数理解析研究所講究録
第2040巻 2017年 10-20

Elementary Triangular Partitioned Cellular Automata

Kenichi Morita
Hiroshima University

1 Introduction

A three-neighbor triangular partitioned cellular automaton (TPCA) is a CA such that cells are triangular, and each cell has three parts. A TPCA is called an *elementary TPCA* (ETPCA), if it is rotation-symmetric, and each part of a cell has only two states. The class of ETPCAs is one of the simplest subclasses of two-dimensional CAs, since the local function of each ETPCA is described by only four local rules. There are 256 ETPCAs as in the case of one-dimensional elementary CAs [13]. Among them there are 36 *reversible ETPCAs* (RETPCAs), and 9 *conservative* RETPCAs. A particular conservative RETPCA T_{0157}, where 0157 is an identification number in the class of EPCAs, was first investigated in [5], and its computational universality was shown. In [9], it was shown that 6 conservative RETPCAs are computationally universal, while the remaining three are non-universal. Hence, universality of all the conservative RETPCAs has been clarified. On the other hand, in [6], a non-conservative RETPCA T_{0347} is studied. In spite of its extreme simplicity, it shows quite interesting behavior like the Game of Life CA [3, 4]. In particular, a glider, which is a space-moving pattern, and glider guns exist in T_{0347}. Using gliders to represent signals, computational universality of T_{0347} is also proved.

In this paper, after giving basic definitions on ETPCAs, we present three kinds of "dualities," and classify 256 ETPCAs. By this, we obtain 82 equivalence classes of ETPCAs. We then give a survey on the past results. In particular, it is explained how computational universality of ETPCAs is shown.

2 Preliminaries

We give several definitions that are needed in the later sections.

2.1 Triangular partitioned cellular automaton

A three-neighbor *triangular partitioned cellular automaton* (TPCA) is a CA defined on the cellular space shown in Fig. 1. In a TPCA, each cell has three parts, and the next state of a cell is determined by the states of the adjacent parts of the three neighbor cells as shown in Fig. 2.

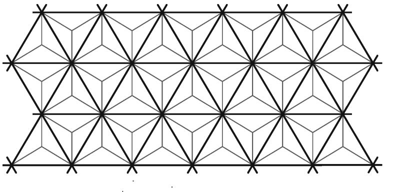

Figure 1: Cellular space of a three-neighbor TPCA.

All the cells of a TPCA are identical copies of a finite state machine, and each cell has three parts, i.e., the left, downward, and right parts, whose state sets are L, D and R, respectively. However, the directions of the cells are not the same, i.e., there are up-triangle cells, down-triangle cells.

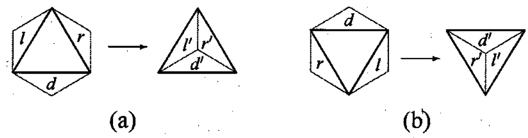

Figure 2: Pictorial representations of the local rule $f(l,d,r) = (l',d',r')$, where $(l,d,r),(l',d',r') \in L \times D \times R$. They are (a) for up-triangle cells, and (b) for down-triangle cells.

We now place cells of a TPCA on $\mathbb{Z}^2$ as shown in Fig. 3. We assume that if the coordinates of an up-triangle cell is (x,y), then $x+y$ must be even. It should be noted, if we define an TPCA on $\mathbb{Z}^2$, there arises a problem that the neighborhood is slightly non-uniform. Namely, for an up-triangle cell, its neighbors are the west, south and east adjacent cells (Fig. 2 (a)), while for a down-triangle cell, its neighbors are the east, north and west adjacent cells (Fig. 2 (b)). Although such non-uniformity can be dissolved by defining a TPCA on a Cayley graph, here we define a TPCA on $\mathbb{Z}^2$.

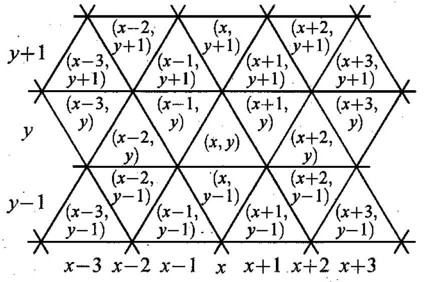

Figure 3: The x-y coordinates in the cellular space of TPCA.

Definition 1 *A* deterministic triangular partitioned cellular automaton (TPCA) *is a system defined by*

$$T = (\mathbb{Z}^2, (L,D,R), ((-1,0),(0,-1),(1,0)), ((1,0),(0,1),(-1,0)), f, (\#,\#,\#)).$$

Here, $\mathbb{Z}^2$ is the set of all two-dimensional points with integer coordinates at which cells are placed. Each cell has three parts, i.e., the left, downward and right parts, where L, D and R are non-empty finite set of states of these parts. The state set Q of each cell is thus given by $Q = L \times D \times R$. The triplet $((-1,0),(0,-1),(1,0))$ is a neighborhood *for up-triangle cells, and $((1,0),(0,1),(-1,0))$ is a neighborhood for down-triangle cells. The item $f : Q \to Q$ is a* local function, *and $(\#,\#,\#) \in Q$ is a* quiescent state *that satisfies $f(\#,\#,\#) = (\#,\#,\#)$. We also allow a TPCA that has no quiescent state.*

If $f(l,d,r) = (l',d',r')$ holds for $(l,d,r),(l',d',r') \in Q$, then this relation is called a *local rule* of the TPCA T (Fig. 2). Configurations of a TPCA, and the global function induced by the local function are defined as below.

Definition 2 *Let T be a TPCA. A* configuration *of T is a function $\alpha : \mathbb{Z}^2 \to Q$. The set of all configurations of T is denoted by* Conf(T), *i.e.,* $\mathrm{Conf}(T) = \{\alpha \mid \alpha : \mathbb{Z}^2 \to Q\}$. *Let $\mathrm{pr}_L : Q \to L$ be the* projection function *such that $\mathrm{pr}_L(l,d,r) = l$ for all $(l,d,r) \in Q$. The projection functions $\mathrm{pr}_D : Q \to D$ and $\mathrm{pr}_R : Q \to R$ are also defined similarly. The* global function $F : \mathrm{Conf}(T) \to \mathrm{Conf}(T)$ *of T is defined as the one that satisfies the following condition.*

$$\forall \alpha \in \mathrm{Conf}(T), \forall (x,y) \in \mathbb{Z}^2 :$$
$$F(\alpha)(x,y) = \begin{cases} f(\mathrm{pr}_L(\alpha(x-1,y)), \mathrm{pr}_D(\alpha(x,y-1)), \mathrm{pr}_R(\alpha(x+1,y))) & \text{if } x+y \text{ is even} \\ f(\mathrm{pr}_L(\alpha(x+1,y)), \mathrm{pr}_D(\alpha(x,y+1)), \mathrm{pr}_R(\alpha(x-1,y))) & \text{if } x+y \text{ is odd} \end{cases}$$

From this definition, we can see that the next state of the up-triangle cell is determined by the present states of the left part of the west neighbor cell, the downward part of the south neighbor cell, and the right part of the east neighbor cell. On the other hand, the next state of the down-triangle cell is determined by the present states of the left part of the east neighbor cell, the downward part of the north neighbor cell, and the right part of the west neighbor cell. Therefore, for a local rule $f(l,d,r) = (l',d',r')$, there are two kinds of pictorial representations as shown in Fig. 2 (a) and (b). Namely, Fig. 2 (a) is for up-triangle cells, while Fig. 2 (b) is for down-triangle cells.

In [11], it is shown injectivity of the global function is equivalent to that of the local function in one-dimensional PCAs. The following lemma is proved in a similar manner as this.

Lemma 1 *[10] Let T be a TPCA, f be its local function, and F be its global function. Then, F is injective iff f is injective.*

Definition 3 *Let T be a TPCA. The TPCA T is called* reversible *if its local (or equivalently global) function is injective.*

Definition 4 *Let* $T = (\mathbb{Z}^2,(L,D,R),((-1,0),(0,-1),(1,0)),((1,0),(0,1),(-1,0)),f,(\#,\#,\#))$ *be a TPCA. The TPCA T is called* rotation-symmetric *(or* isotropic*) if the conditions (1) and (2) holds.*

(1) L=D=R

(2) $\forall (l,d,r),(l',d',r') \in L\times D\times R:\ f(l,d,r) = (l',d',r') \Rightarrow f(d,r,l) = (d',r,l')$

2.2 Elementary triangular partitioned cellular automaton (ETPCAs)

Definition 5 *Let* $T = (\mathbb{Z}^2,(L,D,R),((-1,0),(0,-1),(1,0)),((1,0),(0,1),(-1,0)),f,(0,0,0))$ *be a TPCA. It is called an* elementary triangular partitioned cellular automaton *(ETPCA), if* $L = D = R = \{0,1\}$*, and it is rotation-symmetric.*

The set of states of a cell of an ETPCA is $L\times D\times R = \{0,1\}^3$, and thus a cell has eight states. When drawing figures of T's local rules and configurations, we indicate the states 0 and 1 of each part of by a blank and a particle (i.e., •), respectively.

Since ETPCA is rotation-symmetric, and each part of a cell has the state set $\{0,1\}$, its local function is defined by only four local rules. Hence, an ETPCA can be specified by a four-digit number $wxyz$, as shown in Fig. 4, such that $w,z \in \{0,7\}$ and $x,y \in \{0,1,\ldots,7\}$. Thus, there are 256 ETPCAs. Note that w and z must be 0 or 7 because an ETPCA is deterministic and rotation-symmetric. The ETPCA with the number $wxyz$ is denoted by T_{wxyz}.

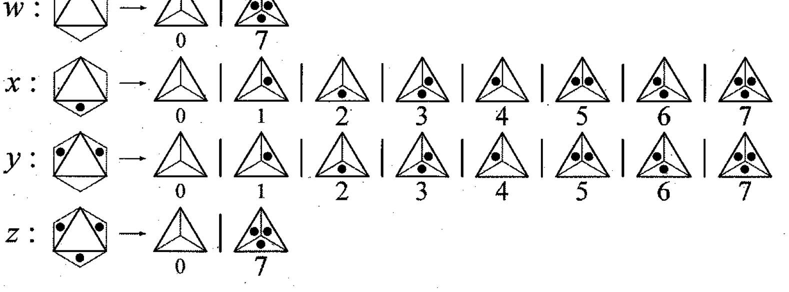

Figure 4: Representing an ETPCA by a four-digit number $wxyz$, where $w,z \in \{0,7\}$ and $x,y \in \{0,1,\ldots,7\}$. Vertical bars indicate alternatives of a right-hand side of a rule.

A *reversible ETPCA* (RETPCA) is an ETPCA whose local function is injective (Definition 3). Thus, it is easy to show the following.

Lemma 2 *Let T_{wxyz} be an ETPCA. It is reversible iff the following conditions (1) and (2) hold.*
(1) $(w,z) \in \{(0,7),(7,0)\}$
(2) $(x,y) \in \{1,2,4\} \times \{3,5,6\} \cup \{3,5,6\} \times \{1,2,4\}$

A conservative ETPCA is a one such that the total number of particles (i.e., •'s) is conserved in each local rule. Hence, it is defined as follows.

Definition 6 *Let T_{wxyz} be an ETPCA. It is called a* conservative ETPCA *if the following condition holds.*

$$w = 0 \wedge x \in \{1,2,4\} \wedge y \in \{3,5,6\} \wedge z = 7$$

From Lemma 2 and Definition 6, it is clear the following holds.

Lemma 3 *Let T be an ETPCA. If T is conservative, then it is reversible.*

By the above lemma, conservative ETPCAs are called *conservative RETPCAs* hereafter. We can see that there are 36 RETPCAS (by Lemma 2), and among them there are nine conservative RETPCAs (by Definition 6).

2.3 Dualities in ETPCAs

As seen above, there are 256 ETPCAs. However, there are some "equivalent" ETPCAs, and thus the number of essentially different ETPCAs is much smaller. Here, we introduce three kinds of dualities, and classify the ETPCAs based on them [10].

2.3.1 Duality under reflection

Definition 7 *Let T and $\hat{T}$ be ETPCAs, and f and $\hat{f}$ be their local functions. We say T and $\hat{T}$ are* dual under reflection, *if the following holds, and it is written as $T \underset{\text{refl}}{\longleftrightarrow} \hat{T}$.*

$$\forall (l,d,r),(l',d',r') \in \{0,1\}^3 : f(l,d,r) = (l',d',r') \Leftrightarrow \hat{f}(r,d,l) = (r',d',l')$$

By this definition, we can see that the local rules of $\hat{T}$ are the mirror images of those of T. Therefore, an evolution process of T's configurations is simulated in a straightforward manner by the mirror images of the T's configurations in $\hat{T}$. For example, $T_{0137} \underset{\text{refl}}{\longleftrightarrow} T_{0467}$ holds (Fig. 5).

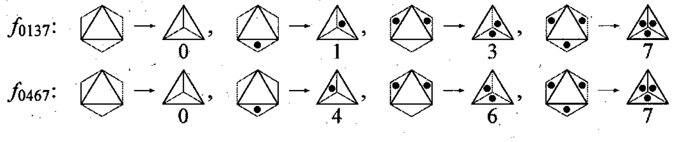

Figure 5: The local functions of T_{0137} and T_{0467} that are dual under reflection

2.3.2 Duality under complementation

For $x \in \{0,1\}$, let $\bar{x}$ denote $1-x$, i.e., the complement of x.

Definition 8 *Let T and $\overline{T}$ be ETPCAs, and f and $\overline{f}$ be their local functions. We say T and $\overline{T}$ are* dual under complementation, *if the following holds, and it is written as $T \underset{\text{comp}}{\longleftrightarrow} \overline{T}$.*

$$\forall (l,d,r),(l',d',r') \in \{0,1\}^3 : f(l,d,r) = (l',d',r') \Leftrightarrow \overline{f}(\bar{l},\bar{d},\bar{r}) = (\overline{l'},\overline{d'},\overline{r'}))$$

By this, we can see that the local rules of $\overline{T}$ are the 0-1 exchange (i.e., taking their complements) of those of T. Therefore, an evolution process of T's configurations is simulated in a straightforward manner by the complemented images of the T's configurations in $\overline{T}$. For example, $T_{0157} \underset{\text{comp}}{\longleftrightarrow} T_{0267}$ (Fig. 6).

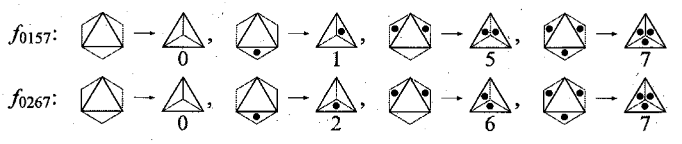

Figure 6: The local functions of T_{0157} and T_{0267} that are dual under complementation

2.3.3 Duality under odd-step complementation

Definition 9 *Let T be an ETPCA such that its local function f satisfies the following.*
(1) $\forall(l,d,r),(l',d',r') \in \{0,1\}^3 : f(l,d,r) = (l',d',r') \Rightarrow f(\overline{l},\overline{d},\overline{r}) = (\overline{l'},\overline{d'},\overline{r'})$
Let $\tilde{T}$ be another ETPCA, and $\tilde{f}$ be its local function. We say T and $\tilde{T}$ are dual under odd-step complementation, *if the following holds, and it is written as $T \underset{\text{osc}}{\longleftrightarrow} \tilde{T}$.*

$$\forall(l,d,r),(l',d',r') \in \{0,1\}^3 : f(l,d,r) = (l',d',r') \Leftrightarrow \tilde{f}(l,d,r) = (\overline{l'},\overline{d'},\overline{r'})$$

Since the ETPCA T satisfies the condition (1) in Definition 9, we can see that for each local rule $f(l,d,r) = (l',d',r')$ of T, there are two local rules $\tilde{f}(l,d,r) = (\overline{l'},\overline{d'},\overline{r'})$ and $\tilde{f}(\overline{l},\overline{d},\overline{r}) = (l',d',r')$ of $\tilde{T}$ (hence $\tilde{T}$ also satisfies (1)). Let F and $\tilde{F}$ be the global function of T and $\tilde{T}$, respectively. If the initial configuration of T is $\alpha : \mathbb{Z}^2 \to \{0,1\}^3$, then we assume α is also given to $\tilde{T}$ as its initial configuration. Since there is a local rule $\tilde{f}(l,d,r) = (\overline{l'},\overline{d'},\overline{r'})$ for each $f(l,d,r) = (l',d',r')$, the configuration $\tilde{F}(\alpha)$ is the complement of the configuration $F(\alpha)$. Furthermore, since there is a local rule $\tilde{f}(\overline{l},\overline{d},\overline{r}) = (l',d',r')$ for each $f(l,d,r) = (l',d',r')$, the configuration $\tilde{F}^2(\alpha)$ is the same as $F^2(\alpha)$. In this way, at an even step $\tilde{T}$ gives the same configuration as T, while at an odd step $\tilde{T}$ gives the complemented configuration of T. For example, $T_{0347} \underset{\text{osc}}{\longleftrightarrow} T_{7430}$ holds (Fig. 7). Figure 8 shows an example of their evolution processes.

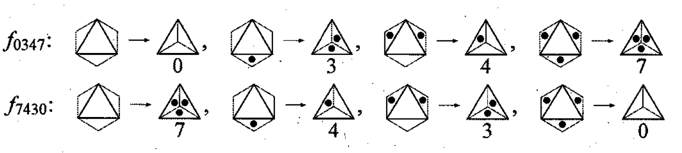

Figure 7: The local functions of T_{0347} and T_{7430}, which are dual under odd-step complementation

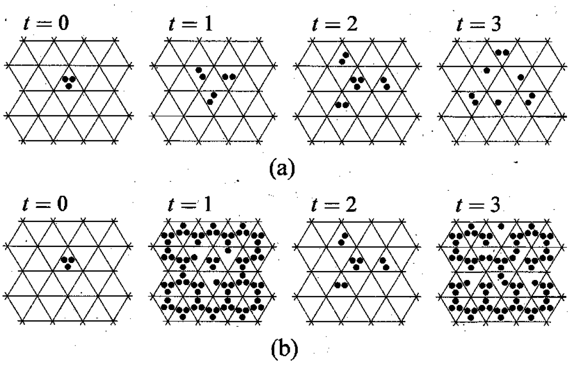

Figure 8: Example of evolution processes in (a) T_{0347}, and (b) T_{7430}, which are dual under odd-step complementation

Note that, in Definition 9, the ETPCA T must satisfy the condition (1). Therefore, the relation $\underset{\text{osc}}{\longleftrightarrow}$ is defined on the ETPCAs of the form T_{wxyz} such that $w+z=7$ and $x+y=7$. Hence, only the following

16 ETPCAs have their dual counterparts under odd-step complementation.

$$T_{0077} \underset{\text{osc}}{\longleftrightarrow} T_{7700},\quad T_{0167} \underset{\text{osc}}{\longleftrightarrow} T_{7610},\quad T_{0257} \underset{\text{osc}}{\longleftrightarrow} T_{7520},$$
$$T_{0347} \underset{\text{osc}}{\longleftrightarrow} T_{7430},\quad T_{0437} \underset{\text{osc}}{\longleftrightarrow} T_{7340},\quad T_{0527} \underset{\text{osc}}{\longleftrightarrow} T_{7250},$$
$$T_{0617} \underset{\text{osc}}{\longleftrightarrow} T_{7160},\quad T_{0707} \underset{\text{osc}}{\longleftrightarrow} T_{7070}$$

2.3.4 Equivalence classes of ETPCAs

If ETPCAs T and T' are dual under reflection, complementation, or odd-step complementation, then they can be regarded as essentially the same ETPCAs. Here, we classify the 256 ETPCAs into equivalence classes based on the three dualities. We define the relation $\longleftrightarrow$ as follows: For any ETPCAs T and T',

$$T \longleftrightarrow T' \Leftrightarrow (T \underset{\text{refl}}{\longleftrightarrow} T' \vee T \underset{\text{comp}}{\longleftrightarrow} T' \vee T \underset{\text{osc}}{\longleftrightarrow} T')$$

holds. Now, let $\longleftrightarrow^*$ be the reflexive and transitive closure of $\longleftrightarrow$. Then, $\longleftrightarrow^*$ is an equivalence relation, since $\longleftrightarrow$ is symmetric. By this, 256 ETPCAs are classified into 82 equivalence classes. Table 1 shows the classification result.

Table 1: Total numbers and numbers of equivalence classes of ETPCAs, RETPCAs, and conservative RETPCAs

	Total number	Equivalence classes
ETPCAs	256	82
RETPCAs	36	12
Conservative RETPCAs	9	4

3 Conservative RETPCAs

There are nine conservative RETPCAs (Definition 6). We have the following four equivalence classes under the relation $\longleftrightarrow^*$.

$$\{T_{0157}, T_{0457}, T_{0237}, T_{0267}\}, \{T_{0137}, T_{0467}\}, \{T_{0167}, T_{0437}\}, \{T_{0257}\}$$

It has been proved that the RETPCAs in the first two classes are computationally universal, while those in the last two classes are non-universal [5, 9].

To prove computational universality of a reversible CA, it is sufficient to show that any reversible logic circuit composed of switch gates (Fig. 9 (a)), inverse switch gates (Fig. 9 (b)), and delay elements can be simulated in it (Lemma 8).

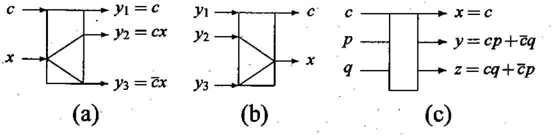

Figure 9: (a) Switch gate. (b) Inverse switch gate, where $c = y_1$ and $x = y_2 + y_3$ under the assumption $(y_2 \to y_1) \wedge (y_3 \to \overline{y_1})$. (c) Fredkin gate.

Lemma 8 can be derived, e.g., in the following way. First, a Fredkin gate (Fig. 9 (c)) can be constructed out of switch gates and inverse switch gates (Lemma 4). Second, any *reversible sequential machine* (RSM), in particular, a rotary element (RE), which is a 2-state 4-symbol RSM, is composed only of Fredkin gates and delay elements (Lemma 5). Third, any *reversible Turing machine* is constructed out of REs (Lemma 6). Finally, any (irreversible) Turing machine is simulated by a reversible one (Lemma 7). Thus, Lemma 8 follows. Note that the circuit that realizes a reversible Turing machine constructed by this method becomes an infinite (but ultimately periodic) circuit.

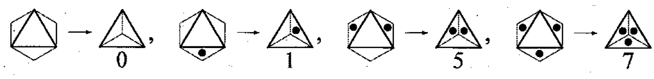

Figure 10: Local function of T_{0157}

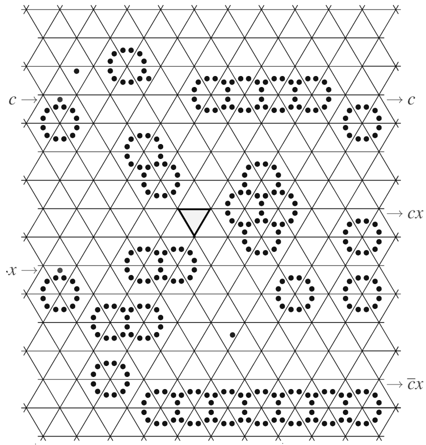

Figure 11: Switch gate module implemented in T_{0157} [10]. Two signals from c and x interact at the cell indicated by bold lines.

Lemma 4 [2] *A Fredkin gate can be simulated by a circuit composed of switch gates and inverse switch gates, which produces no garbage signals.*

Lemma 5 [10] *Any RSM (in particular RE) can be simulated by a circuit composed of Fredkin gates and delay elements, which produces no garbage signals.*

Lemma 6 [10] *Any reversible Turing machine can be simulated by a garbage-less circuit composed only of REs.*

Lemma 7 [1] *Any (irreversible) Turing machine can be simulated by a garbage-less reversible Turing machine.*

Lemma 8 *A reversible CA is computationally universal, if any circuit composed of switch gates, inverse switch gates, and delay elements is simulated in it.*

First, consider the conservative RETPCA T_{0157} (Fig. 10). Its universality was first proved in [5] by showing that switch gate and inverse switch gate modules, delay elements, and signal crossing modules are implemented in its cellular space. Hence, by Lemma 8, it is computationally universal. Figure 11 shows a switch gate module in T_{0157}. There, a single particle works as a signal, which moves along a wire. Two signals coming from the input ports c and x interact at the cell indicated by bold lines.

Second, consider the RETPCA T_{0137} (Fig. 12). In [9] it is shown that switch gate and inverse switch gate modules, delay elements, and signal crossing modules are implemented in its cellular space. Hence, it is also computationally universal. Figure 13 gives a switch gate module in T_{0137}.

Next, consider T_{0257}. From Fig. 14, we can interpret that in each local rule, all the coming particles make U-turns. Hence, *every* configuration of T_{0257} is of period 2. Therefore, it is trivially non-universal.

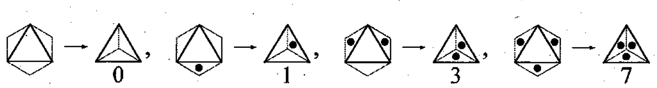

Figure 12: Local function of T_{0137}

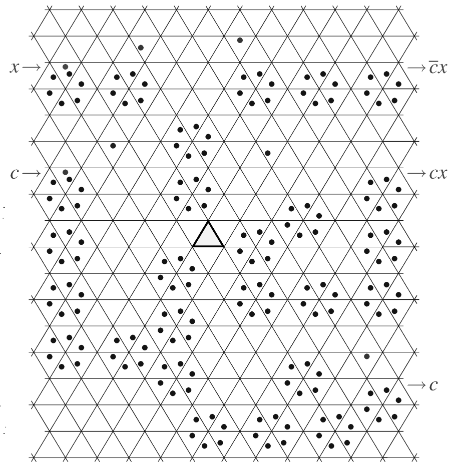

Figure 13: Switch gate module implemented in T_{0137} [9].

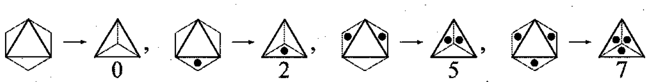

Figure 14: Local function of T_{0257}

Likewise in T_{0167} (T_{0437}, respectively), all the coming particles make right-turns (left-turns). Hence, *every* configuration is of period 6, and thus T_{0167} and T_{0437} are non-universal.

By above, and by the dualities, we have the following.

Theorem 1 *[5, 9] The conservative RETPCAs T_{0157}, T_{0457}, T_{0237}, T_{0267}, T_{0137}, and T_{0467} are computationally universal. On the other hand, T_{0167}, T_{0437}, and T_{0257} are non-universal.*

4 Non-conservative RETPCA T_{0347}

Here, we focus on a specific non-conservative RETPCA T_{0347} [6]. Its local function is given in Fig. 15. In spite of its extreme simplicity of the local function, it exhibits quite interesting behavior similar the case of the Game of Life CA [3, 4].

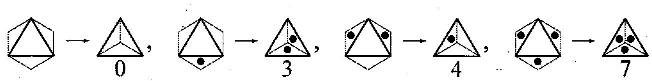

Figure 15: Local function of T_{0347}

There are several useful patterns in T_{0347}. The most useful pattern in T_{0347} is a *glider* shown in Fig. 16 (a). It swims in the cellular space like a fish or an eel. It travels a unit distance, the side-length of a triangle, in 6 steps. By rotating it appropriately, it can move in any of the six directions. A *block* is a pattern shown in Fig. 16 (b). It is a stable pattern. Combining several blocks, right-turn, U-turn, and left-turn of a glider will be implemented. A *fin* is a pattern that simply rotates with period 6 (Fig. 16 (c)). It can also travel around a block, or a sequence of blocks. A *rotator* is a pattern shown in Fig. 16 (d). Like a fin, it rotates around some point, and its period is 42.

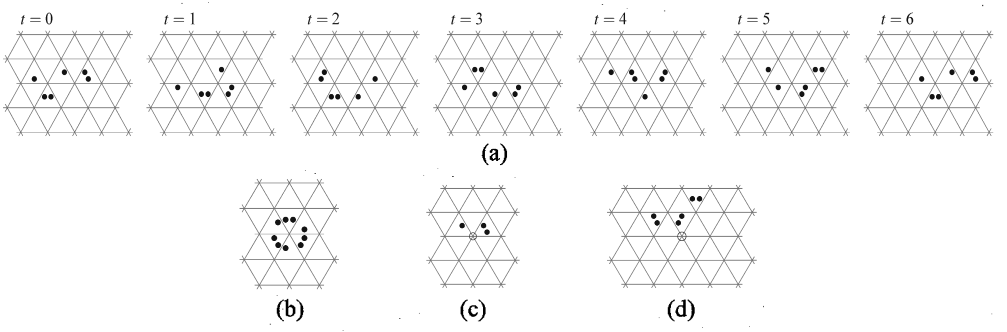

Figure 16: Useful patterns in T_{0347}. (a) Glider, (b) block, (c) fin, and (d) rotator. A fin and a rotator go around the point ○.

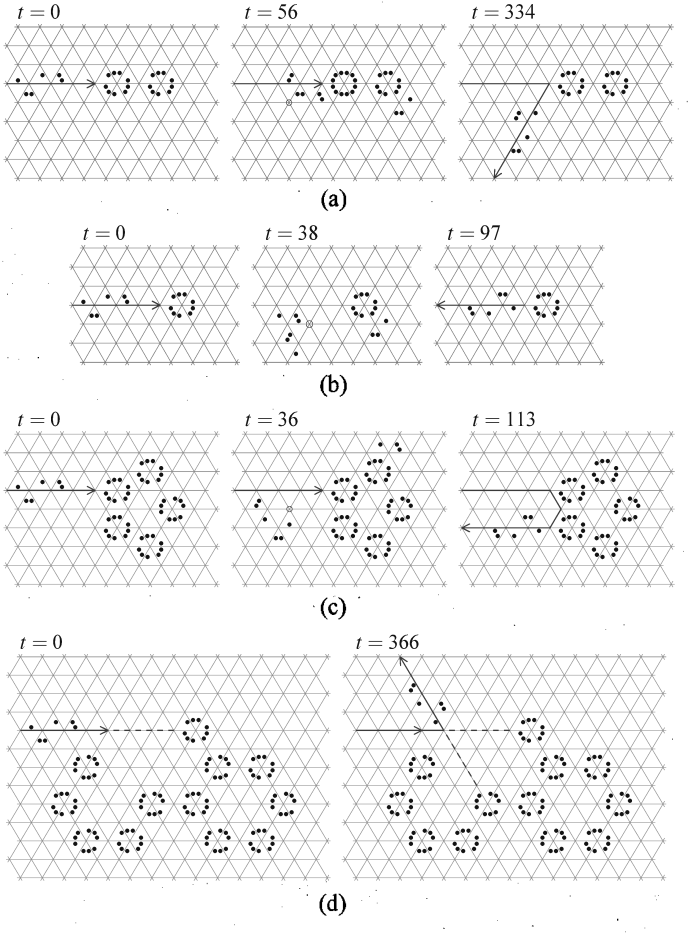

Figure 17: Turn modules in T_{0347} [6]. (a) Right-turn module composed of two blocks, (b) backward-turn module, (c) U-turn module, and (d) left-turn module.

We can see that the move direction of a glider is controlled by such interactions of these objects. We first collide a glider with two blocks (Fig. 17 (a)). Then, the glider is split into a rotator and a fin ($t = 56$). The fin travels around the blocks three times without interacting with the rotator. At the end of the fourth round, they meet to form a glider, which goes to the south-west direction ($t = 334$). Hence, two blocks act as a 120°-*right-turn module*. It is also possible to make a right-turn module with a different delay time using three or five blocks. If we collide a glider with a single block as shown in Fig. 17 (b), then the glider makes backward turn. Hence, a single block acts as a *backward-turn module*. Figure 17 (c) shows a *U-turn module*. A *left-turn module* is given in Fig. 17 (d).

We now show computational universality of T_{0347}. It is possible to implement a switch gate and an inverse switch gate in T_{0347} using gliders as signals. The operation of a switch gate itself is realized by colliding two gliders as shown in Fig. 18. Using many turn modules to adjust the collision timing and the directions of the input gliders, we can construct a *switch gate* as shown in Fig. 19.

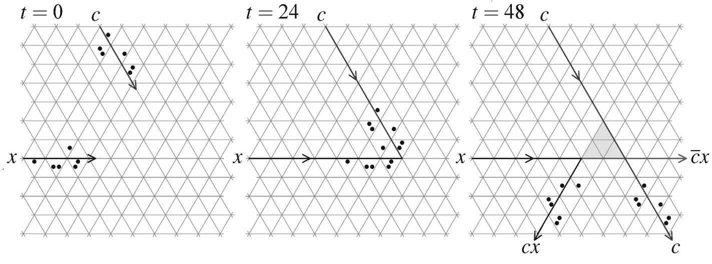

Figure 18: Switch operation realized by collision of two gliders

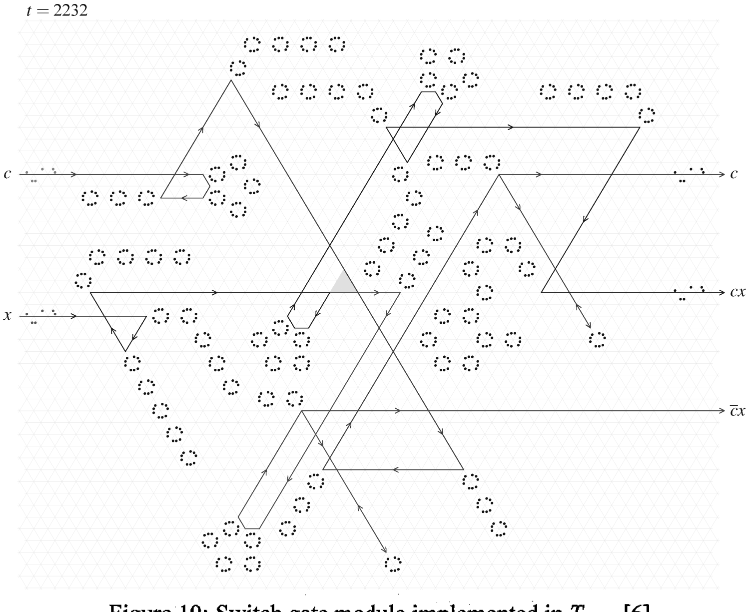

Figure 19: Switch gate module implemented in T_{0347} [6]

By above, and by the dualities, we have the following.

Theorem 2 *[6] The non-conservative RETPCAs* T_{0347}, T_{0617}, T_{7430} *and* T_{7160} *are computationally universal.*

5 Concluding remarks

In this paper, we studied the class of ETPCAs. So far, we had ten computationally universal RETPCAs. However, there are 256 ETPCAs in total, and thus investigation of other universal or interesting (R)ETPCAs is left for the future study.

Generally, it is difficult to follow evolution processes of ETPCAs using only paper and pencil. In [7, 8] evolution processes of configurations of T_{0157}, T_{0267}, T_{0137}, and T_{0347} can be seen by simulation movies. As for T_{0347}, an emulator file (Reversible_World.zip) on the CA simulator *Golly* [12] is also available at the Rule Table Repository of Golly (http://github.com/GollyGang/ruletablerepository).

Acknowledgement. This work was supported by JSPS KAKENHI Grant Number 15K00019.

References

[1] Bennett, C.H.: Logical reversibility of computation. IBM J. Res. Dev. **17**, 525–532 (1973). DOI 10.1147/rd.176.0525

[2] Fredkin, E., Toffoli, T.: Conservative logic. Int. J. Theoret. Phys. **21**, 219–253 (1982). DOI 10.1007/BF01857727

[3] Gardner, M.: Mathematical games: The fantastic combinations of John Conway's new solitaire game "life". Sci. Am. **223 (4)**, 120–123 (1970)

[4] Gardner, M.: Mathematical games: On cellular automata, self-reproduction, the Garden of Eden and the game "life". Sci. Am. **224 (2)**, 112–117 (1971)

[5] Imai, K., Morita, K.: A computation-universal two-dimensional 8-state triangular reversible cellular automaton. Theoret. Comput. Sci. **231**, 181–191 (2000). DOI 10.1016/S0304-3975(99)00099-7

[6] Morita, K.: An 8-state simple reversible triangular cellular automaton that exhibits complex behavior. In: AUTOMATA 2016 (eds. M. Cook, T. Neary) LNCS 9664, pp. 170–184 (2016). DOI 10.1007/978-3-319-39300-1_14

[7] Morita, K.: Reversible and conservative elementary triangular partitioned cellular automata (slides with simulation movies). Hiroshima University Institutional Repository, http://ir.lib.hiroshima-u.ac.jp/00039997 (2016)

[8] Morita, K.: A reversible elementary triangular partitioned cellular automaton that exhibits complex behavior (slides with simulation movies). Hiroshima University Institutional Repository, http://ir.lib.hiroshima-u.ac.jp/00039321 (2016)

[9] Morita, K.: Universality of 8-state reversible and conservative triangular partitioned cellular automaton. In: ACRI 2016 (eds. S. El Yacoubi, et al.) LNCS 9863, pp. 45–54 (2016). DOI 10.1007/978-3-319-44365-2_5

[10] Morita, K.: Theory of Reversible Computing. Springer (in press)

[11] Morita, K., Harao, M.: Computation universality of one-dimensional reversible (injective) cellular automata. Trans. IEICE Japan **E72**, 758–762 (1989)

[12] Trevorrow, A., Rokicki, T., et al.: Golly: an open source, cross-platform application for exploring Conway's Game of Life and other cellular automata. http://golly.sourceforge.net/ (2005)

[13] Wolfram, S.: A New Kind of Science. Wolfram Media Inc.(2002)

Hiroshima University
Higashi-Hiroshima 739-8527 Japan
e-mail: km@hiroshima-u.ac.jp

広島大学　森田憲一