数理解析研究所講究録
第2051巻 2017年 30-31

On the Flat Folding of Origami

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1. Introduction

In Japan, Origami is one of the most popular toys with a paper for children. Hence, we have been very familiar to Origami.

On the other hand, many studies of Origami have been mathematically researched. In this paper, we introduce some results on the flat folding of Origami. In particular, we make mention of single vertex fold and multiple vertex folds ([1], [2]). Moreover, we touch upon new study about mathematical model of flat origami ([3])

2. Single Vertex Fold

We start with the simplest case. The simplest case for flat origami folds is single vertex fold. We define a single vertex fold to be a creases pattern with only one vertex in the interior of the paper and all crease lines incident to it.

There are two famous theorems for a single vertex fold as follows.

Theorem 2.1 *(Maekawa and Justin (1987)) Let M be the number of mountain creases and V be the number of valley creases adjacent to a vertex in a single vertex fold. Then it holds*

$$M-V=\pm 2.$$

Theorem 2.2 *(Kawasaki(1989), Justin(1989)) Let v be a vertex of degree $2n$ in a single vertex fold and let $\alpha_1, \alpha_2, \cdots, \alpha_{2n}$ be the consecutive angles between the creases. Then v is a flat vertex fold if and only if*

$$\alpha_1-\alpha_2+\alpha_3-\alpha_4+\cdots-\alpha_{2n}=0.$$

3. Multiple Vertex Folds

Next, we describe some results of multiple vertex folds. In our talk at this workshop, we mention the proof of Theorem 3.1.

Theorem 3.1 *(Hull (1994), [2]) Given a multiple vertex flat-fold, let M (resp. V) denote the number of mountain (resp. valley) creases, U (resp. D) denote the number of up (resp. down) vertices, and M_i (resp. V_i) denote the number of interior mountain (resp. valley) creases. Then it holds*

$$M - V = 2U - 2D - M_i + V_i.$$

Theorem 3.2 *(Kawasaki(1997), Justin(1997)) Let us denote $R(m_i)$ to be the reflection in the plain, along a line m_i. Given a multiple vertex fold, let γ be any closed, vertex-avoiding curve drawn on the crease pattern which crosses crease lines $m_1, m_2, \cdots, m_n$, in order. Then, if the crease pattern can fold flat, we have*

$$R(m_1) \times R(m_2) \times \cdots \times R(m_n) = I$$

Where I denotes the identity transformation.

4. Mathematical Model of Flat Origami

Finally, we touch upon hot study about mathematical model of flat origami ([3]).

A is a compact set having the interior, and f is a fold line. A is divided into the parts A_1, A_2 by f. Let us denote R_f to be the reflection in the plain, along a line f. Then, all points $x \in A$ is mapped as follows:

$$(f)x = \begin{cases} R_f(x) & (x \in A_1) \\ x & (x \in A_2) \end{cases}$$

For A_1, A_2, we define $A_2 < A_1$ using a partial order $<$.

Using such a partial order, Nosaka try to describe mathematically several flat foldings of Origami, and give strictly mathematical proofs of above theorems.

References

[1] Thomas C. Hull (2002); The Combinatorics of Flat Folds: A Survey *Origami3: Third International Meeting of Origami Science, Mathematics, and Education*, Thomas Hull, Editor, A K Peters, Natick, Massachusetts, pp. 29–38.

[2] Thomas C. Hull (1994); On the mathematical of flat origamis *Congressus Numerantium*, **100**, pp. 215–224.

[3] Kosuke Nosaka (2016); Mathematical model of flat origami's arts, *Poster Session, International Conference on Mathematical Modeling and Application 2016, Origami-Based Modeling and Analysis*, Meiji University, November, 9–12, 2016.