数理解析研究所講究録
第2070巻 2018年 25-31

DIRECTION OF VORTICITY AND A REFINED BLOW-UP CRITERION FOR THE NAVIER-STOKES EQUATIONS WITH FRACTIONAL LAPLACIAN

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ABSTRACT. We give a refined blow-up criterion for solutions of the 3D Navier-Stokes equations with fractional Laplacian. The criterion is composed by the direction field of the vorticity and its magnitude simultaneously. Our result is an improvement of previous results H. Beirao da Veiga and L. Berselli (2002), and D. Chae (2007).

1. INTRODUCTION

The answer to the problem of global regularity for the three dimensional incompressible Navier-Stokes equations is not known. Furthermore, it is unclear whether or not turbulence relates directly to the problem. One of the important phenomena in turbulence is the energy cascade, which represents motion of energy from large-scale to small-scale [13]. In [17], the energy cascade is observed in steady-state two-dimensional turbulence by direct numerical simulation of the Navier-Stokes equations with fractional Laplacian. From this reason, we are concerned with the following simply generalized Navier-Stokes equations:

$$\partial_t v + (v \cdot \nabla)v = -\nabla\pi - \nu(-\Delta)^{\alpha/2}v \qquad \text{in } \mathbb{R}^3 \times (0,\infty) \tag{1}$$

$$\operatorname{div} v = 0 \qquad \text{in } \mathbb{R}^3 \times (0,\infty) \tag{2}$$

$$v(x,0) = v_0(x) \qquad \text{in } \mathbb{R}^3, \tag{3}$$

where $v = v(x,t) = (v_1(x,t), v_2(x,t), v_3(x,t))$ is the velocity of the fluid flows, $\pi = \pi(x,t)$ is the pressure, $\nu > 0$ is the viscosity constant, and $v_0(x)$ is a given initial velocity field satisfying $\operatorname{div} v_0 = 0$. Furthermore, a fractional power of the Laplace operator, $(-\Delta)^{\alpha/2}$ $(\alpha > 0)$, is defined through the Fourier transform

$$\mathcal{F}\left[(-\Delta)^{\alpha/2} f(x)\right](\xi) = |\xi|^{\alpha} \mathcal{F}_{[f]}(\xi),$$

where $\mathcal{F}_{[f]}$ is the Fourier transform of f. We denote the system (1)$\sim$(3) by $(\mathrm{NS})_\alpha$. When $\alpha = 2$, the $(\mathrm{NS})_\alpha$ reduce to the usual Navier-Stokes equations. In this paper we are concerned with the case $0 < \alpha \leq 2$.

The system $(\mathrm{NS})_\alpha$ was first considered by J.L. Lions, who showed that if $\alpha \geq 5/2$, $(\mathrm{NS})_\alpha$ have a unique global smooth solution [14]. And N.H. Katz and N. Pavlović [12] showed that if $2 < \alpha < 5/2$, the Hausdorff dimension of the singular set at the time of first possible blow-up is at most $5 - 2\alpha$. In the case $0 < \alpha < 2$, $(\mathrm{NS})_\alpha$ is completely different from the cases mentioned above. For the case, the existence

2000 *Mathematics Subject Classification.* 35Q30 (35B65 76D03 76D05).

Key words and phrases. Navier-Stokes, fractional powers of the Laplacian, blow-up, vorticity direction.

of weak solutions have not yet proved rigorously. So, main theorem in this paper should be understood as the continuation principle for local in time strong solutions.

In [6, 10], they claim that the vorticity plays an important role in the regularity conditions for the Navier-Stokes equations. Taking rot of (1), we obtain the following equation:

$$\partial_t \omega + \nu(-\Delta)^{\alpha/2}\omega = (\omega \cdot \nabla)v - (v \cdot \nabla)\omega, \tag{4}$$

where the vorticity ω is defined by

$$\omega = \operatorname{rot} v.$$

The velocity can be written in terms of vorticity through the Biot-Savart law:

$$v(x,t) = \frac{-1}{4\pi}\int_{\mathbb{R}^3} \nabla_y \left(\frac{1}{|x-y|}\right) \times \omega(y,t)dy, \tag{5}$$

which follows from (2), with decaying vorticity near infinity. The following continuation principle for the 3D Euler equations, namely the $(\mathrm{NS})_2$ with $\nu = 0$ on the class,

$$E_s(T) := C([0,T);H^s) \cap C^1([0,T);H^{s-1}) \ (s > 3/2+1),$$

is proved by J.T. Beale, T. Kato and A. Majda;

Proposition 1.1 ([1]). *Let $s > 3/2+1, T > 0$. Suppose v is a solution of the Euler equations, corresponding to initial data $v_0 \in H^s(\mathbb{R}^3)$ in the class $E_s(T)$. If we have a priori estimate for vorticity,*

$$\int_0^T \|\omega(t)\|_\infty dt < \infty,$$

then we have $\limsup_{t\nearrow T}\|v(t)\|_{H^s} < \infty$, in particular there is no singularity up to T.

Same continuation principle for $(\mathrm{NS})_\alpha$ with any $\alpha \in (0,2]$ is proved. D. Chae improved this result for $(\mathrm{NS})_\alpha$ with $\alpha \in (0,2]$;

Lemma 1.2 ([5, 9]). *Let $s > 3/2+1, T > 0$. Suppose v is a solution of $(\mathrm{NS})_\alpha$ with $\alpha \in (0,2]$, and belongs to $E_s(T)$. If the vorticity $\omega(x,t)$ satisfies*

$$\omega \in L^p(0,T;L^q), \ \frac{3}{q} + \frac{\alpha}{p} \leq \alpha, \tag{6}$$

where $6/\alpha < q \leq \infty$. Then we have $\limsup_{t\nearrow T}\|v(t)\|_{H^s} < \infty$, in particular there is no singularity up to T.

On the other hand, P. Constantin and C. Fefferman [8] first proved that if the following estimate on the disturbance of the direction vector of vorticity to the Navier-Stokes equations holds in regions of high vorticity, then the solution is regular.

$$\frac{\sqrt{1-(\xi(x,t)\cdot\xi(x+h,t))^2}}{|h|} \leq C,$$

where $\xi(x,t)$ is the direction vector of vorticity.

H. Beirão da Veiga and L. Berselli [2, 3] improved this result for the Navier-Stokes equations as follows.

Assumption (A1). *There exist* $\beta \in [1/2, 1]$, *a positive constant* K, *and* $g \in L^a(0, T; L^b)$, *where*

$$\frac{2}{a} + \frac{3}{b} = \beta - \frac{1}{2} \text{ with } a \in \left[\frac{4}{2\beta - 1}, \infty\right],$$

such that

$$\eta_\beta(x, h, t) := \frac{\sqrt{1 - (\xi(x, t) \cdot \xi(x + h, t))^2}}{|h|^\beta} \leq g(t, x)$$

holds in the region where the vorticity at both x *and* $x + h$ *is larger than* K.

Assumption (A2). *There exist* $\beta \in (0, 1/2]$ *and a positive constant* K *and* C *such that*

$$\eta_\beta(x, h, t) \leq C$$

holds in the region where the vorticity at both x *and* $x + h$ *is larger than* K. *Furthermore,*

$$\omega \in L^2(0, T; L^r) \text{ where } r = \frac{3}{\beta + 1}.$$

Proposition 1.3 ([2, 3]). *Suppose that* v *is a weak solution of the Navier-Stokes equations with* $v_0 \in H^1_\sigma$, *which means the sobolev spaces of solenoidal vector fields. And suppose that Assumption* (A1) *or* (A2) *is satisfied. Then the solution is regular in* $(0, T]$.

In fact, D. Chae [5] have already proved that if the behavior of direction of vorticity to $(\mathrm{NS})_\alpha$ in the whole space is restricted by using some Triebel-Lizorkin norm $\|\cdot\|_{\dot{\mathcal{F}}^\beta_{b,p}}$ (see [[5]-pp.374]), then there is no singularity.

Proposition 1.4 ([5]). *Let* $v(x, t)$ *be a some solution of* $(\mathrm{NS})_\alpha$ *and* $\omega(x, t) = \nabla \times v(x, t)$. *Let* $\xi(x, t)$ *be its direction vector of vorticity, defined for* $\omega(x, t) \neq 0$. *Suppose there exists* $\beta \in (0, 1)$, $p \in (3/(3-\beta), \infty]$, $b \in (1, \infty]$, $r \in (1, 3/\beta)$ *satisfying*

$$\frac{\beta}{3} < \frac{1}{b} + \frac{1}{r} < \frac{\alpha + \beta}{3}, \quad \frac{1}{r} + \frac{1}{p} < 1 + \frac{\beta}{3}$$

and $a, q \in [1, \infty]$ *such that the following holds.*

$$\xi \in L^a(0, T; \dot{\mathcal{F}}^\beta_{b,p}) \text{ and } \omega \in L^q(0, T; L^r)$$

$$\text{with } \frac{\alpha}{q} + \frac{\alpha}{a} + \frac{3}{r} + \frac{3}{b} \leq \alpha + \beta.$$

Then, there is no singularity up to T.

Here, M. Tanahashi et al [15, Introduction] state that "Recently, from the results of direct numerical simulations, it is shown that there are high vorticity regions in homogeneous turbulence, which are supposed to be a candidate of fine scale structure in turbulence." Therefore, it is more important to obtain some continuation principle under some condition in regions of high vorticity. From this reason, we prove the following theorem;

Theorem. *Let* $s > 3/2 + 1, T > 0$. *Let* v *be a some solution of* $(\mathrm{NS})_\alpha$ *with* $\alpha \in (0, 2]$. *Let* $\beta \in (0, 1]$, $a, b \in [1, \infty]$, $q \in (1, \infty]$, $r \in (1, 3/\beta)$ *satisfy*

$$\frac{\beta}{3} < \frac{1}{r} + \frac{1}{b} \leq \frac{\beta + \alpha}{3}, \quad \frac{\alpha}{q} + \frac{\alpha}{a} + \frac{3}{r} + \frac{3}{b} \leq \alpha + \beta.$$

For fixed $K > 0$, define $\Omega(K) := \{(x,t) \in \mathbb{R}^3 \times (0,T) | |\omega(x,t)| > K\}$, where $\omega(x,t) := \nabla \times v(x,t)$.

Suppose v belongs to $E_s(T)$. And suppose that
(B1) $\omega \in L^q(0,T;L^r)$,
(B2) *there exist $g \in L^a(0,T;L^b)$, $K > 0$ such that $\eta_\beta(x,h,t) \leq g(x,t)$ for $(x,t), (x+h,t) \in \Omega(K)$.*

Then we have $\limsup_{t \nearrow T} \|v(t)\|_{H^s} < \infty$, in particular there is no singularity up to T.

Roughly speaking, the above theorem means that the assumption of regularity of the direction vector of vorticity in regions of high vorticity induces regularity of velocity to $(\mathrm{NS})_\alpha$ as is the case with the usual Navier-Stokes equations.

Remark 1.5. *Since the Theorem includes the special case with $\alpha = 2$, $a = b = \infty$ and $q = 2$, the main theorem is the generalization of Proposition 1.3 for $(\mathrm{NS})_\alpha$.*

On the other hand, the proof of this theorem is not simple generalization of them. This is because, when $\alpha < 3/2$, taking L^2 inner product of (4) by ω, we are not able to absorb the convection term into the viscosity term. To overcome this difficulty, we take L^2 inner product of (4) by $\omega|\omega|^{p-2}$, where $p \geq 3/\alpha (> 2)$.

From this reason, we prove this theorem not in the L^2-framework, but L^p. Moreover, in the usual Navier-Stokes equations case, we have some energy estimate by using initial data, which we can obtain by taking L^2 inner product of $(\mathrm{NS})_2$ by v. However, we can not obtain available estimate on $\int_0^T \|\omega(t)\|_p^p dt$, by taking L^2 inner product of $(\mathrm{NS})_\alpha$ by $v|v|^{p-2}$. From this reason, we need to use Lemma 1.2.

Remark 1.6. *When we consider the critical case of the Theorem with $K = 0$ and $a = b = \infty$, the main theorem becomes the special case of Proposition 1.4.*

We use the fact that $F^\beta_{\infty,\infty}$ norm is equivalent to C^β norm for $\beta \in (0,1)$, where $F^\beta_{\infty,\infty}$ norm is the Triebel-Lizorkin norm defined in Section 2.8 *of* [16] *and C^β norm is the Hölder-Zygmund norm. The details can be found in Remark 1.4 of* [5].

In the proof of the main theorem, splitting the vorticity to the high vorticity part and low part based on K, we need to estimate the low vorticity parts of the convection term, J_2, J_3(see equation(11)). In usual Navier-Stokes equations case, J_2 can be estimated by $\|\omega\|_2^{14/5}$, furthermore, H. Beirão da Veiga and L. Berselli [3] estimate the J_2 term by using the energy estimate on $\|\omega\|_2$.

On the other hand, we can not obtain any available estimate on $\|\omega\|_p$ as we mentioned above. Therefore, in $(\mathrm{NS})_\alpha$ case, we need to estimate J_2 by $\|\omega\|_r^{\frac{\alpha}{\alpha+\beta-\frac{3}{b}-\frac{3}{r}}} \|\omega\|_p^p$.

2. Estimate of the convection term

In this section we recall important results proved by P. Constantin [7]. Defining the strain matrix S as $(1/2(\partial_{x_j} v_i + \partial_{x_i} v_j))_{ij}$, by using (5) we obtain the following formula

$$
\begin{aligned}
(7) \qquad S &= \frac{3}{4\pi} P.V. \int_{\mathbb{R}^3} \frac{1}{2}(\hat{y} \otimes (\hat{y} \times \omega(x+y,t)) + (\hat{y} \times \omega(x+y,t)) \otimes \hat{y}) \frac{dy}{|y|^3} \\
&=: S_{[\omega]}(x,t).
\end{aligned}
$$

The integral is in the sense of principal value and $\hat{y}$ is the direction vector of $y = y/|y|$. The tensor product $\otimes$ denotes the matrix

$$(a \otimes b)_{ij} = a_i b_j \ (a = (a_i)_i, b = (b_i)_i \in \mathbb{R}^3),$$

and $\times$ is cross products. Since

$$(\omega \cdot \nabla)v \cdot \omega = S\omega \cdot \omega (=: I),$$

considering (7), the convection term can be written by

$$\begin{aligned}&(\omega(x,t) \cdot \nabla)v(x,t) \cdot \omega(x,t)\\ &= \frac{3}{4\pi} P.V. \int_{\mathbb{R}^3} (\xi(x,t) \cdot \hat{y})(\xi(x+y,t) \times \xi(x,t) \cdot \hat{y})|\omega(x+y,t)| \frac{dy}{|y|^3} |\omega|^2,\end{aligned}$$

where $\xi := \omega(x,t)/|\omega(x,t)|$. In the next section, we show the sketch of proof of main theorem in this paper.

3. Proof of the main Theorem

Let $p \geq \max\{6/\alpha - 2, 3/\alpha, 2\}$ be a finite number. Taking $L^2(\mathbb{R}^3)$ inner product of (4) by $\omega(x,t)|\omega(x,t)|^{p-2}$, we have

$$\begin{aligned}\frac{1}{p}\partial_t \|\omega\|_p^p + \nu \int_{\mathbb{R}^3} (-\Delta)^{\alpha/2}\omega \cdot \omega|\omega|^{p-2} dx &= \int_{\mathbb{R}^3} (\omega \cdot \nabla)v \cdot \omega|\omega|^{p-2} dx\\ &=: J.\end{aligned} \tag{8}$$

The viscosity term on the left hand side is estimated by

$$\int_{\mathbb{R}^3} |\omega|^{p-2}\omega \cdot (-\Delta)^{\alpha/2}\omega dx \geq \frac{2}{p}\int_{\mathbb{R}^3} \left|(-\Delta)^{\alpha/4}|\omega|^{p/2}\right|^2 dx \geq \frac{\nu C_\alpha}{p}\|\omega\|^p_{\frac{3p}{3-\alpha}}, \tag{9}$$

where we used Lemma 3.3 in [11] for the estimate of the fractional derivative in the first inequality, and used the Sobolev imbedding in the second inequality.

Let K be a positive constant in main Theorem and split $\omega(x)$ into $\omega(x) = \omega_{(1)} + \omega_{(2)}$, where

$$\omega_{(1)} = \begin{cases}\omega(x,t), \text{ if } |\omega(x,t)| \leq K\\ 0, \text{ if } |\omega(x,t)| > K\end{cases},$$

$$\omega_{(2)} = \begin{cases}0, \text{ if } |\omega(x,t)| \leq K\\ \omega(x,t), \text{ if } |\omega(x,t)| > K\end{cases}.$$

Let us decompose $S_{[\omega]}(x,t) = S_{(1)} + S_{(2)} = S_{[\omega_{(1)}]}(x,t) + S_{[\omega_{(2)}]}(x,t)$. Note that, by the Calderón-Zygmund inequality [4],

$$\|S_{(i)}\|_\zeta \leq \|\omega_{(i)}\|_\zeta \ (\zeta \in (1,\infty), i = 1,2). \tag{10}$$

Let us decompose the convection term J into the following three parts.

$$\begin{aligned}J_1 &:= \int_{\mathbb{R}^3} S_{(2)}\xi \cdot \xi|\omega_{(2)}|^p dx\\ J_2 &:= \int_{\mathbb{R}^3} S_{(1)}\xi \cdot \xi|\omega_{(2)}|^p dx\\ J_3 &:= J - J_1 - J_2.\end{aligned} \tag{11}$$

A direct calculation yields

$$|J_3| \leq C_{J_3} K\|\omega\|_p^p, \tag{12}$$

by using the Hölder inequality and (10) with $\zeta = p$.

Next, by the Hölder inequality we get

$$|J_2| \le \|\omega_{(1)}\|_{\frac{\mu}{\mu-1}} \|\omega_{(2)}\|_{p\mu}^p,$$

where $\mu \in (1, min\{\frac{3p}{3-\alpha}, \frac{r}{r-1}\})$. Moreover, using L^p-interpolation inequality and the Young inequality, we obtain

$$|J_2| \le C_{J_2} \|\omega_{(1)}\|_r^{\frac{\alpha r(\mu-1)}{3-3\mu+\mu\alpha}} \|\omega\|_p^p + C_1 \|\omega\|_{\frac{3p}{3-\alpha}}^p. \tag{13}$$

The most difficult term is J_1, and here we need the help of assumption (B2). From (7) we get

$$S_{(2)}(x,t)\omega_{(2)}(x,t) \cdot \omega_{(2)}(x,t)$$
$$= \frac{3}{4\pi} P.V. \int_{\mathbb{R}^3} (\xi(x,t) \cdot \hat{y})(\xi(x+y,t) \times \xi(x,t) \cdot \hat{y}) |\omega_{(2)}(x+y,t)| \frac{dy}{|y|^3} |\omega_{(2)}|^2.$$

Using assumption (B2) and the Hölder inequality, we get

$$|J_1| \le \frac{3}{4\pi} \|g\|_b \left\|I_\beta(|\omega|)(x)\right\|_q \|\omega\|_{pk}^p,$$

where b, q, k satisfy

$$\frac{1}{b} + \frac{1}{q} + \frac{1}{k} = 1, \ b, q, k \ge 1, \tag{14}$$

and $I_\beta(\cdot), 0 < \beta < 3$, is the operator defined by the Riesz potential as follows.

$$I_\beta(|\omega|)(x) := \gamma(\beta) \int_{\mathbb{R}^3} \frac{\omega(x+y)}{|y|^{3-\beta}} dy, \ \gamma(\beta) := 2^\beta \pi^{3/2} \frac{\Gamma(\frac{\beta}{2})}{\Gamma(\frac{3-\beta}{2})}.$$

On the other hand, using the standard L^p-interpolation inequality and the Young inequality, we have

$$|J_1| \le C_{J_1} \|g\|_b^{\frac{\alpha}{\alpha+\beta-\frac{3}{b}-\frac{3}{r}}} \|\omega\|_r^{\frac{\alpha}{\alpha+\beta-\frac{3}{b}-\frac{3}{r}}} \|\omega\|_p^p + C_2 \|\omega\|_{\frac{3p}{3-\alpha}}^p, \tag{15}$$

where

$$\frac{1}{q} = \frac{1}{r} - \frac{\beta}{3}, \ r > 1.$$

Here, we choose μ satisfying $\mu < \frac{3+r(\alpha+\beta-\frac{3}{b}-\frac{3}{r})}{3+r(\alpha+\beta-\frac{3}{b}-\frac{3}{r})-\alpha}$. Combining (8) with (9), (12), (13) and (15), we derive

$$\partial_t \|\omega\|_p^p + \nu C_\alpha \|\omega\|_{\frac{3p}{3-\alpha}}^p \le C \|g\|_b^{\frac{\alpha}{\alpha+\beta-\frac{3}{b}-\frac{3}{r}}} \|\omega\|_r^{\frac{\alpha}{\alpha+\beta-\frac{3}{b}-\frac{3}{r}}} \|\omega\|_p^p. \tag{16}$$

The Gronwall lemma applied to (16) with the Hölder inequality provides

$$\|\omega(x,t)\|_p^p < \infty, \ \text{(by assumption (B1), (B2))}. \tag{17}$$

Lastly, integrating (16) over $[0,T]$, we obtain

$$\int_0^T \|\omega\|_{\frac{3p}{3-\alpha}}^p dt < \infty.$$

Hence, applying Lemma 1.2, we find that

$$limsup_{t \nearrow T} \|v(t)\|_{H^s} < \infty.$$

□

Acknowledgement.

The author thanks Prof. T. Yoneda for kind help for his research on the Navier-Stokes equation, and Mr. T. Kashiwabara for useful conversation.

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