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Conjecture about Regularity of Prefix Square Roots of Regular Languages

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Zsolt Fazekas, Robert Mercas, Daniel Reidenbach gave the conjecture in [2] which gives necessary and sufficient condition for the primitive prefix square root of a regular language L to be regular. The author gives a counterexample of their conjecture and gives a new conjecture.

1. Preliminary

An *alphabet* V is a finite and nonempty set of symbols, called *letters*. Every finite sequence of letters of V is called a *word* over V. Words over V together with the operation of concatenation with the *empty* word ε form a free monoid V^*. We denote $V^+ = V^* - \{\varepsilon\}$.

Let $w = a_1a_2 \cdots a_n$ where $a_1, a_2, \cdots, a_n \in V$. The *length* of a word w is n and denoted by $|w|$ and the length of the empty word ε is 0.

For a positive integer p,

$$V^{\le p} = \{w \in V^* \mid |w| \le p\},$$

$$V^p = \{w \in V^* \mid |w| = p\}.$$

For a word $w = xyz$ for $x, y, z \in V^*$, a *prefix* of w is x, a *factor* of w is y and a *suffix* of w is z.

For a word $w \in V^+$, the following operations are defined in [1]:

- prefix square reduction: $\Box\!\mid(w) = \{uv \mid w = uuv, \text{ for } u \in V^+, v \in V^*\}$

- suffix square reduction: $|\square(w) = \{uv \mid w = vuu, \text{ for } u \in V^{+}, v \in V^{*}\}$
- prefix-suffix square reduction: $\square|\square(w) = \square|(w) \cup |\square(w)$

For simplicity, we restrict the argument to prefix square reduction.
We define the bounded version for a fixed positive integer p:

- *p-prefix square reduction*: ${}_{p}\square|(w) = \{uv \mid w = uuv, \text{ for } u \in V^{\leq p}, v \in V^{*}\}$

For a language L, we have language: $\square|(L) = \bigcup_{w \in L} \square|(w)$.

The following languages are defined:

$$\square|^{0}(w) = \{w\},$$

$$\square|^{k+1}(w) = \square|(\square|^{k+1}(w)) \quad \text{for any } k \geq 0$$

$$\square|^{*}(w) = \bigcup_{k \geq 0} \square|^{k}(w) \ .$$

For a word w, the *primitive prefix square root* of w is the set $\{u \mid u \in \square|^{*}(w)$ and $\square|(u) = u\}$ and it is denoted by $\sqrt[\square|]{w}$. The primitive *bounded* prefix square root of w is the set $\{u \mid u \in {}_{p}\square|^{*}(w)$ and ${}_{p}\square|(u) = u\}$ and it is denoted by $\sqrt[p\square|]{w}$. For a language L, we define $\sqrt[\square|]{L} = \bigcup_{w \in L} \sqrt[\square|]{w}$ and $\sqrt[p\square|]{L} = \bigcup_{w \in L} \sqrt[p\square|]{w}$.

2. Conjectures

Zsolt Fazekas, Robert Mercas, Daniel Reidenbach gave the following conjecture in [2].

Conjecture (in [2]). Let L be a regular language. The primitive prefix square root of L is regular if and only if there exists some positive integer p such that $\sqrt[\square|]{L} = \sqrt[p\square|]{L}$.

But, I give here the following counterexample and new conjecture.

Example. Let $L = aab^{+}aab^{+}c$ where $a,b,c \in V$. The language L is regular. On the other hand, the primitive prefix square root of L is $\sqrt[\square|]{L} = ab^{+}aab^{+}c \cup ab^{+}c$ and this language is regular.

But, there is no positive integer p such that $\sqrt[\Box]{L} = \sqrt[p\Box]{L}$.

Now, we define a new term to describe our new conjecture: For a word w , if xx is a non-trivial prefix of w and x is prefix square free, then we say that xx is the *minimal* prefix square of w .

Conjecture . Let L be a regular language. The primitive prefix square root of L is regular if and only if there exists positive integer N such that, for every word $w \in \Box^{*}(L)$, the length of the minimal prefix square of w is smaller than N .

References

[1] P. Bottoni, A. Labella and V. Mitrana, Theor. Comput. Sci., **682**, (2017), pp 49-56.

[2] Szilárd Zsolt Fazekas, Robert Mercas, Daniel Reidenbach, On the Prefix-Suffix Duplication Reduction, International Journal of Foundations of Computer Science (in print).

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