Mathematica in Teaching Basics of Stereoscopy

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Abstract. *We use Wolfram Mathematica in teaching the basics of stereoscopy. 3D graphics in Mathematica allows teachers and students to easily construct simple stereo images and manipulate constructed images to demonstrate different effects and distortions introduced by incorrect methods of constructing stereo images. We demonstrate basic stereoscopic effects and typical mistakes made by beginners. In particular, we demonstrate some basic tricks one can do with changing the position of the stereo window. Compactness of the Mathematica code facilitates students' experiments and helps in understanding basic concepts and ideas of stereoscopy and computer vision.*

1   Introduction

Soon after the invention of photography in 1839, a first suggestion for a stereo camera was given by Brewster in 1847. By 1860 viewing stereo photos was a popular pastime [1]. Later - in the 20th century - interest waned. Recently, with the development of technology, interest in 3D imaging has increased: 3D cameras are available; 3D lenses exist for cameras and smartphones; 3D cartoons and movies can be watched on an ordinary TV or computer screen using anaglyph (red-cyan) glasses; there are special 3D TVs, 3D movie cinemas, glasses-free 3D displays, etc.; and, finally, the recent development of virtual reality has stimulated interest in 3D imaging.

Modern Computer Algebra Systems (CAS) - for example, *Mathematica*, Maple, Maxima, and others - provide not only opportunity for analytic and numerical calculations, but also convenient tools for image construction, manipulation and animations. It makes CAS a good resource to use in the classroom. We use Wolfram *Mathematica* in teaching the basics of stereoscopy.

We demonstrate basic stereoscopic effects and typical mistakes made by beginners. In particular, we demonstrate the importance of correctly choosing the stereo window and its manipulation. A stereo image can be represented as a stereopair (for parallel and cross-eye views). By changing the parallax of the left and right parts of the stereogram, one can move the image (or some part of it) in front of or behind the window frame. Our manipulations with stereo images are based on a classic textbook by J.G. Ferwerda [1].
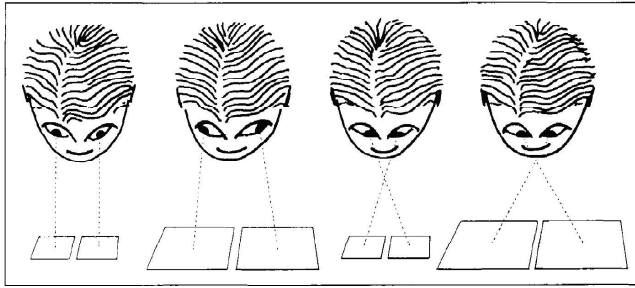
Figure 1: Viewing a stereo pair. Left-to-right: parallel view; diverging view lines; cross-eye view; cross-eye view when viewing large pictures. Adopted from [1].

2 Viewing stereo pairs

There are special spectacles, glasses and masks to view stereo pairs, but it is also possible to view stereo pairs without special equipment. Two basic methods can be used for (glasses-) free view: the method with parallel view lines and with transverse view lines (cross-eye) [1], see Figure 1. One should bear in mind that view lines should not diverge for parallel view (i.e. size of each image should not exceed 6 cm). This restriction is waved for cross-eye view, though viewing too large pictures can also cause problems.

One can practice viewing stereo pairs using Figure 2. Though it was intended by the author of [1] for practicing cross-eye views, it can be used for practicing parallel views as well. If one looks at Figure 2 cross-eye (i.e., using cross-eye view), pyramids will be seen. If one uses parallel view, shallow (inverted) pyramids will be seen.

For many people it is easy to start with cross-eye view: place a finger (or tip of the pencil, etc.) between your eyes and the image and look attentively with both eyes at it. Then start paying attention to what your finger (pencil) is actually pointing at, without moving your eyes. In the general case the image you will see will be double. By moving your finger closer to (or further from) the image, you can change relative positions of these double images. Once two "squares with crosses" coincide, you will see a stereo image. One can make a 3D image by using nearby squares (1 and 2, 2 and 3, etc.) or every other (1 and 3, 2 and 4, etc.) - in the last case the observed pyramids will be steeper.

To practice a parallel view one can try "to look through the picture" or use a tubular view using folded paper (or just use palms) to restrict the field of view so that each eye can see only the proper part of the image.

After practicing (if one needs experience) by completing exercises successfully (see Figures 2, 3), one can try to view a stereo image from a "real" stereo pair in Figure 3. In this figure and further on the text a stereo image is given as a set of three pictures: LRL, where L stands for the left-eye view and R for the right-eye view. The left two images (LR) should be used for the parallel view, and the right two (RL) for the cross-eye view.
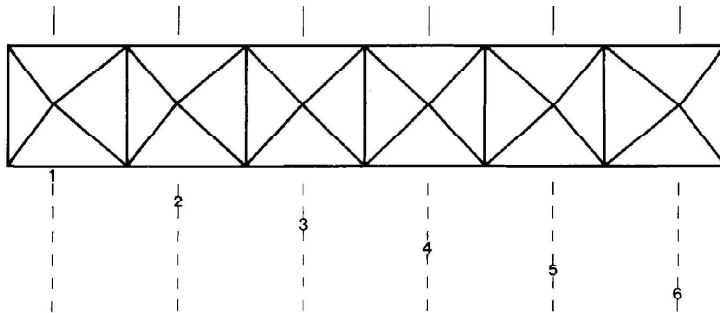
Figure 2: The stereo exercise field. Adopted from [1].

Figure 3: Stereo image for parallel and cross-eye view. Left pair of images should be used for parallel view, right two images - for cross-eye view.

3   Experiments with stereo window

Here, we demonstrate some basic tricks one can do by changing the position of the stereo window; and some basic mistakes of beginners (and not only beginners - similar mistakes can be seen in big-screen, multi-million-dollar budget movies!).

One of the standard "Wow!" effects with 3D images is when most of the image appears behind the frame, but part of the image is hanging out in front (see Figure 4). One has to be careful: that part shouldn't be cut by the frame! It is especially important in action scenes when objects which are moving around can easily be cut by the frame border. For example, in Figure 5 lower part of the image stays in front of the stereo window, and it looks impressive until one notices that at the same time it is cut (shadowed) by the lower border of the stereo window frame that is behind it.

The position of the object with respect to the stereo window can be changed by shifting horizontally the left and right parts of the stereogram ([1], see also [2] and references therein), see Figure 6. *Mathematica* provides good tools for image processing and manipulation [3]. One can easily manipulate with relative parallax of the left and right images to change the position of the image with respect to the stereo window. For example, compare series of Figures 3, 7, 8. On Fugure 3, image

Figure 4: Flower is hanging out in the middle while rest of the image is behind stereo window.

Figure 5: Example of the bad "Wow!" effect: lower part of the image is in front of the stereo window, but it is cut by the lower border of window frame that is behind it.

is behind stereo window. On Fugure 7 it is closer with smaller pine hanging out a little. Figure 8 gives an example of bad positioning of stereo window: foreground is in front of the stereo window, but it is cut by the window frame that is supposed to be behind it. See also Figures 9 - 11.

4   Conclusions

Mathematica allows easily construct simple "artificial" stereo images that can be used for experiments. Examples are given on Figures 12 and 13.

All experiments described above can be also performed with animation. *Mathematica* also makes easy demonstration of different distortions that could be introduced by using improper choice of line-of-sight, etc. Build-in methods for solving systems of linear and non-linear equations make constructing random dot stereograms (autostereograms) [4] easy. They also introduce some entertainment elements that help keep the interest of students on the educational process.
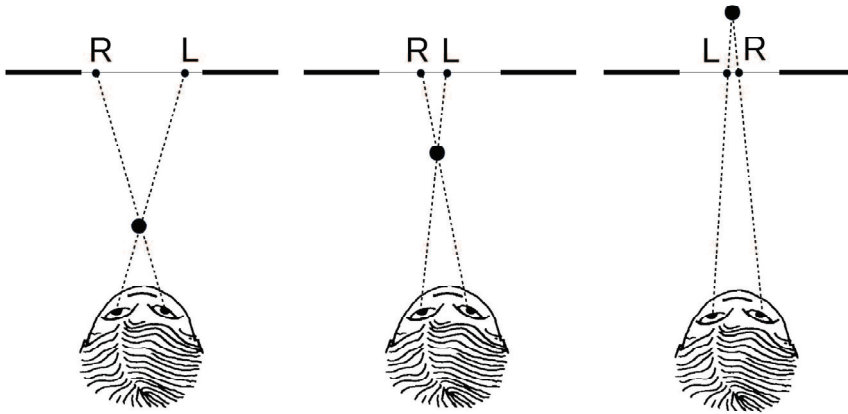
Figure 6: One can change position of the object with respect to the window by shifting horizontally (left or right) left and right parts of the stereogram.

Figure 7: Same as Figure 3 but relative parallax changed so that image is closer to stereo window.

References

[1] *Ferwerda J.G.* The World Of 3-D: A Practical Guide To Stereo Photography. 3-D Production (2003)

[2] *T*he Stereographics Developer's Handbook - Background on Creating Images for CrystalEyes® and SimulEyes®, StereoGraphics Corporation (1997)

[3] *Hastings C., Mischo K., Morrison M.* Hands-on Start to Wolfram MATHEMATICA and Programming with the Wolfram Language. Second edition. Wolfram Media (2016)

[4] *Terrel M.S., Terrel R.E.* Behind the Scenes of a Random Dot Stereogram. The American Mathematical Monthly, Vol. 101, No. 8. pp. 715-724. (1994)

Figure 8: Same as Figures 3 and 7 but relative parallax changed so that foreground on the image is in front of the stereo window.

Figure 9: Monument (of Peter the Great) is behind stereo window.

Figure 10: Hand of the monument stays out a little while most of the image is behind stereo window.

Figure 11: The monument is in front of stereo window, but feet are cut by the frame that is supposed to be behind it.
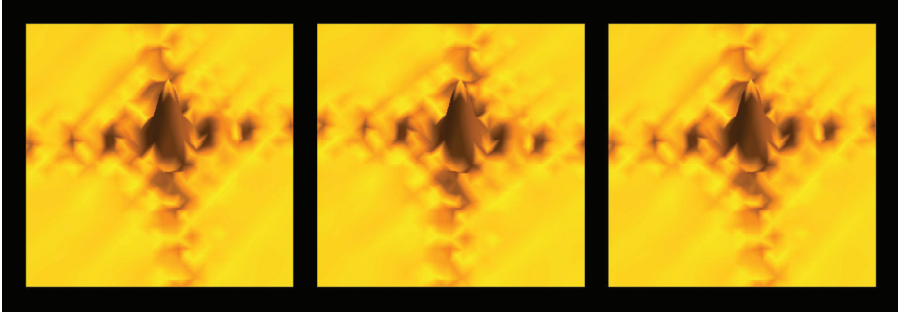
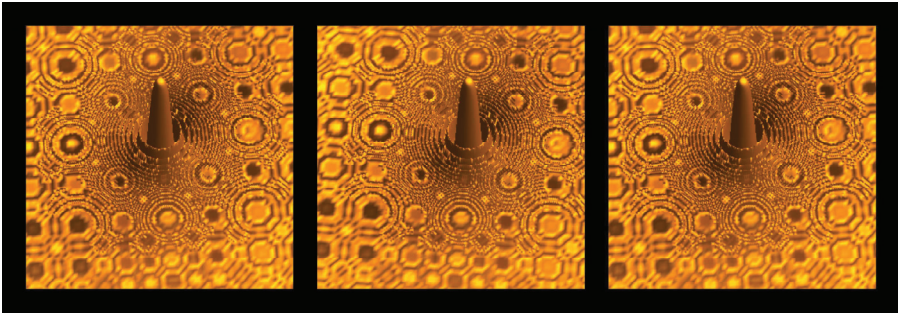Figure 12: Example of the 3D image generated in *Mathematica*.

Figure 13: Another example of the 3D image generated in *Mathematica*.