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サプライチェインネットワークにおける ロバストな均衡モデルについて

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1 Introduction

Supply chains are connections of process including procurement of raw materials, production, shipping and selling for a product. Supply chain management is one of main objects in management science, and hence many researchers have proposed mathematical models to optimize or analyze supply chain networks. In this paper, we focus on competitive supply chain networks. Because supply chains involve many decision-makers (called players) and they independently decide their behaviors, competitive situations often occur. To analyze, Nagurney et al. [10] proposed a supply chain network equilibrium (SCNE) model which is constructed by the manufacturers, the retailers and the demand markets. Yamada et al. [18] extended the SCNE model by considering the behaviors of the distribution centers. Hammond and Beullens [5] developed a model for closed-loop supply chain with the manufacturers and the consumer markets considering returns of the products and evaluated the effect of a legislation. Yang et al. [19] extended this closed-loop model to a general closed-loop supply chain network with the raw material suppliers, the manufacturers, the retailers, the recovery centers and the demand markets. Li and Nagurney [8] treated a quality competition with the suppliers, the producing firms and the demand markets and evaluated the quality of the products. Nagurney [9] considered a freight service provision network model with the disaster relief organizations, the freight service providers and the demand points. Nagurney et al. [11] dealt with a network model for post-disaster humanitarian relief by NGOs which is constructed by NGOs, victims and donors.

Recently, particular attention is paid to the risk management of supply chains. Supply chain has many vulnerabilities as to natural disasters, business fluctuations, demand and cost uncertainties and so on. Wu et al. [17] suggested that supply chains face the many kinds of risk factor and there are many cases that supply chains were disrupted, so it is necessary to deal with these risks. Kleindorfer and Saad [7] pointed out the necessities of three tasks which are "Specifying sources of risk and vulnerabilities, Assessment, and Mitigation" (SAM). Tang [16] defined "supply chain risk management (SCRM)" as "the management of supply chain risks through coordination or collaboration among the supply chain partners so as to ensure profitability and continuity". Also, there are many researches analyzing the optimal behavior as to risks by using a mathematical model. Dong et al. [4] expanded the SCNE model in [10] to a equilibrium model with a demand uncertainty. Bertsimas and Thiele [3] analyzed inventory management problems under the demand and cost uncertainties. Qiang et al. [13] developed a supply chain network model with uncertainty in costs and demand. In addition, they proposed a supply chain network performance measure. Pishvaee et al. [12] dealt with a market to market closed-loop supply chain network. In their research, demands, returns and transportation costs are assumed as uncertain factors. Baghalian et al. [1] developed a multi-supply chain network model considering demand uncertainties and disruption risks.

These researches analyzed a supply chain network with uncertainties in demand or cost. But they did not consider a model which has uncertainties in the other players' strategies. In actually, there are many cases which players in the supply chain can not know the others' strategies exactly. The aim of this paper is to consider the case that there are uncertainties in competitors' strategies in the SCNE model in [10]. In this paper, we assume that some firms can not know the exact value of competitors' strategies. We also assume that under these uncertainties, they minimize their cost for the worst case. This approach is called robust optimization or robust methodology. According to Bertsimas and Thiele [3], "the robust methodology can be understood as a "reasonable worst-case" approach". So we call the model considered in this paper "robust SCNE model".

2 Robust SCNE model

In this section, we develop a robust supply chain network equilibrium model. The network that we consider is shown in Figure 1. In this model, there are m manufacturers, n retailers (or wholesalers) and o demand markets. The manufacturers produce products and delivery them to the retailers. The retailers buy the products from the manufacturers and sale them to the demand markets. The demand markets buy the products from the retailers. Each manufacturer minimizes his total cost by deciding the amount of the products shipped to the retailers. Each retailer minimizes his total cost by deciding the amount of the products ordered to each manufacturer and the products sold to each demand market. Each demand market decides the purchase volume and the market price which satisfies the equilibrium conditions described later. We assume that each manufacturer and retailer cannot know exactly strategies of the other manufacturers and retailers respectively. We also assume that they minimize their cost for the worst case. We formulate their problems as second-order cone programming problems and also reformulate them as a VIP.

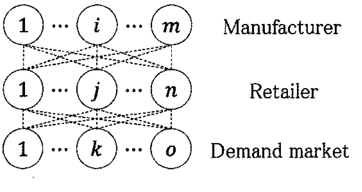

Figure 1: Supply chain network equilibrium model.

First, we define the variables for this model. For $i = 1, \cdots, m,\ j = 1, \cdots, n,\ k = 1, \cdots, o$:

$q_{ij}(\geq 0)$: the amount of the products shipped by manufacturer i to retailer j,
$w_{jk}(\geq 0)$: the amount of the products sold by retailer j to demand market k,
$p_k(\geq 0)$: the market price at demand market k.

As for these variables, we define some vectors as follows:

$$\begin{aligned}
q_{i\cdot} &:= (q_{i1}, \cdots, q_{in})^T, \\
q_{\cdot j} &:= (q_{1j}, \cdots, q_{mj})^T, \\
w_{j\cdot} &:= (w_{j1}, \cdots, w_{jo})^T, \\
w_{\cdot k} &:= (w_{1k}, \cdots, w_{nk})^T, \\
q_{-i\cdot} &:= (q_{1\cdot}^T, \cdots, q_{i-1\cdot}^T, q_{i+1\cdot}^T, \cdots, q_{m\cdot}^T)^T, \\
q_{\cdot -j} &:= \left(q_{\cdot 1}^T, \cdots, q_{\cdot j-1}^T, q_{\cdot j+1}^T, \cdots, q_{\cdot n}^T\right)^T, \\
w_{-j\cdot} &:= \left(w_{1\cdot}^T, \cdots, w_{j-1\cdot}^T, w_{j+1\cdot}^T, \cdots, w_{n\cdot}^T\right)^T, \\
w_{\cdot -k} &:= \left(w_{\cdot 1}^T, \cdots, w_{\cdot k-1}^T, w_{\cdot k+1}^T, \cdots, w_{\cdot o}\right)^T, \\
p &:= (p_1, \cdots, p_o)^T.
\end{aligned}$$

Note that $q_{i\cdot}$ is a variable of manufacturer i, $q_{\cdot j}$ and $w_{j\cdot}$ are variables of retailer j, and $w_{\cdot k}$ and p_k are variables of demand market k. As for the price of each product, we define as follows:

ρ_{ij} : the price of the product charged by retailer j to manufacturer i,
π_j : the price of the product charged by demand market k to retailer j.

We also use the following notations:

$$\begin{aligned}
\rho_{i\cdot} &:= (\rho_{i1}, \cdots, \rho_{in})^T, \\
\rho_{\cdot j} &:= (\rho_{1j}, \cdots, \rho_{mj})^T.
\end{aligned}$$

2.1 The problems solved by the manufacturers

We consider the problem solved by the manufacturers. We define the following functions for manufacturer i $(i = 1, \cdots, m)$:

$f_i(q_{i\cdot}, q_{-i\cdot})$: a production cost function of manufacturer i,
$c_{ij}(q_{ij})$: a transaction cost between manufacturer i and retailer j $(j = 1, \cdots, n)$.

We assume that c_{ij} is convex. A product function depends on not only the decision variable of manufacturer i (namely, $q_{i\cdot}$) but also that of the other manufacturers (namely, $q_{-i\cdot}$), that is, if the manufacturers procure the raw materials of the products from a same supplier, a competition for the raw materials may occur and affect the production cost. In this paper, we define $f_i(q_{i\cdot}, q_{-i\cdot})$ as follows:

$$f_i(q_{i\cdot}, q_{-i\cdot}) := \left(\sum_{j=1}^{n} q_{ij}\right)\left(a_{ii} + b_{i1}\sum_{j=1}^{n} q_{1j} + \cdots + b_{ii}\sum_{j=1}^{n} q_{ij} + \cdots + b_{im}\sum_{j=1}^{n} q_{mj}\right),$$

where a_{ii} and b_{is} $(s = 1, \cdots, m)$ are nonnegative scalars. Let a_i denote a column vector in n dimensions whose all components are a_{ii} and $B_{is} \in \mathbb{R}^{n\times n}$ denote a matrix whose all components are b_{is} . Then it follows that

$$f_i(q_{i\cdot}, q_{-i\cdot}) = a_i^T q_{i\cdot} + q_{i\cdot}^T B_{ii} q_{i\cdot} + \sum_{\substack{l=1 \\ l\neq i}}^{m} q_{i\cdot}^T B_{il} q_{l\cdot}\ .$$

The total sales of manufacturer i is $\rho_{i\cdot}^T q_{i\cdot}$. When we denote the total cost of manufacturer i by $\Psi_i(q_{i\cdot}, q_{-i\cdot})$, the total cost of him is as follows:

$$\Psi_i(q_{i\cdot}, q_{-i\cdot}) = -\rho_{i\cdot}^T q_{i\cdot} + \sum_{j=1}^{n} c_{ij}(q_{ij}) + a_i^T q_{i\cdot} + q_{i\cdot}^T B_{ii} q_{i\cdot} + \sum_{\substack{l=1 \\ l\neq i}}^{m} q_{i\cdot}^T B_{il} q_{l\cdot} \ .$$

We assume that manufacturer i cannot know the exact value of $q_{-i\cdot}$ but estimates $q_{l\cdot}$ as $\tilde{q}_{l\cdot} := q_{l\cdot} + M_{il}\Delta u_l$ $(l = 1, \cdots, i-1, i+1, \cdots, m)$, where $M_{il} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix and $\Delta u_l \in \mathbb{R}^n$ satisfies $\|\Delta u_l\| \leq 1$. We also assume that under this uncertainty, the manufacturer minimizes his cost based on the robust approach, that is, he minimizes his cost for the worst case. Then, manufacturer i's cost function for the worst case $\tilde{\Psi}_i(q_{i\cdot}, q_{-i\cdot})$ is

$$\tilde{\Psi}_i(q_{i\cdot}, q_{-i\cdot}) = \max\left\{\Psi_i(q_{i\cdot}, \tilde{q}_{-i\cdot}) \mid \tilde{q}_{-i\cdot} \in U_{-i}\right\},$$

where $\tilde{q}_{-i\cdot} := \left((\tilde{q}_{1\cdot})^T, \cdots, (\tilde{q}_{i-1\cdot})^T, (\tilde{q}_{i+1\cdot})^T, \cdots, (\tilde{q}_{m\cdot})^T\right)^T, U_{-i} := \prod_{\substack{l=1 \\ l\neq i}}^{m} U_{il}$ and $U_{il} := \{\tilde{q}_{l\cdot} = q_{l\cdot} + M_{il}\Delta u_l \mid \|\Delta u_l\| \leq 1\}$. Generally, $\tilde{\Psi}_i(q_{i\cdot}, q_{-i\cdot})$ is defined by the supremum of $\Psi_i(q_{i\cdot}, \tilde{q}_{-i\cdot})$. But $\Psi_i(q_{i\cdot}, \tilde{q}_{-i\cdot})$ is continuous and U_{-i} is compact, $\tilde{\Psi}_i(q_{i\cdot}, q_{-i\cdot})$ can be denoted as the maximum of $\Psi_i(q_{i\cdot}, \tilde{q}_{-i\cdot})$. Note that U_{-i} implies the uncertainty set for manufacturer i. So the function which manufacturer i should minimize is

$$\begin{aligned}\tilde{\Psi}_i(q_{i\cdot}, q_{-i\cdot}) = & -\rho_{i\cdot}^T q_{i\cdot} + \textstyle\sum_{j=1}^{n} c_{ij}(q_{ij}) + a_i^T q_{i\cdot} \\ & + q_{i\cdot}^T B_{ii} q_{i\cdot} + \textstyle\sum_{\substack{l=1 \\ l\neq i}}^{m} q_{i\cdot}^T B_{il} q_{l\cdot} + \textstyle\sum_{\substack{l=1 \\ l\neq i}}^{m} \max\limits_{\|\Delta u_l\|\leq 1} q_{i\cdot}^T B_{il} M_{il} \Delta u_l \ .\end{aligned}$$

Since $\max\limits_{\|\Delta u_l\|\leq 1} q_{i\cdot}^T B_{il} M_{il} \Delta u_l = \|M_{il}^T B_{il}^T q_{i\cdot}\| = \|M_{il} B_{il} q_{i\cdot}\|$, manufacturer i solves the following second-order cone programming problem:

$$\begin{array}{ll} \min\limits_{q_{i\cdot}, s_i} & \hat{\Psi}_i\left(q_{i\cdot}, q_{-i\cdot}, s_i\right) = -\rho_{i\cdot}^T q_{i\cdot} + \sum_{j=1}^{n} c_{ij}(q_{ij}) + a_i^T q_{i\cdot} + q_{i\cdot}^T B_{ii} q_{i\cdot} \\ & \qquad + \sum_{\substack{l=1 \\ l\neq i}}^{m} q_{i\cdot}^T B_{il} q_{l\cdot} + \sum_{\substack{l=1 \\ l\neq i}}^{m} s_{il} \\ \text{s.t.} & 0 \leq q_{i\cdot}, \ \|M_{il} B_{il} q_{i\cdot}\| \leq s_{il} \ (l = 1, \cdots, i-1, i+1, \cdots, m), \end{array} \tag{2.1}$$

where $s_i := (s_{i1}, \cdots, s_{ii-1}, s_{ii+1}, \cdots, s_{im})^T$.

By letting the feasible set of (2.1) be T_i, this set can be written by

$$T_i := \left\{\left(q_{i\cdot}^T, s_i^T\right)^T \mid 0 \leq q_{i\cdot}, \|M_{il} B_{il} q_{i\cdot}\| \leq s_{il} \ (l = 1, \cdots, i-1, i+1, \cdots, m)\right\}.$$

Because T_i is a nonempty convex set and the objective function $\hat{\Psi}_i$ is convex with $q_{i\cdot}$ and s_i, the problem (2.1) is a convex programming problem. So, for given $q_{-i\cdot}$, the optimal condition of (2.1) is

$$\begin{array}{l} \sum_{j=1}^{n} \left\{\left(\frac{\partial f_i(q_{i\cdot}^*, q_{-i\cdot})}{\partial q_{ij}} + \frac{dc_{ij}(q_{ij}^*)}{dq_{ij}} - \rho_{ij}^*\right) \times \left(q_{ij} - q_{ij}^*\right)\right\} \\ + \sum_{\substack{l=1 \\ l\neq i}}^{m} (s_{il} - s_{il}^*) \geq 0, \\ \forall \left(q_{i\cdot}^T, s_i^T\right)^T \in T_i \ . \end{array} \tag{2.2}$$

Here, $(q_{i\cdot}^*, s_i^*)$ denotes the solution. By gathering (2.2) as for all manufacturers, we get the following equilibrium condition:

$$\begin{array}{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{\left(\frac{\partial f_i(q_{i\cdot}^*, q_{-i\cdot}^*)}{\partial q_{ij}} + \frac{dc_{ij}(q_{ij}^*)}{dq_{ij}} - \rho_{ij}^*\right) \times \left(q_{ij} - q_{ij}^*\right)\right\} \\ + \sum_{i=1}^{m} \sum_{\substack{l=1 \\ l\neq i}}^{m} (s_{il} - s_{il}^*) \geq 0, \\ \forall \left(q^T, s^T\right)^T \in T \ , \end{array} \tag{2.3}$$

where $T := \left\{ \left(q^T, s^T\right)^T \mid 0 \leq q, \|M_{il} B_{il} q_{i\cdot}\| \leq s_{il} \ (l = 1, \cdots, i-1, i+1, \cdots, m, \ i = 1, \cdots, m) \right\}$, $q := \left(q_{1\cdot}^T, \cdots, q_{m\cdot}^T\right)^T$ and $s := \left(s_1^T, \cdots, s_m^T\right)^T$.

2.2 The problems solved by the retailers

Next, we consider the problem solved by the retailers. We define the following function for retailer j $(j = 1, \cdots, n)$:

$$h_j(q_{\cdot j}, q_{\cdot -j}) \quad : \quad \text{a handling cost of retailer } j.$$

According to Nagurney et al. [10], the handling cost may include " the display and storage cost associated with product ". In this paper, we define $h_j(q_{\cdot j}, q_{\cdot -j})$ as below:

$$h_j(q_{\cdot j}, q_{\cdot -j}) := \left(\sum_{i=1}^m q_{ij}\right) \left(\delta_{jj} + \gamma_{j1} \sum_{i=1}^m q_{i1} + \cdots + \gamma_{jj} \sum_{i=1}^m q_{ij} + \cdots + \gamma_{jn} \sum_{i=1}^m q_{in}\right), \tag{2.4}$$

where δ_{jj} and γ_{jt} $(t = 1, \cdots, n)$ are nonnegative scalars. Let δ_j denote a column vector in m dimensions whose all components are δ_{jj} and $\Gamma_{jt} \in \mathbb{R}^{m \times m}$ denote a matrix whose all components are γ_{jt} . Then, the function (2.4) is rewritten as

$$h_j\left(q_{\cdot j}, q_{\cdot -j}\right) = \delta_j^T q_{\cdot j} + q_{\cdot j}^T \Gamma_{jj} q_{\cdot j} + \sum_{\substack{r=1 \\ r \neq j}}^n q_{\cdot j}^T \Gamma_{jr} q_{\cdot r} \ .$$

Note that the total sales for the products of retailer j is $\pi_j \sum_{k=1}^o w_{jk}$ and the total stocking cost is $\rho_{\cdot j}^T q_{\cdot j}$. We assume that the retailers must not cause an absence of the goods. That is, for retailer j, the total stock of the products $(= \sum_{i=1}^m q_{ij})$ must not be less than the total sales amounts $(= \sum_{k=1}^o w_{jk})$. If we denote the total cost of retailer j by $\Phi_j(q_{\cdot j}, q_{\cdot -j}, w_{j\cdot})$, the total cost of him is given by

$$\Phi_j(q_{\cdot j}, q_{\cdot -j}, w_{j\cdot}) = -\pi_j \sum_{k=1}^o w_{jk} + \rho_{\cdot j}^T q_{\cdot j} + \delta_j^T q_{\cdot j} + q_{\cdot j}^T \Gamma_{jj} q_{\cdot j} + \sum_{\substack{r=1 \\ r \neq j}}^n q_{\cdot j}^T \Gamma_{jr} q_{\cdot r} \ .$$

We assume that retailer j cannot know the exact value of $q_{\cdot -j}$ but estimates $q_{\cdot r}$ as $\tilde{q}_{\cdot r} := q_{\cdot r} + N_{jr} \Delta v_r$ $(r = 1, \cdots, j-1, j+1, \cdots, n)$, where $N_{jr} \in \mathbb{R}^{m \times m}$ is a symmetric positive definite matrix and $\Delta v_r \in \mathbb{R}^m$ satisfies $\|\Delta v_r\| \leq 1$. We also assume that he minimizes his cost for the worst case by using the robust approach same as the problems solved by the manufacturers. Then, retailer j's cost function for the worst case $\tilde{\Phi}_i(q_{\cdot j}, q_{\cdot -j}, w_{j\cdot})$ is

$$\tilde{\Phi}_j(q_{\cdot j}, q_{\cdot -j}, w_{j\cdot}) = \max\{\Phi_i(q_{\cdot j}, \tilde{q}_{\cdot -j}, w_{j\cdot}) | \ \tilde{q}_{\cdot -j} \in V_{-j}\},$$

where $\tilde{q}_{\cdot -j} := \left((\tilde{q}_{\cdot 1})^T, \cdots, (\tilde{q}_{\cdot j-1})^T, (\tilde{q}_{\cdot j+1})^T, \cdots, (\tilde{q}_{\cdot n})^T\right)^T$, $V_{-j} := \prod_{\substack{r=1 \\ r \neq j}}^n V_{jr}$ and $V_{jr} := \{\tilde{q}_{\cdot r} = q_{\cdot r} + N_{jr} \Delta v_r | \ \|\Delta v_r\| \leq 1\}$. Here, V_{-j} is the uncertainty set for retailer j. So the function which retailer j should minimize is

$$\begin{aligned} \tilde{\Phi}_i(q_{\cdot j}, q_{\cdot -j}, w_{j\cdot}) = & -\pi_j \textstyle\sum_{k=1}^o w_{jk} + \rho_{\cdot j}^T q_{\cdot j} + \delta_j^T q_{\cdot j} \\ & + q_{\cdot j}^T \Gamma_{jj} q_{\cdot j} + \textstyle\sum_{\substack{r=1 \\ r \neq j}}^n q_{\cdot j}^T \Gamma_{jr} q_{\cdot r} + \textstyle\sum_{\substack{r=1 \\ r \neq j}}^n \max\limits_{\|\Delta v_r\| \leq 1} q_{\cdot j}^T \Gamma_{jr} N_{jr} \Delta v_r \ . \end{aligned}$$

Note that $\max\limits_{\|\Delta v_r\|\le 1} q_{\cdot j}^T \Gamma_{jr} N_{jr} \Delta v_r = \|N_{jr}^T \Gamma_{jr}^T q_{\cdot j}\| = \|N_{jr}\Gamma_{jr} q_{\cdot j}\|$. Accordingly, retailer j solves the following second-order cone programming problem:

$$\begin{array}{ll} \min\limits_{q_{\cdot j}, w_{j\cdot}, t_j} & \hat{\Phi}_j\left(q_{\cdot j}, q_{\cdot -j}, w_{j\cdot}, t_j\right) = -\pi_j \sum_{k=1}^{o} w_{jk} + \rho_{\cdot j}^T q_{\cdot j} + \delta_j^T q_{\cdot j} + q_{\cdot j}^T \Gamma_{jj} q_{\cdot j} \\ & \qquad + \sum_{\substack{r=1 \\ r\neq j}}^{n} q_{\cdot j}^T \Gamma_{jr} q_{\cdot r} + \sum_{\substack{r=1 \\ r\neq j}}^{n} t_{jr} \\ \text{s.t.} & 0 \le q_{\cdot j},\ 0 \le w_{j\cdot}, \sum_{k=1}^{o} w_{jk} \le \sum_{i=1}^{m} q_{ij},\ \|N_{jr}\Gamma_{jr} q_{\cdot j}\| \le t_{jr} \\ & \qquad (r = 1, \cdots, j-1, j+1, \cdots, n), \end{array} \tag{2.5}$$

where $t_j := (t_{j1}, \cdots, t_{jj-1}, t_{jj+1}, \cdots, t_{jn})^T$. Let S_j denote the feasible set of (2.5), and then S_j is given by

$$S_j := \left\{ \left(q_{\cdot j}^T, w_{j\cdot}^T, t_j^T\right)^T \ \middle|\ 0 \le q_{\cdot j}, 0 \le w_{j\cdot}, \sum_{k=1}^{o} w_{jk} \le \sum_{i=1}^{m} q_{ij}, \|N_{jr}\Gamma_{jr} q_{\cdot j}\| \le t_{jr} \right. \\ \left. (r = 1, \cdots, j-1, j+1, \cdots, n) \right\}.$$

Because S_j is a nonempty convex set and the objective function $\hat{\Phi}_j$ is convex with $q_{\cdot j}, w_{j\cdot}$ and t_j, (2.5) is a convex programming problem. By using a Lagrange multiplier $\xi_j \in \mathbb{R}_+$, for given $q_{\cdot -j}$, the optimal condition for (2.5) is given by the following (see [2], for example) :

$$\begin{array}{l} \sum_{i=1}^{m} \left\{ \left(\rho_{ij}^* + \frac{\partial h_j(q_{\cdot j}^*, q_{\cdot -j})}{\partial q_{ij}} - \xi_j^* \right) (q_{ij} - q_{ij}^*) \right\} \\ + \sum_{k=1}^{o} \left\{ \left(\xi_j^* - \pi_j^* \right) \left(w_{jk} - w_{jk}^* \right) \right\} \\ + \left(\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} w_{jk}^* \right) (\xi_j - \xi_j^*) \\ + \sum_{\substack{r=1 \\ r\neq j}}^{n} \left(t_{jr} - t_{jr}^* \right) \ge 0, \\ \forall \left(q_{\cdot j}^T, w_{j\cdot}^T, \xi_j, t_j^T \right)^T \in \hat{S}_j, \end{array} \tag{2.6}$$

where

$$\hat{S}_j := \left\{ \left(q_{\cdot j}^T, w_{j\cdot}^T, \xi_j, t_j^T\right)^T \ \middle|\ 0 \le q_{\cdot j}, 0 \le w_{j\cdot}, 0 \le \xi_j, \|N_{jr}\Gamma_{jr} q_{\cdot j}\| \le t_{jr} \right. \\ \left. (r = 1, \cdots, j-1, j+1, \cdots, n) \right\}.$$

Also, by gathering (2.6) as for all retailers, we get the following equilibrium condition:

$$\begin{array}{l} \sum_{j=1}^{n} \sum_{i=1}^{m} \left\{ \left(\rho_{ij}^* + \frac{\partial h_j(q_{\cdot j}^*, q_{\cdot -j}^*)}{q_{ij}} - \xi_j^* \right) (q_{ij} - q_{ij}^*) \right\} \\ + \sum_{j=1}^{n} \sum_{k=1}^{o} \left\{ \left(\xi_j^* - \pi_j^* \right) \left(w_{jk} - w_{jk}^* \right) \right\} \\ + \sum_{j=1}^{n} \left(\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} w_{jk}^* \right) (\xi_j - \xi_j^*) \\ + \sum_{j=1}^{n} \sum_{\substack{r=1 \\ r\neq j}}^{n} \left(t_{jr} - t_{jr}^* \right) \ge 0, \\ \forall \left(q^T, w^T, \xi^T, t^T \right)^T \in \hat{S}, \end{array} \tag{2.7}$$

where $\hat{S} := \left\{ \left(q^T, w^T, \xi^T, t^T\right)^T \ \middle|\ 0 \le q, 0 \le w, 0 \le \xi, \|N_{jr}\Gamma_{jr} q_{\cdot j}\| \le t_{jr}\ (r = 1, \cdots, j-1, j+1, \cdots, n,\ j = 1, \cdots, n) \right\}$, $w := \left(w_{1\cdot}^T, \cdots, w_{n\cdot}^T\right)^T$, $\xi := (\xi_1, \cdots, \xi_n)^T$ and $t := \left(t_1^T, \cdots, t_n^T\right)^T$.

2.3 The conditions for the demand markets

Finally, we consider conditions which should be satisfied in the demand markets. For $k = 1, \cdots, o$, we define the following functions:

$d_k(p)$: a demand function of demand market k ,
$g_{jk}(w_{\cdot k}, w_{\cdot -k})$: a transaction cost between demand market k and retailer j $(j = 1, \cdots, n)$.

We assume that on the equilibrium, the following conditions are satisfied for demand market k $(k = 1, \cdots, o)$:

$$\begin{cases} \pi_j + g_{jk}(w_{\cdot k}, w_{\cdot -k}) = p_k & \text{if} \quad w_{jk} > 0, \\ \pi_j + g_{jk}(w_{\cdot k}, w_{\cdot -k}) \geq p_k & \text{if} \quad w_{jk} = 0, \end{cases} \tag{2.8}$$

$$\begin{cases} d_k(p) = \sum_{j=1}^{n} w_{jk} & \text{if} \quad p_k > 0, \\ d_k(p) \leq \sum_{j=1}^{n} w_{jk} & \text{if} \quad p_k = 0. \end{cases} \tag{2.9}$$

Condition (2.8) means that, when a demand market purchases the products from retailer j, sum of the transaction cost and the price of the product equals to the market price and when the demand market does not buy any products, sum of the transaction cost and the price of the product surpasses the market price. Condition (2.9) means that, when the market price is positive, the market demand equals to the purchase volume of the products from the retailers and when the market price is 0, the market demand belows the purchase volume of the products from the retailers.

For given $w_{\cdot -k}$ and p_{-k}, the conditions (2.8) and (2.9) can be rewritten as follows:

$$\begin{aligned} &\sum_{j=1}^{n} \left\{ (\pi_j^* + g_{jk}(w_{\cdot k}^*, w_{\cdot -k}) - p_k^*)(w_{jk} - w_{jk}^*) \right\} \\ &+ \left(\sum_{j=1}^{n} w_{jk}^* - d_k(p_k^*, p_{-k}) \right) (p_k - p_k^*) \geq 0, \\ &\forall w_{\cdot k} \in \mathbb{R}_+^n, p_k \in \mathbb{R}_+. \end{aligned} \tag{2.10}$$

By gathering (2.10) for all demand markets, we obtain

$$\begin{aligned} &\sum_{k=1}^{o} \sum_{j=1}^{n} \left\{ (\pi_j^* + g_{jk}(w_{\cdot k}^*, w_{\cdot -k}^*) - p_k^*)(w_{jk} - w_{jk}^*) \right\} \\ &+ \sum_{k=1}^{o} \left\{ \left(\sum_{j=1}^{n} w_{jk}^* - d_k(p_k^*, p_{-k}^*) \right) (p_k - p_k^*) \right\} \geq 0, \\ &\forall w \in \mathbb{R}_+^{no}, p \in \mathbb{R}_+^k. \end{aligned} \tag{2.11}$$

2.4 Reformulation as a VIP

By gathering (2.3), (2.7) and (2.11), we get

$$\begin{aligned} &\sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ \left(\frac{\partial f_i(q_{i\cdot}^*, q_{-i\cdot}^*)}{\partial q_{ij}} + \frac{dc_{ij}(q_{ij}^*)}{dq_{ij}} + \frac{\partial h_j(q_{\cdot j}^*, q_{\cdot -j}^*)}{\partial q_{ij}} - \xi_j^* \right) \left(q_{ij} - q_{ij}^* \right) \right\} \\ &+ \sum_{j=1}^{n} \sum_{k=1}^{o} \left\{ \left(g_{jk}(w_{\cdot k}^*, w_{\cdot -k}^*) - p_k^* + \xi_j^* \right) \left(w_{jk} - w_{jk}^* \right) \right\} \\ &+ \sum_{j=1}^{n} \left\{ \left(\sum_{i=1}^{m} q_{ij}{}^* - \sum_{k=1}^{o} w_{jk}{}^* \right) (\xi_j - \xi_j^*) \right\} \\ &+ \sum_{k=1}^{o} \left\{ \left(\sum_{j=1}^{n} w_{jk}^* - d_k(p_k^*, p_{-k}^*) \right) (p_k - p_k^*) \right\} \\ &+ \sum_{i=1}^{m} \left\{ \sum_{\substack{l=1 \\ l \neq i}}^{m} (s_{il} - s_{il}^*) \right\} \\ &+ \sum_{j=1}^{n} \left\{ \sum_{\substack{r=1 \\ r \neq j}}^{n} \left(t_{jr} - t_{jr}^* \right) \right\} \geq 0, \\ &\forall \left(q^T, w^T, \xi^T, p^T, s^T, t^T \right)^T \in K, \end{aligned}$$

where

$$K := \Big\{ \left(q^T, w^T, \xi^T, p^T, s^T, t^T\right)^T \mid 0 \le q, 0 \le w, 0 \le \xi, 0 \le p, \|M_{il}B_{il}q_{i\cdot}\| \le s_{il} \\ (l \neq i, i = 1, \cdots, m), \|N_{jr}\Gamma_{jr}q_{\cdot j}\| \le t_{jr} \ (r \neq j, j = 1, \cdots, n) \Big\}.$$

Therefore the all players' problems are reformulated as the VIP:

$$\text{Find } x^* \in K \text{ such that } F(x^*)^T(x - x^*) \ge 0, \ \forall x \in K \ , \tag{2.12}$$

where

$$x := \begin{pmatrix} q \\ w \\ \xi \\ p \\ s \\ t \end{pmatrix} \quad \text{and} \quad F(x) := \begin{pmatrix} \frac{\partial f_1(q_{1\cdot}, q_{-1\cdot})}{\partial q_{11}} + \frac{dc_{11}(q_{11})}{dq_{11}} + \frac{\partial h_1(q_{\cdot 1}, q_{\cdot -1})}{\partial q_{11}} - \xi_1 \\ \vdots \\ \frac{\partial f_1(q_{1\cdot}, q_{-1\cdot})}{\partial q_{1n}} + \frac{dc_{1n}(q_{1n})}{dq_{1n}} + \frac{\partial h_n(q_{\cdot n}, q_{\cdot -n})}{\partial q_{1n}} - \xi_n \\ \vdots \\ \frac{\partial f_m(q_{m\cdot}, q_{-m\cdot})}{\partial q_{m1}} + \frac{\partial c_{m1}(q_{m1})}{\partial q_{m1}} + \frac{\partial h_1(q_{\cdot 1}, q_{\cdot -1})}{\partial q_{m1}} - \xi_1 \\ \vdots \\ \frac{\partial f_m(q_{m\cdot}, q_{-m\cdot})}{\partial q_{mn}} + \frac{\partial c_{mn}(q_{mn})}{\partial q_{mn}} + \frac{\partial h_n(q_{\cdot n}, q_{\cdot -n})}{\partial q_{mn}} - \xi_n \\ g_{11}(w_{\cdot 1}, w_{\cdot -1}) - p_1 + \xi_1 \\ \vdots \\ g_{1o}(w_{\cdot o}, w_{\cdot -o}) - p_o + \xi_1 \\ \vdots \\ g_{n1}(w_{\cdot 1}, w_{\cdot -1}) - p_1 + \xi_n \\ \vdots \\ g_{no}(w_{\cdot o}, w_{\cdot -o}) - p_o + \xi_n \\ \sum_{i=1}^m q_{i1} - \sum_{k=1}^o w_{1k} \\ \vdots \\ \sum_{i=1}^m q_{in} - \sum_{k=1}^o w_{nk} \\ \sum_{j=1}^n w_{j1} - d_1(p_1, p_{-1}) \\ \vdots \\ \sum_{j=1}^n w_{jo} - d_o(p_o, p_{-o}) \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

3 Numerical experiments

Now we introduce the solution method of VIP (2.12). It is difficult to solve VIP (2.12) directly because this problem has non-differentiable functions in set K. In the paper, we reformulate (2.12) as a second-order cone complementarity problem. Note that K can be written as follows:

$$
\begin{aligned}
K := \Big\{ \left(q^T, w^T, \xi^T, p^T, s^T, t^T\right)^T \ \Big| \ & \left(q^T, w^T, \xi^T, p^T\right)^T \in \mathbb{R}_+^\sigma, \\
& \left(s_{il}, q_{i\cdot}^T B_{il} M_{il}\right)^T \in \mathcal{K}^{1+n} \ (l = 1, \cdots, i-1, i+1, \cdots, m, \ i = 1, \cdots, m), \\
& \left(t_{jr}, q_{\cdot j}^T \Gamma_{jr} N_{jr}\right)^T \in \mathcal{K}^{1+m} \ (r = 1, \cdots, j-1, j+1, \cdots, n, \ j = 1, \cdots, n) \Big\},
\end{aligned}
$$

where $\sigma := mn + no + n + o$, and $\mathcal{K}^{1+n}$ and $\mathcal{K}^{1+m}$ are $1+n$ and $1+m$ dimensional second-order cone respectively. Generally, the $1+\zeta$ dimensional second-order cone $\mathcal{K}^{1+\zeta}$ is defined by

$$
\mathcal{K}^{1+\zeta} := \left\{ y = \left(y_1, y_2^T\right)^T \mid y_1 \geq \|y_2\|, y_1 \in \mathbb{R}, y_2 \in \mathbb{R}^\zeta \right\}.
$$

We define $\mathcal{K}^1$ by $\mathbb{R}_+$. We define $\mathcal{K}$ and $\theta(x)$ as

$$
\mathcal{K} := \mathbb{R}_+^\sigma \times \prod_{i=1}^{m} \prod_{\substack{l=1 \\ l \neq i}}^{m} \mathcal{K}^{n+1} \times \prod_{j=1}^{n} \prod_{\substack{r=1 \\ r \neq j}}^{n} \mathcal{K}^{m+1}, \quad \theta(x) := \begin{pmatrix} q \\ w \\ \xi \\ p \\ s_{12} \\ M_{12} B_{12} q_{1\cdot} \\ \vdots \\ s_{mm-1} \\ M_{mm-1} B_{mm-1} q_{m\cdot} \\ t_{12} \\ N_{12} \Gamma_{12} q_{\cdot 1} \\ \vdots \\ t_{nn-1} \\ N_{nn-1} \Gamma_{nn-1} q_{\cdot n} \end{pmatrix}.
$$

Then, the set K appearing in (2.12) is rewritten by $K = \{x \mid \theta(x) \in \mathcal{K}\}$. Therefore, from KKT conditions of VIP (2.12), VIP (2.12) can be reformulated as the following mixed second-order cone complementarity problem (see [14] and [15]):

$$
\begin{array}{rl}
\text{Find} & (x, \lambda) \in \mathbb{R}^\nu \times \mathbb{R}^\tau \\
\text{such that} & F(x) - \nabla \theta(x) \lambda = 0, \\
& \theta(x) \in \mathcal{K}, \ \lambda \in \mathcal{K}, \ \theta(x)^T \lambda = 0,
\end{array}
\tag{3.1}
$$

where $\nu := \sigma + m(m-1) + n(n-1)$ and $\tau := \sigma + m(m-1)(n+1) + n(n-1)(m+1)$.

We solve (3.1) by ReSNA [6] to analyze the impact of uncertainties on the shipments, the prices, the profits and the total supply chain cost per unit. We choose the parameters same as Nagurney et al. [10] for each cost function and demand function. The number of the manufacturers, the retailers and the demand markets are two respectively. In these numerical experiments, we consider the following two cases with a constant α:

Case 1. Manufacturer 1 does not know the exact behavior of manufacturer 2 and the parameters for the uncertainties are given by

$$M_{12} = \alpha \times \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, M_{21} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Case 2. Neither manufacturer 1 nor manufacturer 2 knows the exact value of the opponent each other and the parameters for the uncertainties are given by

$$M_{12} = \alpha \times \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, M_{21} = \alpha \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

For both the cases, we change α from 0 to 5. When α equals to 0, each manufacturer knows the exact behavior of the opponent. The larger α gets, the larger the uncertainties get too. Also for both the cases, there are not uncertainties between the retailers.

The results of the experiments are shown in Figures 2–9. In Figure 2 and Figure 6, [circles and solid line] and [diamonds and solid line], and [triangles and solid line] and [squares and solid line] are overlapping respectively. The rest four dashed lines are overlapping too. In Figure 3 and Figure 7, the line of [circles and solid line] and [diamonds and solid line], [triangles and solid line] and [squares and solid line], [circles and dashed line] and [diamonds and dashed line], and [triangles and dashed line] and [squares and dashed line] are overlapping respectively. In Figure 4 and Figure 8, [triangles and solid line] and [squares and solid lines] are overlapping.

Seeing Figure 2, the larger the uncertainty gets, the fewer the amount of the products shipped by manufacturer 1 gets. But the amount of the products shipped by manufacturer 2 is increasing. Sum of the amount of the products shipped by retailer 1 and 2 get fewer. In Figure 3, the prices charged by the demand markets to the retailers are getting higher. We see that the profit of manufacturer 1 is decreasing while the profit of manufacturer 2 is decreasing from Figure 4, and the total cost per unit is getting higher from Figure 5. Seeing Figure 6, if both the manufacturers have the uncertainties, they decrease the amounts of the products. But manufacturer 1, who has more uncertainty than manufacturer 2, produces fewer. We see that the price charged by the retailers to manufacturer 2 is getting lower from Figure 6, and the profit of manufacturer 2 is decreasing from Figure 8. Comparing Figure 5 and Figure 9, the total supply chain cost per unit for the case 2 is higher than case 1. In both the cases, the larger the uncertainties get, the more retailers get profit.

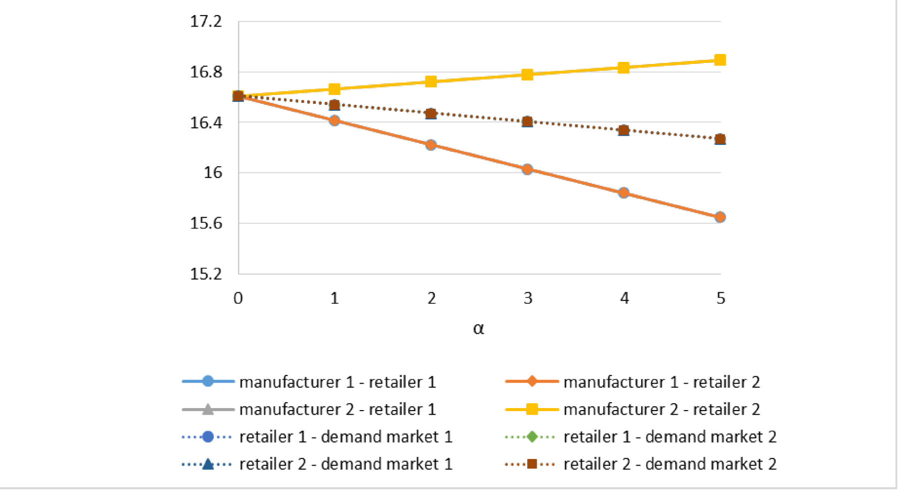

Figure 2: The effects of uncertainties on the shipments (case 1).

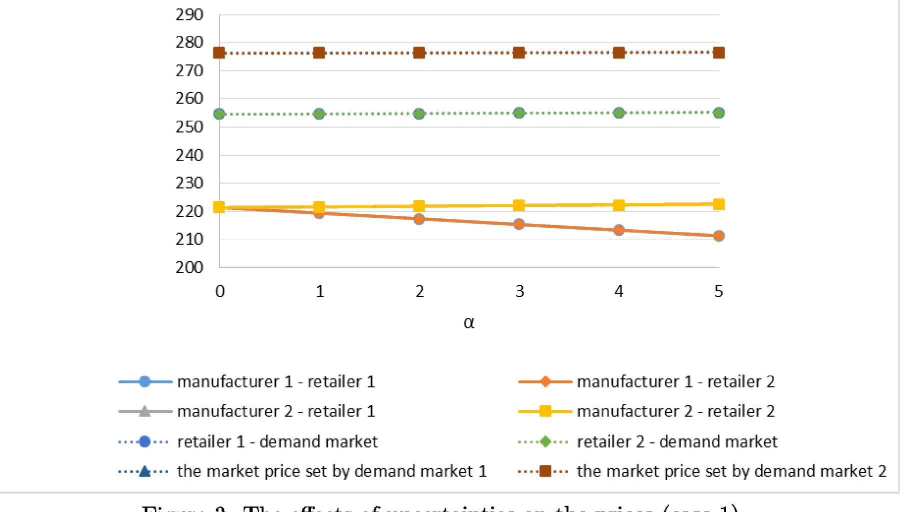

Figure 3: The effects of uncertainties on the prices (case 1).

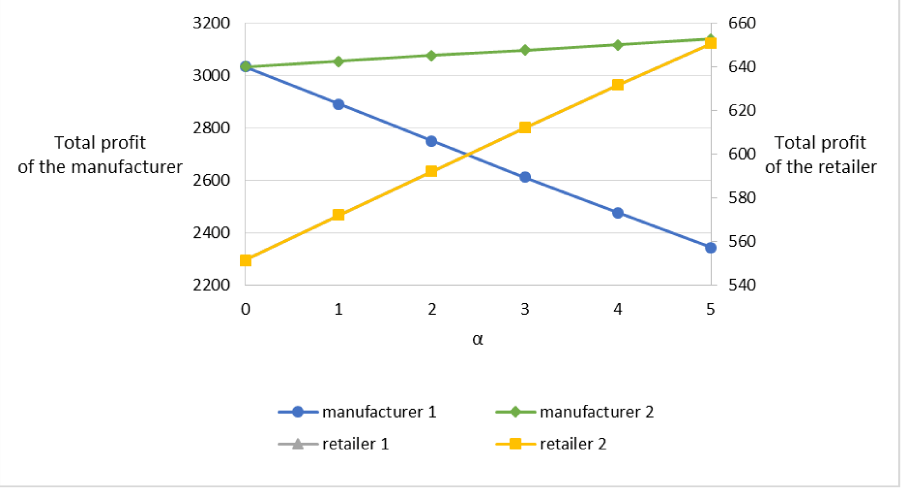

Figure 4: The effects of uncertainties on the profits (case 1).

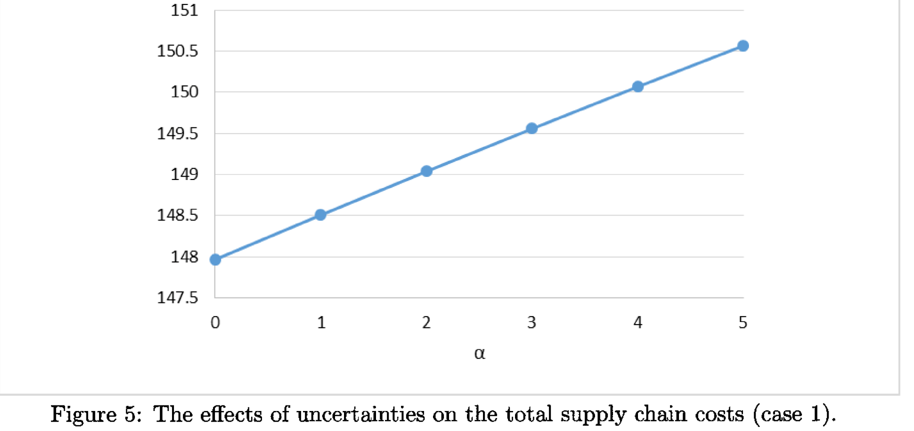

Figure 5: The effects of uncertainties on the total supply chain costs (case 1).

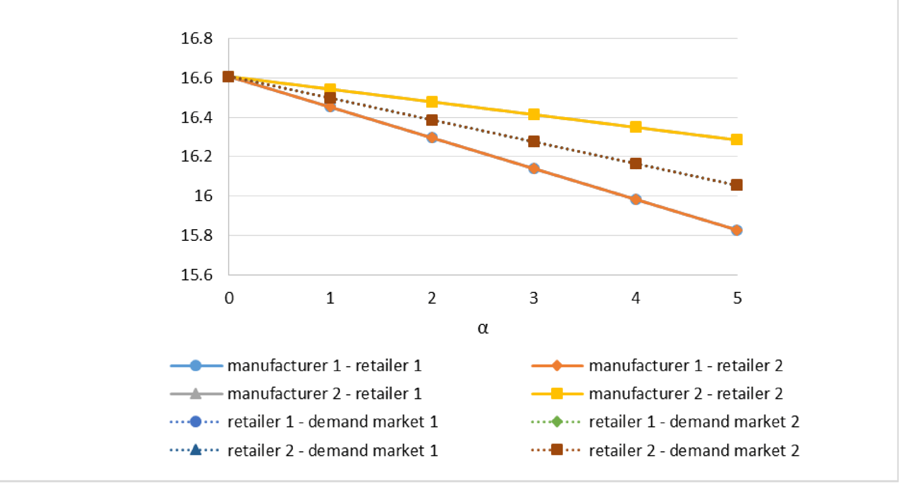

Figure 6: The effects of uncertainties on the shipments (case 2).

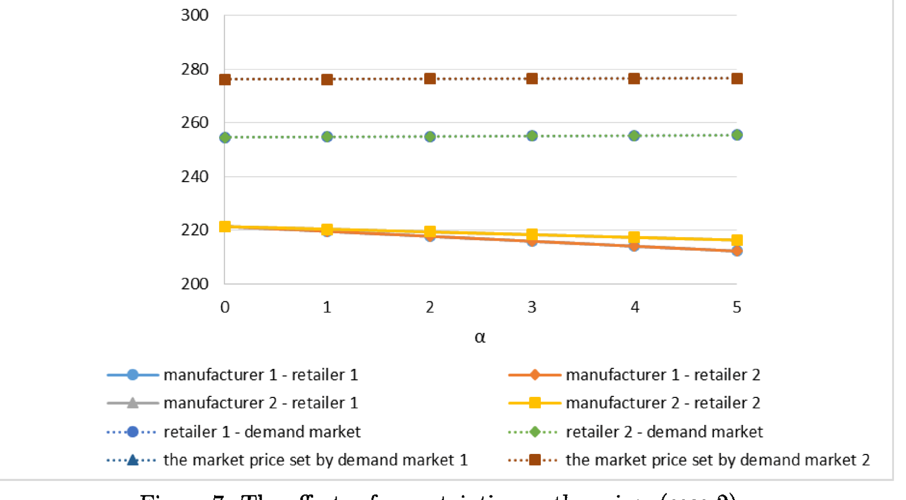

Figure 7: The effects of uncertainties on the prices (case 2).

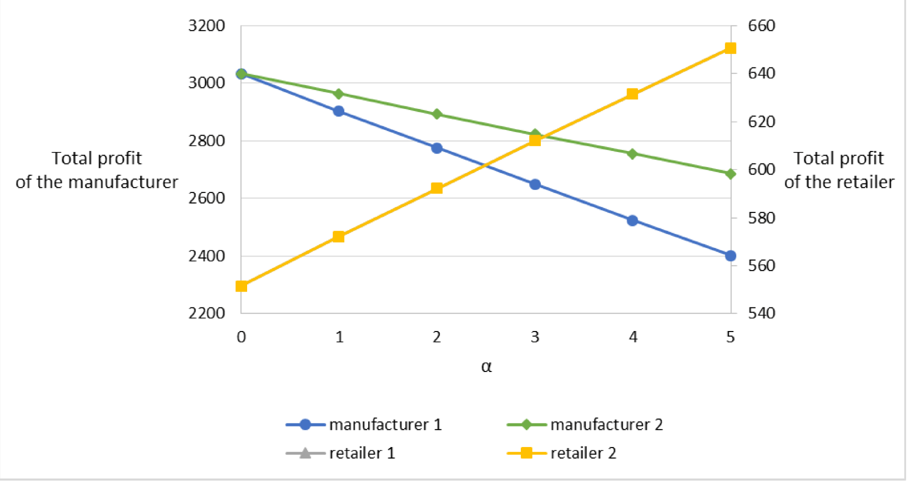

Figure 8: The effects of uncertainties on the profits (case 2).

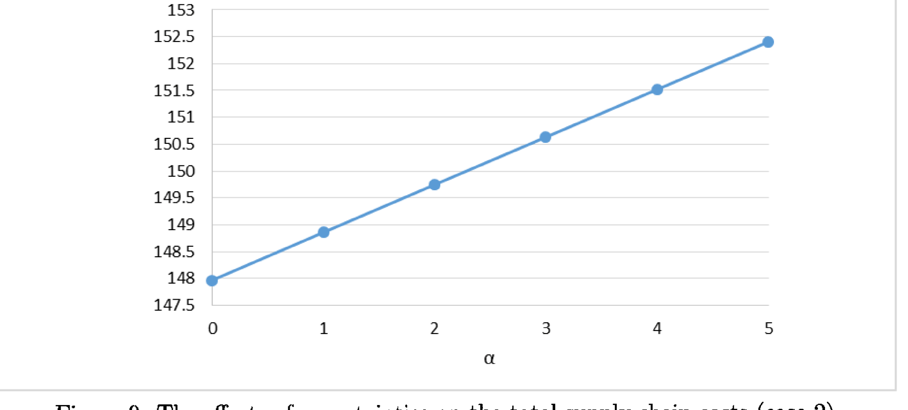

Figure 9: The effects of uncertainties on the total supply chain costs (case 2).

4 Conclusions

In the paper, we have developed a robust SCNE model which some players cannot know the exact value of other players' strategies. In addition, we have given some numerical experiments. For a future research, to consider the model with uncertainties in players' variables and demands such as [4] simultaneously is an interesting topic. Also numerical experiments with realistic parameters is an important issue.

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