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Application of singular stochastic control theory to fish-eating waterfowl population management

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1. Introduction

Phalacrocorax carbo (*P. carbo;* Great Cormorant) is a fish-eating bird having worldwide distribution including Japan, Europe and North America (Fukuda et al., 2000; Bzoma et al., 2003; van Eerden et al, 2012; Doucette et al., 2011) and each individual adult bird eats 500 (g) of fishes per day (Yamamoto, 2008, 2009). Their population in Japan has recently been rapidly increasing, which leads to the excessive predation from the bird to riverine fishes in the country (Yamamoto, 2008, 2009). To overcome this severe situation, local fishery cooperatives and governments have empirically taken various countermeasures, such as gun shooting (sharp shooting especially in Lake Biwa), expulsion by fireworks and guns, and freezing eggs using dry ice (Yamamoto, 2008, 2009). On the other hand, *P. carbo* is not an alien species at least in Japan. Thus, *P. carbo* should not be exterminated in the management policies. Furthermore, they provide ecosystem services, such as nutrient cycling if it is not excessive as mentioned above (Green and Elmberg, 2014; Kameda et al., 2006).

Feeding damage from *P. carbo* to *Plecoglossus altivelis* (*P. altivelis*; Ayu), which is one of the most economically and culturally important inland fishery resources in Japan (Takahashi et al., 2006), is severe in particular. The fish catch of *P. altivelis* accounts for 7.3% (2.4×10^{6} (kg)) in Japanese inland fisheries (Ministry of Agriculture, Forestry, and Fisheries, 2016) and has served as their main source of income. For maintaining population of *P. altivelis*, inland fishery cooperatives have released the farmed fish in rivers. Hence, we need to establish a sustainable management policy of the bird population that can effectively suppress the predation from the bird to riverine fishes while the bird population should not totally be exterminate.

We approach the above-mentioned issue from the perspective of mathematical models. In this paper, a singular stochastic optimal control model (Pham, 2009; Tsujimura and Maeda, 2016) is employed to find a sustainable management policy of the population of *P. carbo* (Yaegashi et al., 2017b, Yaegashi et al., 2017c; Yaegashi et al., 2018). A singular stochastic control model is based on a controlled stochastic differential equation (SDE) (Øksendal, 2003) with a performance index to be maximized or minimized by choosing an appropriate control. A threshold-type optimal policy is derived in the model. Singular stochastic control models have been studied in detail in finance, economics, insurance, and related research areas (Al Motairi and Zervos, 2017; Azcue and Muler, 2014; Cadenillas and Huamán-Aguilar, 2016; Song and Zhu, 2016). They have been employed for finding simple resource management policies subject to stochastic dynamics as well (Lungu and Øksendal, 1997; Alvarez, 1998; Alvarez,1999). However, its application to predator population management is still rare to the authors' knowledge. This paper therefore focuses on an exploratory approach of a singular stochastic control model to a predator management problem. Our approach reduces finding a management policy of the bird population to an exactly-solvable variational inequality. A threshold-type, sustainable predator suppression policy for the bird population is derived from this variational inequality.

The rest of this paper is organized as follows. The mathematical model with one variable is presented in section 2. Section 3 presents the model with two variables and the numerical method for the associated variational inequality. Then, a demonstrating computation result is shown. Section 4 concludes this paper and gives future perspectives of our research.

2. One variable model

In one-variable model, the dynamics of *P. carbo* is only considered during an infinite period (Yaegashi et al., 2017b; Yaegashi et al., 2018).

2.1 Stochastic differential equation

The temporal evolution of the population of *P. carbo* in a habitat is described by a controlled SDE. The SDE is a linear stochastic population growth model driven by a multiplicative noise subject to the population decrease by a countermeasure to the bird. In this paper, the bird population is treated as a continuous variable assuming that it is sufficiently large. The bird population dynamics is assumed to follow the Ito's SDE (Øksendal, 2003; Pham, 2009)

$$\mathrm{d}X_t = X_t\left(\mu \mathrm{d}t + \sigma \mathrm{d}B_t\right) - \mathrm{d}\eta_t,\quad X_{-0} = x \geq 0, \tag{1}$$

with the conventions $\lim_{s \nearrow t} X_s = X_{t-0}$ for $t > 0$ and $\lim_{s \nearrow 0} X_s = X_{-0}$. Here, X_t is the total number of the bird at the time t, $\mu > 0$ is the deterministic growth rate of the population, $\sigma > 0$ is the magnitude of stochastic fluctuation involved in the population dynamics, and B_t is the 1-D standard Brownian motion on the complete probability space (Pham, 2009) whose filtration is right-continuous and satisfies the usual conditions (Karatzas and Shreve, 2012). Hereafter, an assumption on the model parameters of the SDE (1) (Grigoriu, 2014)

$$\mu > \frac{\sigma^2}{2} \tag{2}$$

is employed, which means that the bird population without countermeasures ($\eta_t = 0$ for all t) does not become extinct. The variable η_t represents the right-continuous, adapted process (Pham, 2009) that represents the decrease of the population through a countermeasure such as gun shooting, which directly reduces the bird population. Formally, the increment $\mathrm{d}\eta_t$ is rewritten as

$$\mathrm{d}\eta_t = u_t \mathrm{d}t \tag{3}$$

with a measurable process $u_t \in [0,\infty)$ that represents the killed population by the countermeasure per unit time. We assume $\eta_{-0} = 0$, meaning that η_t is identified as the total bird population that has been killed during the time interval $(0,t)$.

2.2 Performance index

The decision-maker of the present model (a local fishery cooperatives or a local government) manages the population of *P. carbo*. The performance index is an index that should be maximized by the decision-maker through choosing an optimal control $\eta_t = \eta_t^*$. The performance index for an admissible η_t is denoted as $v = v(x;\eta)$, and is set as

$$v(x;\eta) = \mathrm{E}\left[\int_0^\infty e^{-\delta s}\left(RX_s^M - SX_s^m\right)\mathrm{d}s - \int_0^\infty e^{-\delta s}\mathrm{d}\eta_s\right] \quad (4)$$

where $\mathrm{E}[\cdot]$ is the expectation conditioned on $X_{-0} = x \geq 0$, $\delta > 0$ is the discount rate of the profit, S, R, m, and M are model parameters which satisfy $S, R \geq 0$ and $0 < M < 1 < m \leq 2$. The performance index v represents the expected net profit of the decision-maker. The discount rate δ represents the attitude of the decision-maker on management of the bird population; larger δ means that he/she performs the suppression from a longer-term viewpoint. This is because no sustainable management policy may be obtained with small δ. Hereafter, the conditions for the parameters

$$\delta > \mu m + \frac{\sigma^2}{2}m(m-1) \quad (5)$$

are assumed, meaning that the decision-maker manages the bird population from a sufficiently long-term, sustainable viewpoint. The term $-SX_s^m$ quantifies the loss of the riverine fishes by the predation from *P. carbo* per unit time and the term RX_s^M represents the ecosystem services per unit time that *P. carbo* can provide (Zedler and Kercher, 2005). The terms $SX_s^m - RX_s^M$ is unimodal and convex with respect to X_s. When there is no bird population ($X_s = 0$), neither the cost nor the profit arises. The second term in the right-hand side of (4) represents the cost of taking the countermeasure. The parameters S and R are weights on the first and second terms of the performance index v, which depend on the attitude of the decision-maker on the bird population management.

2.3 Variational inequality

The value function $V(x)$ is defined as the maximized performance index v:

$$V(x) = \sup_\eta v(x;\eta) = v(x;\eta^*). \quad (6)$$

Applying the dynamic programming principle (Pham, 2009) leads to the variational inequality

$$\inf\left(\mathcal{E}V + Sx^m - Rx^M, \frac{\mathrm{d}V}{\mathrm{d}x} + 1\right) = 0 \quad \text{in } x > 0 \quad (7)$$

with the degenerate elliptic operator

$$\mathcal{E}V = \delta V - \mu x\frac{\mathrm{d}V}{\mathrm{d}x} - \frac{1}{2}\sigma^2 x^2 \frac{\mathrm{d}^2 V}{\mathrm{d}x^2}. \quad (8)$$

The boundary condition is prescribed as $V = 0$ at $x = 0$. The boundary condition in (7) means that neither the profit nor the loss arises if there is no bird population ($x = 0$). The left part in "min" operator corresponds to the situation where the countermeasure should not be taken, while the right part corresponds to the situation where the countermeasure should immediately be taken.

2.4 Exact solution

An exact solution to the variational inequality (7) can be found in this case, which is the value function defined in (6) and it is a classical solution. With the assumption (5), an application of an analytical technique following Chapter 4.5 of Pham (2009) gives the unique solution

$$V(x)=\begin{cases} ax^k + Ax^m + Bx^M & (0 < x \le \bar{x}) \\ b - x & (x > \bar{x}) \end{cases} \tag{9}$$

with

$$k=\frac{1}{2}\left(1-\frac{2\mu}{\sigma^2}+\sqrt{\left(\frac{2\mu}{\sigma^2}-1\right)^2+\frac{8\delta}{\sigma^2}}\right)(>m) \tag{10}$$

and

$$A=\frac{-S}{\delta-\mu m-\frac{\sigma^2}{2}m(m-1)}<0,\quad B=\frac{R}{\delta-\mu M-\frac{\sigma^2}{2}M(M-1)}>0. \tag{11}$$

Here a, b, and $\bar{x}$ are the unknowns that solve the flowing system of nonlinear equations

$$\begin{cases} a\bar{x}^k + A\bar{x}^m + B\bar{x}^M = b-\bar{x} \\ ka\bar{x}^{k-1} + mA\bar{x}^{m-1} + MB\bar{x}^{M-1} = -1 \\ k(k-1)a\bar{x}^{k-2} + m(m-1)A\bar{x}^{m-2} + M(M-1)B\bar{x}^{M-2} = 0 \end{cases}, \tag{12}$$

and $\bar{x}$ is the threshold for suppression. Combining the second and third equations of (12) leads to the governing algebraic equation of $\bar{x}$ as

$$\bar{x}^{m-1}=\frac{-\left[MB\bar{x}^{M-1}(k-M)+k-1\right]}{mA(k-m)}. \tag{13}$$

By the classical intermediate value theorem, Eq.(13) has a unique solution such that $0<\bar{x}<\infty$. The other unknowns a and b are obtained from the first and second equations of (12) with determined $\bar{x}$. Note that the solution (9) is a classical solution: $V \in C^2(0,\infty)\cap C[0,\infty)$. The solution (9) indicates that the countermeasure should be taken only when the bird population X_t is about to exceed the threshold $\bar{x}$: otherwise, the countermeasure should not be taken.

3. Two variable model

In two-variable model, the population dynamics of *P. carbo* and *P. altivelis* are simultaneously considered during a finite period (Yaegashi et al., 2017c).

3.1 Stochastic differential equations

A predator-prey dynamics between *P. carbo* and *P. altivelis* during a finite period, spring to the coming autumn in a year, is considered. The dynamics of *P. carbo* is renewable while that of *P. altivelis* is not. The time is denoted as $t \in [0,T)$ with the terminal time T, which is the time when all the *P. altivelis* die. The dynamics of *P. altivelis* consist of its total population N_t and the weight W_t, and are assumed to be deterministic. The governing equations of N_t and W_t for $t \ge 0$ are described as

$$\mathrm{d}N_t = -(D + aX_t + \chi_{\tau \ge t}c)N_t\mathrm{d}t \tag{14}$$

and

$$\mathrm{d}W_t = r\left(1-\frac{W_t}{K}\right)W_t\mathrm{d}t \tag{15}$$

where D is the mortality rate of *P. altivelis*, a is a positive constant that modulates the predation pressure from *P. carbo* to *P. altivelis*, τ is the opening time of harvesting *P. altivelis*, $\chi_{\tau\geq t}$ is the indicator function such that $\chi_{\tau\geq t}=1$ for $t\geq\tau$ and $\chi_{\tau\geq t}=0$ otherwise, c is the harvesting pressure by human, r is the intrinsic growth rate of *P. altivelis*, and K is the maximum body weight of *P. altivelis*. In the two-variable model, the population dynamics of *P. carbo* for $t\geq 0$ is described by the following Itô's SDE

$$\mathrm{d}X_t = X_t\left(\mu\mathrm{d}t+\sigma\mathrm{d}B_t-\mathrm{d}\eta_t\right) \tag{16}$$

where η_t represents decrease of the growth rate of the bird by an indirect countermeasure.

3.2 Performance index

The performance index in two-variable model is set as

$$v(t,x,n;\eta)=\mathrm{E}\left[\alpha\int_t^T e^{-\delta s}\left(RX_s^M-SX_s^m\right)N_t\mathrm{d}s+\beta\int_\tau^T e^{-\delta s}cN_tW_t\mathrm{d}s-\gamma\int_t^T e^{-\delta s}\mathrm{d}\eta_s\right] \tag{17}$$

where $\mathrm{E}[\cdot]$ is the expectation conditioned on $X_{-0}=x\geq 0$ and $N_{-0}=N_0=n\geq 0$, and $\alpha>0$, $\beta>0$ and $\gamma>0$ are weight constants. Without loss of generality, $\gamma=1$ is assumed.

3.3 Variational inequality

The value function $V(x)$ is defined as the maximized performance index v:

$$V(t,x,n)=\sup_\eta v(t,x,n;\eta)=v(t,x,n;\eta^*). \tag{18}$$

By applying the dynamic programming principle (Pham, 2009) leads to the variational inequality,

$$\inf\left\{\mathcal{P}V+\alpha\left(Sx^m-Rx^M\right)n-\chi_{\tau\geq t}\beta cnW_t,\ x\frac{\partial V}{\partial x}+1\right\}=0\ \text{ in } [0,T)\times\Omega \tag{19}$$

with the degenerate parabolic operator

$$\mathcal{P}V=-\frac{\partial V}{\partial t}+\delta V-\mu x\frac{\partial V}{\partial x}-\frac{\sigma^2x^2}{2}\frac{\partial^2 V}{\partial x^2}+\left(D+ax+\chi_{\tau\geq t}c\right)n\frac{\partial V}{\partial n} \tag{20}$$

where the domain Ω of (x,n) is defined as $(0,+\infty)\times(0,+\infty)$. The terminal condition $V=0$ is prescribed at $t=T$ and the boundary condition $V=0$ along $x=0$ and $n=0$. The boundary conditions mean that there is no profit and loss when there is no *P. altivelis* or no *P. carbo* ($x=0$ or $n=0$). It is expected that there exists the free boundary C fulfilling the following requirement uniquely exists; at each time $t\in[0,T)$, the domain Ω is divided as $\Omega=\Omega_\mathrm{L}\cup\Omega_\mathrm{R}\cup C$ with $\Omega_\mathrm{L}\cap\Omega_\mathrm{R}=\varnothing$ where Ω_L and Ω_R are sub-domains defined as

$$\Omega_\mathrm{L}=\left\{(x,n)\middle| PV+\alpha\left(Sx^m-Rx^M\right)n-\chi_{\tau\geq t}\beta cnW_t=0,\ x\frac{\partial V}{\partial x}+1>0\right\} \tag{21}$$

and

$$\Omega_\mathrm{R}=\left\{(x,n)\middle| PV+\alpha\left(Sx^m-Rx^M\right)n-\chi_{\tau\geq t}\beta cnW_t>0,\ x\frac{\partial V}{\partial x}+1=0\right\}. \tag{22}$$

In the sub-domain Ω_L the countermeasure should not be performed, while it should immediately be performed in the sub-domain Ω_R.

3.4 Numerical method

The domain Ω is divided into the 1-D domains $x \in (0,L)$ and $n \in (0,N_0)$ with a large truncated parameter $L > 0$ and the number of the initial population of *P. altivelis* N_0 for the sake of numerical computation. For numerically solving the variational inequality (19), a conventional penalty method and the three-stage operator-splitting technique (Glowinski et al., 2016) are adopted as

$$-\frac{\partial V}{\partial t} + \delta V - \mu x \frac{\partial V}{\partial x} - \frac{\sigma^2 x^2}{2} \frac{\partial^2 V}{\partial x^2} + \alpha \left(S x^m - R x^M \right) n + \lambda \min \left(0, x \frac{\partial V}{\partial x} + 1 \right) = 0 \qquad (23)$$

in $(t,x) \in (0,T) \times (0,L)$ and

$$-\frac{\partial V}{\partial t} + \left(D + ax + \chi_{\tau \geq t} c \right) n \frac{\partial V}{\partial n} - \chi_{\tau \geq t} \beta c n W_t = 0 \qquad (24)$$

in $(t,n) \in (0,T) \times (0,N_0)$. Here, λ is the penalty parameter, which should be taken as a large number. Each time step is marched as the following process: Eq.(24) is integrated with the half increment $\Delta t / 2$, then Eq.(23) is integrated with the increment Δt, and finally Eq.(24) is again integrated with the half increment $\Delta t / 2$. A fully implicit discretization is employed at each stage for both Eqs.(23) and (24). The Petrov-Galerkin finite element scheme (Yoshioka et al., 2014) is adopted for spatial discretization of Eq.(23) except for the penalty term. The conventional first order upwind difference method is used for the penalty term in (23) and is also applied to the spatial discretization of (24). For the numerical computation, the boundary conditions are supplemented as $x \frac{\partial V}{\partial x} + 1 = 0$ along $x = L$. No boundary condition is unnecessary along $n = N_0$ considering the characteristics of the first equation of (14). Note that the scheme has preliminary been applied to an exactly solvable, simpler variational inequality for its accuracy verification.

3.5 Results

The 1-D domains $(0,L)$ and $(0,N_0)$ are discretized into 250 elements. The time increment for temporal integration is set as $\Delta t = 0.01$. The parameter values are estimated based on the previous research (Yaegashi et al., 2017a, Yaegashi et al., 2017b) and are set as $D = 3.9 \times 10^{-3}$ (1/day), $a = 1.0 \times 10^{-2}$ (1/day), $c = 1.0 \times 10^{-2}$ (1/day), $N_0 = 1.0 \times 10^{6}$ (-), $r = 3.7 \times 10^{-2}$ (1/day), $K = 6.5 \times 10^{-2}$ (kg), $W_0 = 4.0 \times 10^{-3}$ (kg), $\mu = 4.7 \times 10^{-4}$ (1/day), $\sigma = 4.0 \times 10^{-4}$ (1/day$^{1/2}$), $S = 2.0$ (-), $R = 1.0 \times 10^{-10}$ (-), $m = 4.0$ (-), $M = 0.5$ (-), $T = 180$ (day) and $L = 1.0 \times 10^{4}$ (-). The decision-maker-dependent parameters are $\delta = 1.0 \times 10^{-9}$ (1/day), $\alpha = 1.0 \times 10^{-2}$ (-) and $\beta = 1.0 \times 10^{-7}$ (1/kg). Figure 1 shows the sub-domains Ω_{L} and Ω_{R} and the profiles of the free boundary at $t = 30$ (day); green, $t = 60$ (day); blue, $t = 178$ (day); pink, and $t = 179$ (day); red. Figure 1 indicates that the free boundary C depends on both the predator population x and the fish population n. In addition, the assumption on the existence of the free boundary is satisfied; the domain Ω is indeed divided into the sub-domains Ω_{L} and Ω_{R}. For large x, performing the countermeasure is optimal (Ω_{L}), while for small x not performing the countermeasure is optimal (Ω_{R}). The profiles of the free boundary C implies that the threshold $\bar{x}$ should be decreased as the number of remaining fish n increases for all t against this model parameters. The free boundary C seems not to move between $t = 0$ (day) and $\tau = 60$ (day), the opening time of harvesting *P. altivelis*; green line. Then, the free boundary suddenly moves downward just after τ, and after τ the free boundary C seems not to move until $t = 176$ (day); blue line. After $t = 176$ (day), the free boundary finally begins to move upward around the terminal time; pink line and red line.

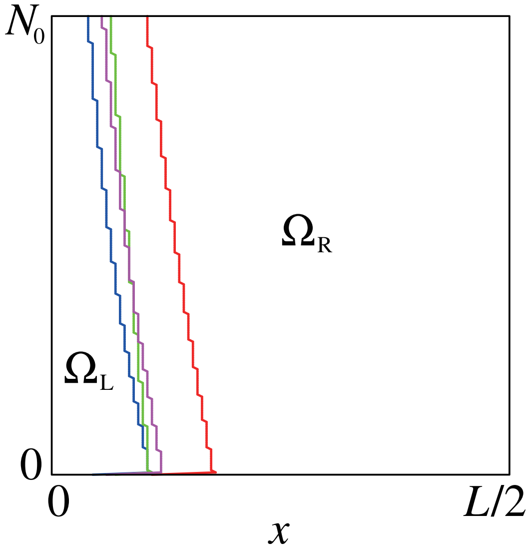

Figure 1. The sub-domains Ω_L and Ω_R and the profiles of the free boundary at $t = 30$ (day); green, $t = 60$ (day); blue, $t = 178$ (day); pink, and $t = 179$ (day); red.

4. Conclusions and future plans

This paper proposed singular stochastic control models for a sustainable population management policy of *P. carbo*. In addition, the numerical method for the associated variational inequality was also presented.

Future research topics are summarized as follows.

1) To estimate the model parameters (especially the ecosystem services)
2) To extend the one-variable model to a seasonaly-dependent counterpart
3) To compare the extended one-variable model and the two-variable model (especially profiles of the free boundary)
4) To extend the population dynamics of *P. carbo* to Verhulst model
5) To extend the model parameters in Verhulst model to time-dependent (μ and σ)
6) To incorporate an age structure into the population dynamics of *P. carbo*
7) To validate the current threshold in Lake Biwa ($\bar{x} = 4{,}000$)
8) To validate the feasibility of a singular control
9) To construct a model with based on an impulse control (Tsujimura and Maeda, 2016)

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References

[1] Al Motairi, H., & Zervos, M. (2017). Irreversible capital accumulation with economic impact. Applied Mathematics & Optimization, 75(3), 525-551.

[2] Alvarez, L.H. (1998). Optimal harvesting under stochastic fluctuations and critical depensation. Mathematical Biosciences, 152(1), 63-85.

[3] Alvarez, L.H. (1999). A class of solvable singular stochastic control problems. Stochastics, 67(1-2), 83-122.

[4] Azcue, P., & Muler, N. (2014). Stochastic optimization in insurance: a dynamic programming approach. Springer New York, Heidelberg, Dordrecht, London, 1-146.

[5] Cadenillas, A., & Huamán-Aguilar, R. (2016). Explicit formula for the optimal government debt ceiling. Annals of Operations Research, 247(2), 415-449.

[6] Doucette, J. L., Wissel, B., & Somers, C.M. (2011). Cormorant–fisheries conflicts: stable isotopes reveal a consistent niche for avian piscivores in diverse food webs. Ecological Applications, 21(8), 2987-3001.

[7] Glowinski, R., Osher, S. J., & Yin, W. (eds.) (2016). Splitting Methods in Communication, Imaging, Science, and Engineering, Springer, New York, 1-94.

[8] Grigoriu, M. (2014). Noise-induced transitions for random versions of Verhulst model. Probabilistic Engineering Mechanics, 38, 136-142.

[9] Green, A.J., & Elmberg, J. (2014). Ecosystem services provided by waterbirds. Biological Reviews, 89(1), 105-122.

[10] Kameda, K., Koba, K., Hobara, S., Osono, T., & Terai, M. (2006). Pattern of natural 15N abundance in lakeside forest ecosystem affected by cormorant-derived nitrogen. Hydrobiologia, 567(1), 69-86.

[11] Karatzas, I., & Shreve, S. (2012). Brownian motion and stochastic calculus (Vol. 113). Springer Science & Business Media, 169-170.

[12] Lungu, E.M., & Øksendal, B. (1997). Optimal harvesting from a population in a stochastic crowded environment. Mathematical Biosciences, 145(1), 47-75.

[13] Ministry of Agriculture, Forestry, and Fisheries. (2016). Statistics of Agriculture, Forestry and Fisheries in Japan during 2016, http://www.maff.go.jp/j/tokei/kouhyou/naisui_gyosei/attach/pdf/index-8.pdf (accessed 20 December 2017). (in Japanese)

[14] Øksendal, B. (2003). Stochastic differential equations. Springer, Berlin, Heidelberg, 1-332.

[15] Pham, H. (2009). Continuous-time stochastic control and optimization with financial applications, Springer, Berlin, Heidelberg, 1-232.

[16] Song, Q., & Zhu, C. (2016). On singular control problems with state constraints and regime-switching: A viscosity solution approach. Automatica, 70, 66-73.

[17] Takahashi, T., Kameda, K., Kawamura, M., & Nakajima, T. (2006). Food habits of great cormorant Phalacrocorax carbo hanedae at Lake Biwa, Japan, with special reference to ayu Plecoglossus altivelis altivelis. Fisheries Science, 72, 477-484.

[18] Tsujimura, M. & Maeda, A. (2016). Stochastic control: Theory and applications, Asakura shoten, Tokyo, 1-145.

[19] van Eerden, M.R., van Rijn, S.,Volponi, S., Paquet, J.Y., & Carss, D. (2012). Cormorant and the European Environment: Exploring Cormorant Ecology on a Continental Scale. COST Action 635 Final Report I: 126.

[20] Yaegashi Y., Yoshioka H., Unami K., & Fujihara M. (2017a). An optimal management strategy for stochastic population dynamics of released Plecoglossus altivelis in rivers.

International Journal of Modeling, Simulation, and Scientific Computing, 8(2), 1750039, p. 16.

[21] Yaegashi Y., Yoshioka H., Unami K., & Fujihara M. (2017b). Optimal policy of predator suppression for sustainable inland fishery management, 12th SDEWES Conference, October 4-8, 2017a, Dubrovnik, Croatia, 0309, p. 11. (Archived Paper)

[22] Yaegashi, Y., Yoshioka, H., Unami, K., & Fujihara, M. (2017c). A two-variable stochastic singular control model for management of fishery resources under predation, Proceedings of The 36th JSST Annual International Conference on Simulation Technology, 198-201.

[23] Yaegashi, Y., Yoshioka, H., Unami, K., & Fujihara, M. (2018). A singular stochastic control model for sustainable population management of the fish-eating waterfowl *Phalacrocorax carbo*, Journal of Environmental Management. (under review)

[24] Yamamoto, M. (2008). What Kind of Bird is the Great Cormorant, National federation of inland water fisheries cooperatives, http://www.naisuimen.or.jp/jigyou/kawau/01-1.pdf (accessed 20 December 2017). (in Japanese)

[25] Yamamoto, M. (2009). Stand face to face with Great Cormorant, National federation of inland water fisheries cooperatives, http://www.naisuimen.or.jp/jigyou/kawau/03-1.pdf (accessed 20 December 2017). (in Japanese)

[26] Yoshioka, H., Unami, K., & Fujihara, M. (2014). Mathematical analysis on a conforming finite element scheme for advection-dispersion-decay equations on connected graphs, J. JSCE, Ser. A2, 70, I265–I274.

[27] Zedler, J.B., & Kercher, S. (2005). Wetland resources: status, trends, ecosystem services, and restorability. Annual Review of Environment and Resources, 30, 39-74.

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