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ADDENDUM TO "AN ABELIAN QUOTIENT OF THE SYMPLECTIC DERIVATION LIE ALGEBRA OF THE FREE LIE ALGEBRA"

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ABSTRACT. In our previous paper, we gave an explicit description of an abelian quotient of the symplectic derivation Lie algebra $\mathfrak{h}_{g,1}$ of the free Lie algebra generated by the fundamental representation of $\mathrm{Sp}(2g,\mathbb{Q})$. This abelian quotient is 1-dimensional and lives in weight 12 part. Here we show that the corresponding quotient map $C : \mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ factors through the Enomoto-Satoh map ES_{12} of degree 12.

1. INTRODUCTION

This article is an addendum to our previous paper [9].

Let H be the fundamental representation over $\mathbb{Q}$ of the symplectic group $\mathrm{Sp}(2g,\mathbb{Q})$. We consider the symplectic derivation Lie algebra $\mathfrak{h}_{g,1}$ of the free Lie algebra $\mathcal{L}(H)$ generated by H. It was shown by Kontsevich [3, 4] that the Lie algebra homology of the Lie algebra $\mathfrak{h}_{\infty,1} := \lim_{g\to\infty} \mathfrak{h}_{g,1}$ obtained by the stabilization is isomorphic to the free graded commutative algebra generated by the stable homology of the Lie algebra $\mathfrak{sp}(2h,\mathbb{Q})$ of $\mathrm{Sp}(2h,\mathbb{Q})$ for sufficiently large h and the totality of the cohomology of the outer automorphism groups $\mathrm{Out}\, F_n$ of free groups of rank $n \geq 2$.

Recently, Bartholdi determined the rational homology of $\mathrm{Out}\, F_7$ with the aid of computers. The result is

$$H_i(\mathrm{Out}\, F_7;\mathbb{Q}) \cong \begin{cases} \mathbb{Q} & (i = 0, 8, 11) \\ 0 & (\text{otherwise}) \end{cases}.$$

By applying Kontsevich's theorem to the result $H_{11}(\mathrm{Out}\, F_7;\mathbb{Q}) \cong \mathbb{Q}$, we have $H_1(\mathfrak{h}_{\infty,1})_{12} \cong \mathbb{Q}$, where $H_1(\mathfrak{h}_{\infty,1})_{12}$ is the weight 12 part of the abelianization of the graded Lie algebra $\mathfrak{h}_{\infty,1}$. In [9], we gave an explicit description of this isomorphism purely in terms of the Lie algebra $\mathfrak{h}_{\infty,1}$. More precisely, we derived a formula of a cocycle $C : \mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ inducing the isomorphism $H_1(\mathfrak{h}_{g,1})_{12} \cong \mathbb{Q}$ for $g \geq 8$. In the paper [6] by Gwénaël Massuyeau and the second named author, the cocycle C is used to construct an invariant of a certain class of sutured 3-manifolds.

One problem regarding the cocycle C is that the present formula for C is huge. In fact, it consists of the linear combination of 647 terms (multiple contractions). Another problem is to understand the meaning of our cocycle C. To cope with these problems, in the same paper, we mentioned the following theorem without a proof.

Theorem 1.1 ([9, Theorem 5.1]). *For $g \geq 6$, the cocycle $C : \mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ factors through* $\mathrm{Im}\, ES_{12}$.

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In the statement, ES_{12} denotes the degree 12 part of the map defined by Enomoto and Satoh in [2]. This map, which we call the *Enomoto-Satoh map* here, is used to understand the structure of the Lie subalgebra of $\mathfrak{h}_{g,1}$ generated by the degree 1 part. The purpose of this article it to give a proof of Theorem 1.1. Our proof is aided by computers.

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2. The symplectic derivation Lie algebra of the free Lie algebra

Following our previous paper [9], we recall the notations used in this article.

The fundamental representation H of $\mathrm{Sp}(2g,\mathbb{Q})$ is $2g$-dimensional and has a natural non-degenerate anti-symmetric bilinear form

$$\mu : H \otimes H \longrightarrow \mathbb{Q}.$$

We fix a symplectic basis $\{a_1, b_1, \ldots, a_g, b_g\}$ of H with respect to μ. That is, they satisfy

$$\mu(a_i, a_j) = \mu(b_i, b_j) = 0, \qquad \mu(a_i, b_j) = \delta_{i,j}$$

for any $1 \leq i, j \leq g$. Let

$$\mathfrak{h}_{g,1} = \bigoplus_{k=0}^{\infty} \mathfrak{h}_{g,1}(k)$$

be the Lie algebra consisting of symplectic derivations of the free Lie algebra

$$\mathcal{L}(H) = \bigoplus_{i=1}^{\infty} \mathcal{L}_i(H)$$

generated by H. $\mathfrak{h}_{g,1}$ is a graded Lie algebra and the degree k part $\mathfrak{h}_{g,1}(k)$ is given by

$$\mathfrak{h}_{g,1}(k) = \mathrm{Ker}\left(H \otimes \mathcal{L}_{k+1}(H) \xrightarrow{[\cdot,\cdot]} \mathcal{L}_{k+2}(H)\right).$$

The symplectic group $\mathrm{Sp}(2g,\mathbb{Q})$ acts naturally on each $\mathfrak{h}_{g,1}(k)$. This action is the restriction of the diagonal actions on $H \otimes \mathcal{L}_{k+1}(H) \subset H \otimes H^{\otimes(k+1)}$, and therefore it is compatible with the stabilization $\mathfrak{h}_{g,1} \hookrightarrow \mathfrak{h}_{g+1,1}$.

Now we concern with the abelianization $H_1(\mathfrak{h}_{g,1}) = \mathfrak{h}_{g,1}/[\mathfrak{h}_{g,1}, \mathfrak{h}_{g,1}]$ of the Lie algebra $\mathfrak{h}_{g,1}$. The grading of $\mathfrak{h}_{g,1}$ gives a direct sum decomposition

$$H_1(\mathfrak{h}_{g,1}) = \bigoplus_{w=0}^{\infty} H_1(\mathfrak{h}_{g,1})_w$$

with

$$H_1(\mathfrak{h}_{g,1})_w := \mathfrak{h}_{g,1}(w) \Big/ \sum_{i=0}^{w} [\mathfrak{h}_{g,1}(i), \mathfrak{h}_{g,1}(w-i)]$$

called the *weight w part*. By a technical reason, we consider the Lie ideal

$$\mathfrak{h}_{g,1}^{+} = \bigoplus_{k=1}^{\infty} \mathfrak{h}_{g,1}(k)$$

of the *positive* degree part. The spaces $H_1(\mathfrak{h}_{g,1})_w$ and $H_1(\mathfrak{h}_{g,1}^+)_w$ are both $\mathrm{Sp}(2g,\mathbb{Q})$-modules and have a relationship (see [7, Section 2], for example) that

$$H_1(\mathfrak{h}_{g,1})_w \cong H_1(\mathfrak{h}_{g,1}^+)_w^{\mathrm{Sp}} = \mathfrak{h}_{g,1}(w)^{\mathrm{Sp}} \Big/ \left(\sum_{i=1}^{w-1}[\mathfrak{h}_{g,1}(i), \mathfrak{h}_{g,1}(w-i)]\right)^{\mathrm{Sp}}$$

for any $w \geq 1$. Here, for an $\mathrm{Sp}(2g,\mathbb{Q})$-module V, we denote by V^{Sp} the invariant subspace for the $\mathrm{Sp}(2g,\mathbb{Q})$-action and by V_{Sp} the coinvariant quotient of V. The general theory of $\mathrm{Sp}(2g,\mathbb{Q})$-representations says that for a finite-dimensional representation V, V^{Sp} and V_{Sp} are isomorphic. Hence $\mathfrak{h}_{g,1}^{\mathrm{Sp}} \cong (\mathfrak{h}_{g,1})_{\mathrm{Sp}}$.

In our paper [9], we proved in a direct way that $H_1(\mathfrak{h}_{g,1}^+)_{12}^{\mathrm{Sp}} \cong \mathbb{Q}$ for $g \geq 8$. More precisely, we gave an explicit description of an $\mathrm{Sp}(2g,\mathbb{Q})$-invariant map (cocycle) $C : \mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ satisfying that

- C is non-trivial for any $g \geq 2$,
- the restriction of C to $\sum_{i=1}^{11}[\mathfrak{h}_{g,1}(i), \mathfrak{h}_{g,1}(12-i)]$ is trivial.

It follows that the cocycle C induces a surjection $\widetilde{C} : H_1(\mathfrak{h}_{g,1}^+)_{12}^{\mathrm{Sp}} \cong \left(H_1(\mathfrak{h}_{g,1}^+)_{12}\right)_{\mathrm{Sp}} \twoheadrightarrow \mathbb{Q}$ for every $g \geq 2$. Moreover it turns out that $\widetilde{C}$ is an isomorphism for $g \geq 8$.

On the other hand, in the study of the structure of the Lie algebra $\mathfrak{h}_{g,1}^+$, the determination of the Lie subalgebra

$$J = \bigoplus_{k=1}^{\infty} J(k), \qquad J(k) \subset \mathfrak{h}_{g,1}(k)$$

of $\mathfrak{h}_{g,1}^+$ generated by the degree 1 part $\mathfrak{h}_{g,1}(1)$ has been considered to be important. To this problem, Enomoto and Satoh [2] provided the following powerful tool. They showed that the $\mathrm{Sp}(2g,\mathbb{Q})$-equivariant map obtained as the composition

$$ES_k : \mathfrak{h}_{g,1}(k) \hookrightarrow H \otimes \mathcal{L}_{k+1}(H) \hookrightarrow H^{\otimes(k+2)} \xrightarrow{\mu\otimes(\mathrm{id}^{\otimes k})} H^{\otimes k} \to \left(H^{\otimes k}\right)_{\mathbb{Z}/k\mathbb{Z}}$$

is not trivial in general, but its restriction to $J(k)$ is trivial for any $k \geq 2$. Here $\left(H^{\otimes k}\right)_{\mathbb{Z}/k\mathbb{Z}}$ denotes the coinvariant quotient of $H^{\otimes k}$ with respect to the action of $\mathbb{Z}/k\mathbb{Z}$ as cyclic purmutations of the entries.

3. Proof of Theorem 1.1

In this section, we explain how we prove Theorem 1.1 describing a relationship between the Enomoto-Satoh map ES_{12} and the cocycle C. Our proof proceeds in the following way with the aid of computers:

(1) Find a generating and coordinate system of $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \cong \mathbb{Q}^{650}$.
(2) Find a coordinate system of $\left(H^{\otimes 12}\right)_{\mathbb{Z}/12\mathbb{Z}}^{\mathrm{Sp}} \cong \mathbb{Q}^{897}$.
(3) Compute the kernel of the map $ES_{12} : \mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \to \left(H^{\otimes 12}\right)_{\mathbb{Z}/12\mathbb{Z}}^{\mathrm{Sp}}$.
(4) Checking that the cocycle C is trivial on $\mathrm{Ker}\, ES_{12}$.

Note that the maps C and ES_{12} are invariant under the stabilization $\mathfrak{h}_{g,1} \hookrightarrow \mathfrak{h}_{g+1,1}$. We may suppose that g is sufficiently large in the proof.

The details of these steps are mentioned in Subsections 3.1–3.4 below. Note that we only mention the method for the computation and omit the explicit computational results. Subsection 3.5 gives an application of our Theorem.

3.1. **Generating and coordinate system of $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}}$.** First we find out a generating and coordinate system of $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \cong \mathfrak{h}_{g,1}(12)_{\mathrm{Sp}}$, which is known to be isomorphic to $\mathbb{Q}^{650}$ for $g \geq 5$ (see [8, Table 3]).

We use Lie spiders to describe elements of $\mathfrak{h}_{g,1}$. A *Lie spider with* $(k+2)$ *legs* is defined by

$$\begin{aligned}
&S(u_1, u_2, u_3, \cdots, u_{k+2}) \\
&:= u_1 \otimes [u_2, [u_3, [\cdots [u_{k+1}, u_{k+2}] \cdots]]] + u_2 \otimes [[u_3, [u_4, [\cdots [u_{k+1}, u_{k+2}] \cdots]]], u_1] \\
&\quad + u_3 \otimes [[u_4, [u_5, [\cdots [u_{k+1}, u_{k+2}] \cdots]]], [u_1, u_2]] + \cdots + u_{k+2} \otimes [[[\cdots [u_1, u_2], \cdots], u_k], u_{k+1}],
\end{aligned}$$

where $u_i \in H$. It is known (see [5], for instance) that Lie spiders with $(k+2)$ legs belong to $\mathfrak{h}_{g,1}(k)$ and generate it. Even if we use Lie spiders, however, it is not easy to describe an element in the invariant part $\mathfrak{h}_{g,1}(k)^{\mathrm{Sp}}$. Instead of writing it directly, we use coordinates for $\mathfrak{h}_{g,1}(k)^{\mathrm{Sp}}$ as in the following way: Every $\mathrm{Sp}(2g,\mathbb{Q})$-invariant linear map $\mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ factors through $\mathfrak{h}_{g,1}(12)_{\mathrm{Sp}}$ and the natural projection $\mathfrak{h}_{g,1}(12) \to \mathfrak{h}_{g,1}(12)_{\mathrm{Sp}}$ is regarded as the projection onto $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}}$. Therefore we get a coordinate system of $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}}$ by finding 650 linearly independent $\mathrm{Sp}(2g,\mathbb{Q})$-invariant linear maps $\mathfrak{h}_{g,1}(12) \to \mathbb{Q}$.

Since $\mathfrak{h}_{g,1}(12)$ is an $\mathrm{Sp}(2g,\mathbb{Q})$-submodule of $H \otimes \mathcal{L}_{13}(H) \subset H^{\otimes 14}$, we have $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \subset \left(H^{\otimes 14}\right)^{\mathrm{Sp}}$. A coordinate for $\left(H^{\otimes 14}\right)^{\mathrm{Sp}}$ is classically known and it is given as follows. Divide the set $\{a, b, c, \ldots, m, n\}$ of 14 letters into 7 pairs, say $(i_1 j_1)(i_2 j_2) \cdots (i_7 j_7)$. Then we consider the map

$$\mu_{(i_1 j_1)(i_2 j_2)\cdots(i_7 j_7)} : H^{\otimes 14} \longrightarrow \mathbb{Q}$$

defined by

$$x_a \otimes x_b \otimes \cdots \otimes x_n \mapsto \mu(x_{i_1}, x_{j_1})\mu(x_{i_2}, x_{j_2}) \cdots \mu(x_{i_7}, x_{j_7}).$$

Here we call this map a *multiple contraction*. It is $\mathrm{Sp}(2g,\mathbb{Q})$-invariant since μ is so. We use multiple contractions restricted to $\mathfrak{h}_{g,1}(12)$ as coordinates of $\mathfrak{h}_{g,1}(12)^{\mathrm{Sp}}$. Note that they are invariant under the stabilization map $\mathfrak{h}_{g,1} \hookrightarrow \mathfrak{h}_{g+1,1}$.

A coordinate system

$$(C_1, C_2, \ldots, C_{650}) : \mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \xrightarrow{\cong} \mathbb{Q}^{650}$$

is given in [9, Appendix A]. We use it to obtain Lie spiders $\xi_1, \xi_2, \ldots, \xi_{650}$ which generate $\mathfrak{h}_{g,1}(12)_{\mathrm{Sp}} \cong \mathbb{Q}^{650}$.

3.2. **Coordinate system of $\left(H^{\otimes 12}\right)^{\mathrm{Sp}}_{\mathbb{Z}/12\mathbb{Z}}$.** It is classically known that $\left(H^{\otimes 12}\right)^{\mathrm{Sp}} \cong \mathbb{Q}^{10395}$ for $g \geq 6$, which gives our bound of genus in Theorem 1.1. Indeed, a coordinate for $\left(H^{\otimes 12}\right)^{\mathrm{Sp}}$ is given by a multiple contraction

$$\mu_{(i_1 j_1)(i_2 j_2)\cdots(i_6 j_6)} : H^{\otimes 12} \longrightarrow \mathbb{Q}$$

with $\{i_1, j_1, \ldots, i_6, j_6\} = \{a, b, \ldots, k, l\}$ defined similarly to the above. The maps obtained in this way (totally $10395 = (12-1)!!$ maps) give the basis of the space of the $\mathrm{Sp}(2g,\mathbb{Q})$-invariant maps $H^{\otimes 12} \to \mathbb{Q}$.

If we take the cyclic invariance of $(H^{\otimes 12})^{\mathrm{Sp}}_{\mathbb{Z}/12\mathbb{Z}}$ into account, we may consider the letters $\{a, b, \ldots, k, l\}$ up to cyclic permutations to get a coordinate of $(H^{\otimes 12})^{\mathrm{Sp}}_{\mathbb{Z}/12\mathbb{Z}} \cong \mathbb{Q}^{897}$. This reduces the amount of computations. With the aid of computers, we take a coordinate system

$$(A_1, A_2, \ldots, A_{897}) : (H^{\otimes 12})^{\mathrm{Sp}}_{\mathbb{Z}/12\mathbb{Z}} \xrightarrow{\cong} \mathbb{Q}^{897}.$$

3.3. **Computation of the map ES_{12}.** Since the map

$$ES_{12} : \mathfrak{h}_{g,1}(12) \longrightarrow (H^{\otimes 12})_{\mathbb{Z}/12\mathbb{Z}}$$

is $\mathrm{Sp}(2g, \mathbb{Q})$-equivariant, it induces a linear map

$$ES_{12} : \mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \longrightarrow (H^{\otimes 12})^{\mathrm{Sp}}_{\mathbb{Z}/12\mathbb{Z}}.$$

Recall that each of multiple contractions $\mu_{(i_1 j_1)(i_2 j_2)\cdots(i_7 j_7)} : \mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ factors through $\mathfrak{h}_{g,1}(12)_{\mathrm{Sp}} \cong \mathfrak{h}_{g,1}(12)^{\mathrm{Sp}}$. Therefore as long as we use the coordinate system constructed above, we may work in the whole $\mathfrak{h}_{g,1}(12)$ without considering the projection onto the $\mathrm{Sp}(2g, \mathbb{Q})$-invariant part.

We compute the matrix $\left(A_i(ES_{12}(\xi_j))\right)_{\substack{1 \le i \le 897 \\ 1 \le j \le 650}}$ representing the map $ES_{12} : \mathfrak{h}_{g,1}(12)^{\mathrm{Sp}} \to (H^{\otimes 12})^{\mathrm{Sp}}_{\mathbb{Z}/12\mathbb{Z}}$. By the standard method, we observe that the image is 284-dimensional and fix a basis of $\mathrm{Ker}\, ES_{12}$.

3.4. **The cocycle C is trivial on $\mathrm{Ker}\, ES_{12}$.** We observe that the map C is trivial on the $\mathrm{Ker}\, ES_{12}$ by applying C to the basis. The data for the map C is in [9, Appendix C]. The computation is straightforward and we finish the proof of Theorem 1.1.

3.5. **Another description of the map C via ES_{12}.** We use Theorem 1.1 to give another description of the $\mathrm{Sp}(2g, \mathbb{Q})$-invariant map $C : \mathfrak{h}_{g,1}(12) \to \mathbb{Q}$ in the form $C = C' \circ ES_{12}$ with an $\mathrm{Sp}(2g, \mathbb{Q})$-invariant map $C' : \mathrm{Im} ES_{12} \subset (H^{\otimes 12})_{\mathbb{Z}/12\mathbb{Z}} \to \mathbb{Q}$. For that, we pick a coordinate system

$$(B_1, B_2, \ldots, B_{287}) : \mathrm{Im}\, ES_{12} \xrightarrow{\cong} \mathbb{Q}^{287}$$

of $\mathrm{Im}\, ES_{12}$ from $(A_1, A_2, \ldots, A_{897})$. Then we write the vector $\left(C(\xi_i)\right)_{1 \le i \le 650}$ as a linear combination of the vectors $\left(B_j(ES_{12}(\xi_i))\right)_{1 \le i \le 650}$ for $1 \le j \le 287$. This gives a formula for the map C'.

We put the resulting formula, which consists of the linear combination of 278 multiple contractions, in Appendix A.

APPENDIX A. A FORMULA FOR $C' : \mathrm{Im} ES_{12} \subset \left(H^{\otimes 12}\right)_{\mathbb{Z}/12\mathbb{Z}} \to \mathbb{Q}$

The following map $C' : \mathrm{Im} ES_{12} \subset \left(H^{\otimes 12}\right)_{\mathbb{Z}/12\mathbb{Z}} \to \mathbb{Q}$ gives the decomposition

$$C = C' \circ ES_{12}.$$

The coefficients of the right hand side of the formula is adjusted so that the greatest common divisor is 1.

$$
\begin{aligned}
&\frac{375557}{9}C' \\
&= 1290664\mu_{(a,b)(c,j)(d,l)(e,k)(f,i)(g,h)} + 397276\mu_{(a,b)(c,i)(d,l)(e,k)(f,j)(g,h)} - 496112\mu_{(a,b)(c,i)(d,k)(e,l)(f,j)(g,h)} \\
&+ 1618892\mu_{(a,b)(c,i)(d,j)(e,l)(f,k)(g,h)} + 1290664\mu_{(a,b)(c,j)(d,l)(e,k)(f,h)(g,i)} - 6268488\mu_{(a,b)(c,k)(d,l)(e,h)(f,j)(g,i)} \\
&- 18703766\mu_{(a,b)(c,l)(d,h)(e,k)(f,j)(g,i)} + 9125052\mu_{(a,b)(c,h)(d,l)(e,k)(f,j)(g,i)} + 1552616\mu_{(a,b)(c,h)(d,k)(e,l)(f,j)(g,i)} \\
&- 2771492\mu_{(a,b)(c,j)(d,l)(e,h)(f,k)(g,i)} - 7140686\mu_{(a,b)(c,l)(d,h)(e,j)(f,k)(g,i)} - 1686524\mu_{(a,b)(c,h)(d,l)(e,j)(f,k)(g,i)} \\
&- 3924514\mu_{(a,b)(c,j)(d,h)(e,l)(f,k)(g,i)} + 456406\mu_{(a,b)(c,h)(d,j)(e,l)(f,k)(g,i)} + 1756034\mu_{(a,b)(c,h)(d,k)(e,j)(f,l)(g,i)} \\
&+ 739894\mu_{(a,b)(c,h)(d,j)(e,k)(f,l)(g,i)} + 7249468\mu_{(a,b)(c,l)(d,i)(e,k)(f,h)(g,j)} + 6655854\mu_{(a,b)(c,i)(d,l)(e,k)(f,h)(g,j)} \\
&+ 3775594\mu_{(a,b)(c,i)(d,k)(e,l)(f,h)(g,j)} + 409618\mu_{(a,b)(c,l)(d,h)(e,k)(f,i)(g,j)} + 4220920\mu_{(a,b)(c,h)(d,l)(e,k)(f,i)(g,j)} \\
&+ 2916972\mu_{(a,b)(c,h)(d,k)(e,l)(f,i)(g,j)} + 1390334\mu_{(a,b)(c,i)(d,l)(e,h)(f,k)(g,j)} + 3233464\mu_{(a,b)(c,l)(d,h)(e,i)(f,k)(g,j)} \\
&- 653498\mu_{(a,b)(c,h)(d,l)(e,i)(f,k)(g,j)} + 1965784\mu_{(a,b)(c,i)(d,h)(e,l)(f,k)(g,j)} + 14344\mu_{(a,b)(c,h)(d,i)(e,l)(f,k)(g,j)} \\
&- 2088812\mu_{(a,b)(c,h)(d,k)(e,i)(f,l)(g,j)} + 1639104\mu_{(a,b)(c,i)(d,h)(e,k)(f,l)(g,j)} - 312336\mu_{(a,b)(c,h)(d,i)(e,k)(f,l)(g,j)} \\
&+ 499190\mu_{(a,b)(c,j)(d,l)(e,i)(f,h)(g,k)} + 5602264\mu_{(a,b)(c,i)(d,l)(e,j)(f,h)(g,k)} + 3557414\mu_{(a,b)(c,i)(d,j)(e,l)(f,h)(g,k)} \\
&+ 751294\mu_{(a,b)(c,j)(d,l)(e,h)(f,i)(g,k)} + 3573290\mu_{(a,b)(c,h)(d,l)(e,j)(f,i)(g,k)} + 1483068\mu_{(a,b)(c,h)(d,j)(e,l)(f,i)(g,k)} \\
&+ 1102526\mu_{(a,b)(c,i)(d,l)(e,h)(f,j)(g,k)} - 1008862\mu_{(a,b)(c,h)(d,l)(e,i)(f,j)(g,k)} + 3910744\mu_{(a,b)(c,i)(d,h)(e,l)(f,j)(g,k)} \\
&+ 12184\mu_{(a,b)(c,h)(d,i)(e,l)(f,j)(g,k)} - 4533704\mu_{(a,b)(c,h)(d,j)(e,i)(f,l)(g,k)} - 2919132\mu_{(a,b)(c,h)(d,i)(e,j)(f,l)(g,k)} \\
&+ 1614572\mu_{(a,b)(c,h)(d,i)(e,k)(f,j)(g,l)} + 1153482\mu_{(a,b)(c,g)(d,l)(e,k)(f,j)(h,i)} + 1016300\mu_{(a,b)(c,g)(d,k)(e,l)(f,j)(h,i)} \\
&+ 17306652\mu_{(a,b)(c,g)(d,j)(e,l)(f,k)(h,i)} + 7843880\mu_{(a,b)(c,g)(d,j)(e,k)(f,l)(h,i)} + 14128782\mu_{(a,b)(c,l)(d,f)(e,k)(g,j)(h,i)} \\
&- 5941360\mu_{(a,b)(c,f)(d,l)(e,k)(g,j)(h,i)} - 9179144\mu_{(a,b)(c,f)(d,k)(e,l)(g,j)(h,i)} + 9684024\mu_{(a,b)(c,e)(d,l)(f,k)(g,j)(h,i)} \\
&+ 13834824\mu_{(a,b)(c,e)(d,k)(f,l)(g,j)(h,i)} + 343152\mu_{(a,b)(c,f)(d,l)(e,j)(g,k)(h,i)} + 3392292\mu_{(a,b)(c,f)(d,j)(e,l)(g,k)(h,i)} \\
&+ 15952512\mu_{(a,b)(c,e)(d,l)(f,j)(g,k)(h,i)} - 15064992\mu_{(a,b)(c,e)(d,j)(f,l)(g,k)(h,i)} + 11787040\mu_{(a,b)(c,f)(d,k)(e,j)(g,l)(h,i)} \\
&+ 8221028\mu_{(a,b)(c,f)(d,j)(e,k)(g,l)(h,i)} + 6344912\mu_{(a,b)(c,e)(d,k)(f,j)(g,l)(h,i)} - 8796504\mu_{(a,b)(c,e)(d,j)(f,k)(g,l)(h,i)} \\
&- 8325972\mu_{(a,b)(c,l)(d,g)(e,k)(f,i)(h,j)} + 1666534\mu_{(a,b)(c,g)(d,l)(e,k)(f,i)(h,j)} - 1213726\mu_{(a,b)(c,g)(d,k)(e,l)(f,i)(h,j)} \\
&+ 3452210\mu_{(a,b)(c,i)(d,l)(e,g)(f,k)(h,j)} - 411362\mu_{(a,b)(c,l)(d,g)(e,i)(f,k)(h,j)} - 9114544\mu_{(a,b)(c,g)(d,l)(e,i)(f,k)(h,j)} \\
&+ 2124382\mu_{(a,b)(c,i)(d,g)(e,l)(f,k)(h,j)} + 2934234\mu_{(a,b)(c,g)(d,i)(e,l)(f,k)(h,j)} + 3030134\mu_{(a,b)(c,g)(d,k)(e,i)(f,l)(h,j)} \\
&+ 1434310\mu_{(a,b)(c,i)(d,g)(e,k)(f,l)(h,j)} + 4274894\mu_{(a,b)(c,g)(d,i)(e,k)(f,l)(h,j)} - 1698112\mu_{(a,b)(c,l)(d,f)(e,k)(g,i)(h,j)} \\
&- 5941360\mu_{(a,b)(c,f)(d,l)(e,k)(g,i)(h,j)} - 9179144\mu_{(a,b)(c,f)(d,k)(e,l)(g,i)(h,j)} + 25334996\mu_{(a,b)(c,e)(d,l)(f,k)(g,i)(h,j)} \\
&+ 3826852\mu_{(a,b)(c,e)(d,k)(f,l)(g,i)(h,j)} - 8479950\mu_{(a,b)(c,f)(d,l)(e,i)(g,k)(h,j)} - 7224682\mu_{(a,b)(c,i)(d,f)(e,l)(g,k)(h,j)} \\
&- 3740452\mu_{(a,b)(c,f)(d,i)(e,l)(g,k)(h,j)} + 8574888\mu_{(a,b)(c,e)(d,l)(f,i)(g,k)(h,j)} + 10263832\mu_{(a,c)(b,d)(e,l)(f,i)(g,k)(h,j)} \\
&+ 13703390\mu_{(a,b)(c,d)(e,l)(f,i)(g,k)(h,j)} - 1190743\mu_{(a,b)(c,i)(d,e)(f,l)(g,k)(h,j)} - 4880762\mu_{(a,b)(c,e)(d,i)(f,l)(g,k)(h,j)} \\
&+ 12225840\mu_{(a,c)(b,d)(e,i)(f,l)(g,k)(h,j)} + 46196626\mu_{(a,b)(c,d)(e,i)(f,l)(g,k)(h,j)} + 3470830\mu_{(a,b)(c,f)(d,k)(e,i)(g,l)(h,j)} \\
&- 623750\mu_{(a,b)(c,f)(d,i)(e,k)(g,l)(h,j)} - 5293180\mu_{(a,b)(c,e)(d,k)(f,i)(g,l)(h,j)} - 1253580\mu_{(a,c)(b,e)(d,i)(f,k)(g,l)(h,j)} \\
&- 477168\mu_{(a,b)(c,e)(d,i)(f,k)(g,l)(h,j)} + 8592720\mu_{(a,c)(b,d)(e,i)(f,k)(g,l)(h,j)} + 12525688\mu_{(a,b)(c,d)(e,i)(f,k)(g,l)(h,j)}
\end{aligned}
$$

$+ 7129396\mu_{(a,b)(c,j)(d,l)(e,g)(f,i)(h,k)} - 4825650\mu_{(a,b)(c,g)(d,l)(e,j)(f,i)(h,k)} - 2594164\mu_{(a,b)(c,g)(d,j)(e,l)(f,i)(h,k)}$
$- 3638832\mu_{(a,b)(c,i)(d,l)(e,g)(f,j)(h,k)} - 7393126\mu_{(a,b)(c,g)(d,l)(e,i)(f,j)(h,k)} + 2137528\mu_{(a,b)(c,i)(d,g)(e,l)(f,j)(h,k)}$
$+ 472080\mu_{(a,b)(c,g)(d,i)(e,l)(f,j)(h,k)} + 47496\mu_{(a,b)(c,g)(d,j)(e,i)(f,l)(h,k)} + 2641496\mu_{(a,b)(c,g)(d,i)(e,j)(f,l)(h,k)}$
$+ 12497276\mu_{(a,b)(c,l)(d,f)(e,j)(g,i)(h,k)} - 5036040\mu_{(a,b)(c,f)(d,l)(e,j)(g,i)(h,k)} + 11543148\mu_{(a,b)(c,j)(d,f)(e,l)(g,i)(h,k)}$
$+ 77018\mu_{(a,b)(c,f)(d,j)(e,l)(g,i)(h,k)} - 2423104\mu_{(a,b)(c,e)(d,l)(f,j)(g,i)(h,k)} + 8254378\mu_{(a,b)(c,e)(d,j)(f,l)(g,i)(h,k)}$
$- 3687724\mu_{(a,c)(b,d)(e,j)(f,l)(g,i)(h,k)} - 31878504\mu_{(a,b)(c,d)(e,j)(f,l)(g,i)(h,k)} + 3424310\mu_{(a,b)(c,l)(d,f)(e,i)(g,j)(h,k)}$
$- 5935666\mu_{(a,b)(c,f)(d,l)(e,i)(g,j)(h,k)} + 1154126\mu_{(a,b)(c,i)(d,f)(e,l)(g,j)(h,k)} - 2243848\mu_{(a,b)(c,f)(d,i)(e,l)(g,j)(h,k)}$
$+ 3621020\mu_{(a,b)(c,e)(d,l)(f,i)(g,j)(h,k)} - 13052782\mu_{(a,b)(c,i)(d,e)(f,l)(g,j)(h,k)} - 4032152\mu_{(a,c)(b,e)(d,i)(f,l)(g,j)(h,k)}$
$- 13677132\mu_{(a,b)(c,e)(d,i)(f,l)(g,j)(h,k)} + 4171976\mu_{(a,c)(b,d)(e,i)(f,l)(g,j)(h,k)} - 6056536\mu_{(a,b)(c,d)(e,i)(f,l)(g,j)(h,k)}$
$- 1911360\mu_{(a,b)(c,f)(d,j)(e,i)(g,l)(h,k)} + 682640\mu_{(a,b)(c,f)(d,i)(e,j)(g,l)(h,k)} + 981588\mu_{(a,c)(b,e)(d,j)(f,i)(g,l)(h,k)}$
$- 3558062\mu_{(a,b)(c,e)(d,j)(f,i)(g,l)(h,k)} - 1787376\mu_{(a,c)(b,e)(d,i)(f,j)(g,l)(h,k)} - 12251666\mu_{(a,b)(c,e)(d,i)(f,j)(g,l)(h,k)}$
$- 1610864\mu_{(a,c)(b,d)(e,i)(f,j)(g,l)(h,k)} + 656456\mu_{(a,b)(c,d)(e,i)(f,j)(g,l)(h,k)} + 1614572\mu_{(a,b)(c,g)(d,i)(e,k)(f,j)(h,l)}$
$+ 1558298\mu_{(a,b)(c,f)(d,k)(e,j)(g,i)(h,l)} + 1600878\mu_{(a,b)(c,f)(d,j)(e,k)(g,i)(h,l)} + 22976278\mu_{(a,b)(c,e)(d,k)(f,j)(g,i)(h,l)}$
$- 588484\mu_{(a,c)(b,e)(d,j)(f,k)(g,i)(h,l)} + 6222068\mu_{(a,b)(c,e)(d,j)(f,k)(g,i)(h,l)} + 3479476\mu_{(a,b)(c,f)(d,k)(e,i)(g,j)(h,l)}$
$+ 1614572\mu_{(a,b)(c,f)(d,i)(e,k)(g,j)(h,l)} + 12603078\mu_{(a,b)(c,e)(d,k)(f,i)(g,j)(h,l)} - 3087616\mu_{(a,c)(b,e)(d,i)(f,k)(g,j)(h,l)}$
$- 12908734\mu_{(a,b)(c,e)(d,i)(f,k)(g,j)(h,l)} + 1660520\mu_{(a,b)(c,f)(d,j)(e,i)(g,k)(h,l)} + 4254520\mu_{(a,b)(c,f)(d,i)(e,j)(g,k)(h,l)}$
$+ 1298080\mu_{(a,c)(b,e)(d,j)(f,i)(g,k)(h,l)} - 6831626\mu_{(a,b)(c,e)(d,j)(f,i)(g,k)(h,l)} - 491456\mu_{(a,c)(b,e)(d,i)(f,j)(g,k)(h,l)}$
$- 10277730\mu_{(a,b)(c,e)(d,i)(f,j)(g,k)(h,l)} + 7404092\mu_{(a,b)(c,h)(d,f)(e,l)(g,k)(i,j)} + 2851892\mu_{(a,b)(c,f)(d,h)(e,l)(g,k)(i,j)}$
$- 9632582\mu_{(a,b)(c,d)(e,l)(f,h)(g,k)(i,j)} - 23611652\mu_{(a,b)(c,h)(d,e)(f,l)(g,k)(i,j)} - 15701684\mu_{(a,b)(c,e)(d,h)(f,l)(g,k)(i,j)}$
$- 656456\mu_{(a,b)(c,d)(e,h)(f,l)(g,k)(i,j)} - 19720916\mu_{(a,b)(c,h)(d,f)(e,k)(g,l)(i,j)} - 6915532\mu_{(a,b)(c,f)(d,h)(e,k)(g,l)(i,j)}$
$- 23611652\mu_{(a,b)(c,h)(d,e)(f,k)(g,l)(i,j)} - 9433196\mu_{(a,b)(c,e)(d,h)(f,k)(g,l)(i,j)} - 656456\mu_{(a,b)(c,d)(e,h)(f,k)(g,l)(i,j)}$
$- 8997096\mu_{(a,b)(c,f)(d,l)(e,g)(h,k)(i,j)} - 54489266\mu_{(a,b)(c,g)(d,f)(e,l)(h,k)(i,j)} - 4183224\mu_{(a,b)(c,f)(d,g)(e,l)(h,k)(i,j)}$
$- 656456\mu_{(a,b)(c,g)(d,e)(f,l)(h,k)(i,j)} + 32282276\mu_{(a,b)(c,e)(d,g)(f,l)(h,k)(i,j)} - 56980426\mu_{(a,b)(c,d)(e,g)(f,l)(h,k)(i,j)}$
$+ 22955196\mu_{(a,b)(c,f)(d,e)(g,l)(h,k)(i,j)} + 19060956\mu_{(a,b)(c,e)(d,f)(g,l)(h,k)(i,j)} - 16968008\mu_{(a,b)(c,f)(d,k)(e,g)(h,l)(i,j)}$
$- 27409342\mu_{(a,b)(c,g)(d,f)(e,k)(h,l)(i,j)} - 15653072\mu_{(a,b)(c,f)(d,g)(e,k)(h,l)(i,j)} - 328228\mu_{(a,b)(c,g)(d,e)(f,k)(h,l)(i,j)}$
$+ 7901608\mu_{(a,b)(c,e)(d,g)(f,k)(h,l)(i,j)} - 9632582\mu_{(a,b)(c,d)(e,g)(f,k)(h,l)(i,j)} + 22955196\mu_{(a,b)(c,f)(d,e)(g,k)(h,l)(i,j)}$
$+ 19060956\mu_{(a,b)(c,e)(d,f)(g,k)(h,l)(i,j)} + 2742672\mu_{(a,b)(c,j)(d,l)(e,h)(f,g)(i,k)} - 9303668\mu_{(a,b)(c,h)(d,l)(e,j)(f,g)(i,k)}$
$+ 1202176\mu_{(a,b)(c,h)(d,j)(e,l)(f,g)(i,k)} - 8002612\mu_{(a,b)(c,j)(d,l)(e,g)(f,h)(i,k)} - 8239598\mu_{(a,b)(c,g)(d,l)(e,j)(f,h)(i,k)}$
$- 11381244\mu_{(a,b)(c,g)(d,j)(e,l)(f,h)(i,k)} - 10117118\mu_{(a,b)(c,h)(d,l)(e,g)(f,j)(i,k)} - 7979112\mu_{(a,b)(c,g)(d,l)(e,h)(f,j)(i,k)} -$
$2052638\mu_{(a,b)(c,h)(d,g)(e,l)(f,j)(i,k)} + 2520874\mu_{(a,b)(c,g)(d,h)(e,l)(f,j)(i,k)} + 8237528\mu_{(a,b)(c,h)(d,j)(e,g)(f,l)(i,k)}$
$+ 734\mu_{(a,b)(c,g)(d,j)(e,h)(f,l)(i,k)} + 5496806\mu_{(a,b)(c,h)(d,g)(e,j)(f,l)(i,k)} - 1671318\mu_{(a,b)(c,g)(d,h)(e,j)(f,l)(i,k)}$
$+ 12209420\mu_{(a,b)(c,e)(d,j)(f,l)(g,h)(i,k)} + 56980426\mu_{(a,b)(c,d)(e,j)(f,l)(g,h)(i,k)} + 6521452\mu_{(a,b)(c,f)(d,l)(e,h)(g,j)(i,k)}$
$+ 12718464\mu_{(a,b)(c,h)(d,f)(e,l)(g,j)(i,k)} + 5422702\mu_{(a,b)(c,f)(d,h)(e,l)(g,j)(i,k)} + 11469008\mu_{(a,b)(c,e)(d,l)(f,h)(g,j)(i,k)}$
$- 4686892\mu_{(a,b)(c,h)(d,e)(f,l)(g,j)(i,k)} - 2287746\mu_{(a,c)(b,e)(d,h)(f,l)(g,j)(i,k)} + 6135330\mu_{(a,b)(c,e)(d,h)(f,l)(g,j)(i,k)}$
$+ 7361116\mu_{(a,c)(b,d)(e,h)(f,l)(g,j)(i,k)} + 25639556\mu_{(a,b)(c,d)(e,h)(f,l)(g,j)(i,k)} - 5871470\mu_{(a,b)(c,f)(d,j)(e,h)(g,l)(i,k)}$
$- 2204182\mu_{(a,b)(c,f)(d,h)(e,j)(g,l)(i,k)} + 12974296\mu_{(a,c)(b,e)(d,j)(f,h)(g,l)(i,k)} + 31622134\mu_{(a,b)(c,e)(d,j)(f,h)(g,l)(i,k)}$
$- 4327564\mu_{(a,c)(b,e)(d,h)(f,j)(g,l)(i,k)} - 5635944\mu_{(a,b)(c,e)(d,h)(f,j)(g,l)(i,k)} - 5826752\mu_{(a,c)(b,d)(e,h)(f,j)(g,l)(i,k)}$
$+ 42662304\mu_{(a,b)(c,d)(e,h)(f,j)(g,l)(i,k)} - 16440132\mu_{(a,b)(c,f)(d,l)(e,g)(h,j)(i,k)} - 24712568\mu_{(a,b)(c,g)(d,f)(e,l)(h,j)(i,k)}$
$- 2503576\mu_{(a,b)(c,f)(d,g)(e,l)(h,j)(i,k)} + 4917432\mu_{(a,b)(c,g)(d,e)(f,l)(h,j)(i,k)} + 10534536\mu_{(a,b)(c,e)(d,g)(f,l)(h,j)(i,k)}$

$$
\begin{aligned}
&+ 9414766\mu_{(a,c)(b,d)(e,g)(f,l)(h,j)(i,k)} - 27380296\mu_{(a,b)(c,d)(e,g)(f,l)(h,j)(i,k)} + 13487068\mu_{(a,b)(c,f)(d,e)(g,l)(h,j)(i,k)} \\
&+ 15166716\mu_{(a,b)(c,e)(d,f)(g,l)(h,j)(i,k)} + 36669318\mu_{(a,b)(c,f)(d,j)(e,g)(h,l)(i,k)} + 17647624\mu_{(a,b)(c,f)(d,g)(e,j)(h,l)(i,k)} \\
&+ 25742042\mu_{(a,b)(c,g)(d,e)(f,j)(h,l)(i,k)} + 57907214\mu_{(a,b)(c,e)(d,g)(f,j)(h,l)(i,k)} + 607636\mu_{(a,b)(c,d)(e,g)(f,j)(h,l)(i,k)} \\
&+ 79227298\mu_{(a,b)(c,f)(d,e)(g,j)(h,l)(i,k)} - 11035736\mu_{(a,c)(b,e)(d,f)(g,j)(h,l)(i,k)} + 32193992\mu_{(a,b)(c,e)(d,f)(g,j)(h,l)(i,k)} \\
&+ 79542486\mu_{(a,b)(c,d)(e,f)(g,j)(h,l)(i,k)} - 8627756\mu_{(a,b)(c,e)(d,k)(f,j)(g,h)(i,l)} - 2010616\mu_{(a,b)(c,e)(d,j)(f,k)(g,h)(i,l)} \\
&+ 1616284\mu_{(a,b)(c,f)(d,h)(e,k)(g,j)(i,l)} + 2928544\mu_{(a,b)(c,e)(d,k)(f,h)(g,j)(i,l)} - 172804\mu_{(a,c)(b,e)(d,h)(f,k)(g,j)(i,l)} \\
&+ 7192562\mu_{(a,b)(c,e)(d,h)(f,k)(g,j)(i,l)} + 11522200\mu_{(a,c)(b,e)(d,j)(f,h)(g,k)(i,l)} - 1126152\mu_{(a,b)(c,e)(d,j)(f,h)(g,k)(i,l)} \\
&+ 476236\mu_{(a,c)(b,e)(d,h)(f,j)(g,k)(i,l)} + 2628946\mu_{(a,b)(c,e)(d,h)(f,j)(g,k)(i,l)} - 11630390\mu_{(a,b)(c,f)(d,g)(e,k)(h,j)(i,l)} \\
&+ 5571528\mu_{(a,c)(b,e)(d,g)(f,k)(h,j)(i,l)} + 22262008\mu_{(a,b)(c,e)(d,g)(f,k)(h,j)(i,l)} + 5975692\mu_{(a,c)(b,d)(e,g)(f,k)(h,j)(i,l)} \\
&- 55187992\mu_{(a,b)(c,d)(e,g)(f,k)(h,j)(i,l)} - 2323830\mu_{(a,b)(c,f)(d,e)(g,k)(h,j)(i,l)} + 2942724\mu_{(a,b)(c,e)(d,f)(g,k)(h,j)(i,l)} \\
&- 2997632\mu_{(a,b)(c,e)(d,j)(f,g)(h,k)(i,l)} + 4432316\mu_{(a,c)(b,e)(d,g)(f,j)(h,k)(i,l)} + 18207838\mu_{(a,b)(c,e)(d,g)(f,j)(h,k)(i,l)} \\
&- 6490952\mu_{(a,c)(b,d)(e,g)(f,j)(h,k)(i,l)} - 14855756\mu_{(a,b)(c,d)(e,g)(f,j)(h,k)(i,l)} + 17506008\mu_{(a,b)(c,f)(d,e)(g,j)(h,k)(i,l)} \\
&+ 4143640\mu_{(a,b)(c,e)(d,f)(g,j)(h,k)(i,l)} - 623488\mu_{(a,b)(c,g)(d,i)(e,l)(f,h)(j,k)} + 5771140\mu_{(a,b)(c,h)(d,i)(e,g)(f,l)(j,k)} \\
&+ 22674240\mu_{(a,b)(c,g)(d,i)(e,h)(f,l)(j,k)} + 7740730\mu_{(a,b)(c,g)(d,h)(e,i)(f,l)(j,k)} - 10413578\mu_{(a,b)(c,e)(d,l)(f,i)(g,h)(j,k)} \\
&+ 10741806\mu_{(a,b)(c,e)(d,i)(f,l)(g,h)(j,k)} + 4961644\mu_{(a,b)(c,e)(d,h)(f,l)(g,i)(j,k)} + 56323970\mu_{(a,b)(c,d)(e,h)(f,l)(g,i)(j,k)} \\
&- 5573888\mu_{(a,b)(c,f)(d,i)(e,h)(g,l)(j,k)} + 4222468\mu_{(a,b)(c,f)(d,h)(e,i)(g,l)(j,k)} - 19238764\mu_{(a,b)(c,e)(d,i)(f,h)(g,l)(j,k)} \\
&- 80198942\mu_{(a,b)(c,d)(e,i)(f,h)(g,l)(j,k)} + 1343224\mu_{(a,b)(c,e)(d,h)(f,i)(g,l)(j,k)} - 656456\mu_{(a,b)(c,d)(e,h)(f,i)(g,l)(j,k)} \\
&- 39771243\mu_{(a,b)(c,d)(e,g)(f,l)(h,i)(j,k)} - 15572360\mu_{(a,b)(c,f)(d,i)(e,g)(h,l)(j,k)} - 19559660\mu_{(a,b)(c,f)(d,g)(e,i)(h,l)(j,k)} \\
&- 32259550\mu_{(a,b)(c,e)(d,i)(f,g)(h,l)(j,k)} + 6663100\mu_{(a,b)(c,e)(d,g)(f,i)(h,l)(j,k)} - 9632582\mu_{(a,b)(c,d)(e,g)(f,i)(h,l)(j,k)} \\
&- 5323592\mu_{(a,b)(c,e)(d,f)(g,i)(h,l)(j,k)} - 79542486\mu_{(a,b)(c,d)(e,f)(g,i)(h,l)(j,k)} - 79279166\mu_{(a,b)(c,e)(d,h)(f,g)(i,l)(j,k)} \\
&- 4018940\mu_{(a,b)(c,e)(d,g)(f,h)(i,l)(j,k)} + 44500\mu_{(a,c)(b,e)(d,h)(f,k)(g,i)(j,l)} + 6449982\mu_{(a,b)(c,e)(d,h)(f,k)(g,i)(j,l)} \\
&+ 14437012\mu_{(a,c)(b,e)(d,i)(f,h)(g,k)(j,l)} + 5974860\mu_{(a,c)(b,e)(d,h)(f,i)(g,k)(j,l)} + 22594296\mu_{(a,b)(c,e)(d,g)(f,k)(h,i)(j,l)} \\
&+ 40210908\mu_{(a,b)(c,e)(d,f)(g,k)(h,i)(j,l)} + 124700\mu_{(a,b)(c,e)(d,f)(g,i)(h,k)(j,l)}
\end{aligned}
$$

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ADDENDUM

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