| < \epsilon^2\ (k = 2, \ldots, d) \end{array} \right\}. \tag{3.18}$$

As a consequence, we see that a holomorphic microfunction of Whitney class at p is represented by a holomorphic function with an apparent parameter on the cone $\widehat{V}^{(*)}$ which extends to $\widehat{X}$ as a C^∞ function and which is cohomologically independent of the variable η, that is, its η derivative becomes zero as a cohomology class.

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