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CHARACTERIZATIONS OF G-LIKE CONTINUA CONTAINING INDECOMPOSABLE SUBCONTINUA AND POSITIVE TOPOLOGICAL ENTROPY

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1. Introduction

The theory of indecomposable continua is one of the most interesting branches of continuum theory. Also, we know that many interesting connections between dynamical systems and continuum theory have been studied by many mathematicians and many indecomposable continua are frequently appeared in chaotic dynamical systems (see References). Such continua play important roles in order to investigate behaviors of the dynamics. We are interested in the following fact that chaotic topological dynamics should imply the existence of complicated topological structures of underlying spaces. In many cases, such continua (=compact connected metric spaces) are indecomposable continua. Also, in many cases, the composants of such indecomposable continua are strongly related to stable or unstable (connected) sets of the dynamics. For instance, in continuum theory and the theory of dynamical systems, the Knaster continuum (= Smale's horse shoe), the pseudo-arc, solenoids and Wada's lakes (= Plykin attractors) etc., are well-known as such indecomposable continua. In the theory of indecomposable continua, the notion of "crookedness" has been essential and well-known. In this article, we introduce a new notion of "free tracing property by free $\mathcal{P}$-chains" and by use the notion we study chaotic dynamics and indecomposability of continua.

2. Preliminaries

In this article, we assume that all spaces are separable metric spaces and all maps are continuous. Let $\mathbb{N}$ be the set of natural numbers, $\mathbb{R}$ the real line, and $I = [0, 1]$ the unit interval. A *graph* is a compact connected 1-dimensional polyhedron. A graph T is a *tree* if T contains no simple closed curve. For a set A, $|A|$ denotes the cardinality of the set A. For a family $\mathcal{A}$ of subsets of a space, $\bigcup \mathcal{A}$ denotes the union of all elements of $\mathcal{A}$, i.e.,

$$\bigcup \mathcal{A} = \bigcup_{A \in \mathcal{A}} A \ (= \bigcup \{A|\ A \in \mathcal{A}\}).$$

For a subset A of a space X, $\overline{A}$ denotes the closure of A in X. A subset E of X is an F_σ-set of X if E is a countable union of closed sets of X.

A *continuum* is a compact connected metric space. We say that a continuum is *nondegenerate* if it has more than one point. A continuum is *indecomposable* [24] if it is nondegenerate and it is not the union of two proper subcontinua. For any continuum H, the set $c(p)$ of all points of the continuum H, which can be joined with the point p by a proper subcontinuum of H, is said to be the *composant* of the point $p \in H$, i.e.,

$$c(p) = \bigcup\{C|\ C \text{ is a proper subcontinuum of } H \text{ containing the point } p\}.$$

Note that for an indecomposable continuum H, the following conditions are equivalent;

(1) the two points p, q belong to same composant of H;
(2) $c(p) \cap c(q) \neq \emptyset$;
(3) $c(p) = c(q)$.

So, we know that if H is an indecomposable continuum, the family

$$\{c(p)|\ p \in H\}$$

of all composants of H is a family of uncountable mutually disjoint sets $c(p)$ which are connected and dense F_σ-sets in H. Note that a (nondegenerate) continuum X is indecomposable if and only if there are three distinct points of X such that any subcontinuum of X containing any two points of the three points coincides with X, i.e., X is irreducible between any two points of the three points [24].

Let H be an indecomposable continuum. We say that a subset Z of H is *transversal for composants of* H if no distinct two points of Z belong to the same composant of H, i.e., if x, y are any distinct points of Z and E is any subcontinuum of H containing x and y, then $E = H$. Note that a subset Z of H is *transversal for composants* of H if and only if Z is vertically embeded with respect to composants of H (see [11]). In [27], Mazurukiewicz proved that if H is an indecomposable continuum, then there is a Cantor set Z in H which is transversal for composants of H.

Let X_i $(i \in \mathbb{N})$ be a sequence of compact metric spaces and let $f_{i,i+1} : X_{i+1} \to X_i$ be a map for each $i \in \mathbb{N}$. The *inverse limit* of the inverse sequence $\{X_i, f_{i,i+1}\}_{i=1}^\infty$ is the space

$$\varprojlim\{X_i, f_{i,i+1}\} = \{(x_i)_{i=1}^\infty \mid x_i = f_{i,i+1}(x_{i+1}) \text{ for each } i \in \mathbb{N}\} \subset \prod_{i=1}^\infty X_i$$

which has the topology inherited as a subspace of the product space $\prod_{i=1}^\infty X_i$. For a map $f : X \to X$, put

$$\varprojlim(X, f) = \{(x_i)_{i=1}^\infty \mid x_i = f(x_{i+1}) \text{ for each } i \in \mathbb{N}\}.$$

A map g from X onto G is an *ϵ-map* $(\epsilon > 0)$ if for every $y \in G$, the diameter of $g^{-1}(y)$ is less than ϵ. For any collection $\mathcal{P}$ of graphs, X is $\mathcal{P}$-like if for any $\epsilon > 0$ there exist an element $G \in \mathcal{P}$ and an ϵ-map from X onto G. A continuum$X$ is G-like if X is $\mathcal{P}$-like, where $\mathcal{P} = \{G\}$. Note that X is G-like if only if X is homeomorphic to the inverse limit of an inverse sequence of G, i.e.,

$$X = \varprojlim\{G_i, f_{i,i+1}\},$$

where $G_i = G$ and $f_{i,i+1} : G_{i+1} \to G_i$ is an onto map for each $i \in \mathbb{N}$. Arc-like continua (=chainable continua) are those which are G-like for $G = I$, and circle-like continua are those which are S-like, where S is a simple closed curve. Our focus in this article is on G-like continua where G is any graph.

Let $\mathcal{U}$ be a collection of subsets of X. The nerve $N(\mathcal{U})$ of $\mathcal{U}$ is the polyhedron whose vertices are elements of $\mathcal{U}$ and there is a simplex

$< U_1, U_2, ..., U_k >$ with distinct vertices $U_1, U_2, ..., U_k \in \mathcal{U}$ if

$$\bigcap_{i=1}^{k} U_i \neq \emptyset.$$

In this paper, we consider the only case that nerves are graphs.

If $\{C_1, \ldots, C_n\}$ is a subcollection of $\mathcal{U}$, we call it a *chain* if $C_i \cap C_{i+1} \neq \emptyset$ for $1 \leq i < n$ and $\overline{C_i} \cap \overline{C_j} \neq \emptyset$ implies that $|i - j| \leq 1$. We say that $\{C_1, \ldots, C_n\}$ is a *free chain in* $\mathcal{U}$ if it is a chain and, moreover, for all $1 < i < n$ we have that $C \in \mathcal{U}$ with $\overline{C} \cap \overline{C_i} \neq \emptyset$ implies that $C = C_i$, $C = C_{i-1}$ or $C = C_{i+1}$. By the *mesh* of a finite collection $\mathcal{U}$ of sets, we means the largest of diameters of elements of $\mathcal{U}$. Note that for a graph G, a continuum X is G-like if and only if for any $\epsilon > 0$, there is a finite open cover $\mathcal{U}$ of X such that $N(\mathcal{U})$ is homeomorphic to G and the mesh of $\mathcal{U}$ is less than ϵ. The Knaster continuum [21] (=Smale's horse shoe) and the pseudo-arc (=hereditarily indecomposable arc-like continuum) are arc-like continua, solenoids are circle-like continua and the Wada' lake [35] (=Plykin attractor [32]) is a $(S_1 \vee S_2 \vee S_3)$-like continuum, where $S_1 \vee S_2 \vee S_3$ denotes the one point union of 3 circles. Such spaces are typical indecomposable continua which often appear in continuum theory and chaotic dynamical systems. The reader may refer to [24] and [31] for standard facts concerning continuum theory.

3. Free tracing property by free G-chains

Let X be a continuum and $m \in \mathbb{N}$. Suppose that A_i $(1 \leq i \leq m)$ are nonempty m open sets in X and x_i $(1 \leq i \leq m)$ are m distinct points of X. We identify the order $A_1 \to A_2 \to \cdots \to A_m$ and the converse order $A_m \to A_{m-1} \to \cdots \to A_1$. Then we consider the equivalence class

$$[A_1 \to A_2 \to \cdots \to A_m] = \{A_1 \to A_2 \to \cdots \to A_m; A_m \to A_{m-1} \to \cdots \to A_1\}.$$

Suppose that $\mathcal{U}$ is a finite open cover of X. We say that a chain $\{C_1, \cdots, C_n\} \subseteq \mathcal{U}$ *follows from the pattern* $[A_1 \to A_2 \to \cdots \to A_m]$ [11] if there exist

$$1 \leq k_1 < k_2 < \cdots < k_m \leq n \text{ or } 1 \leq k_m < k_{m-1} < \cdots < k_1 \leq n$$

such that $C_{k_i} \subset A_i$ for each $i = 1, 2, ..., m$. In this case, more precisely we say that the chain $[C_{k_1} \to C_{k_2} \to \cdots \to C_{k_m}]$ follows from the pattern $[A_1 \to A_2 \to \cdots \to A_m]$. Similarly, we say that a chain $\{C_1, \ldots, C_n\} \subseteq \mathcal{U}$ *follows from the pattern* $[x_1 \to x_2 \to \cdots \to x_m]$ [11] if there exist

$$1 \leq k_1 < k_2 < \cdots < k_m \leq n \text{ or } 1 \leq k_m < k_{m-1} < \cdots < k_1 \leq n$$

such that $x_i \in C_{k_i}$ for each $i = 1, 2, ..., m$, where

$$[x_1 \to x_2 \to \cdots \to x_m] = \{x_1 \to x_2 \to \cdots \to x_m;\ x_m \to x_{m-1} \to \cdots \to x_1\}.$$

More precisely, we say that the chain $[C_{k_1} \to C_{k_2} \to \cdots \to C_{k_m}]$ follows from the pattern $[x_1 \to x_2 \to \cdots \to x_m]$.

Let $\mathcal{P}$ be a collection of graphs and let Z be a subset of a $\mathcal{P}$-like continuum X. We say that Z has *the free tracing property by (resp. free)* $\mathcal{P}$*-chains* if for any $\epsilon > 0$, any $m \in \mathbb{N}$ and any order $x_1 \to x_2 \to \cdots \to x_m$ of any m distinct points x_i $(i = 1, 2, ..., m)$ of Z, there is an open cover $\mathcal{U}$ of X such that the mesh of $\mathcal{U}$ is less than ϵ, the nerve $N(\mathcal{U})$ of $\mathcal{U}$ is homeomorphic to an element of $\mathcal{P}$ and there is a (resp. free) chain in $\mathcal{U}$ which follows from the pattern $[x_1 \to x_2 \to \cdots \to x_m]$.

Especially, for a G-like continuum X, we say that a subset Z of X has *the free tracing property by (resp. free) G-chains* if Z has the free tracing property by (resp. free) $\mathcal{P}$-chains, where $\mathcal{P} = \{G\}$.

In the special the case that X itself is a graph G, for points x_i $(i = 1, 2, ..., m)$ of G, we can similarly define that an edge of G follows from the pattern $[x_1 \to x_2 \to \cdots \to x_m]$.

4. Characterizations of indecomposable continua and free tracing property

A continuum X is *tree-like* if X is $\mathcal{T}$-like, where $\mathcal{T}$ is the collection of all trees. For the case that X is a tree-like continuum, we have the following characterization theorem.

Theorem 4.1. ([12]) *Let $\mathcal{T}$ be the collection of all trees and let X be a $\mathcal{T}$-like continuum, i.e., X is tree-like. Suppose that D is a subset of X with $|D| \geq 3$. Then the following conditions are equivalent.*
(1) *For any order $x_1 \to x_2 \to x_3$ of three distinct points x_i $(i = 1, 2, 3)$ of D and any $\epsilon > 0$, there is an open cover $\mathcal{U}$ of X such that the mesh of $\mathcal{U}$ is less than ϵ, the nerve $N(\mathcal{U})$ of $\mathcal{U}$ is homeomorphic to an element of $\mathcal{T}$ and there is a chain in $\mathcal{U}$ which follows from the pattern $[x_1 \to x_2 \to x_3]$.*
(2) *D has the free tracing property by $\mathcal{T}$-chains.*
(3) *The minimal continuum H in X containing D is indecomposable and Z is transversal for composants of H.*

For the special case of arc-like continua, we have the following characterization theorem.

Theorem 4.2. ([12]) *Let X be an arc-like continuum. Suppose that Z is a subset of X with $|Z| \geq 3$. Then the following conditions are equivalent.*
(1) *X is indecomposable and Z is transversal for composants of X.*
(2) *Z has the free tracing property by free I-chains and X is the minimal continuum containing Z.*

Next result is the main theorem in this section.

Theorem 4.3. ([12]) *Suppose that X is any G-like continuum for a graph G and H is a subcontinuum of X. Then the following conditions (1), (2) and (3) are equivalent.*
(1) *H is indecomposable.*
(2) *There is a Cantor set Z in H such that Z has the free tracing property by free G-chains and H is the unique minimal continuum containing Z. In particular, Z is transversal for composants of H.*
(3) *There is a dense F_σ-set Z_∞ of H such that*

$$Z_\infty = \bigcup_{i \in \mathbb{N}} Z_i$$

is the countable union of Cantor sets Z_i in H, Z_∞ has the free tracing property by free G-chains and H is the unique minimal continuum containing Z_i for each $i \in \mathbb{N}$. In particular, Z_∞ is transversal for composants of H.

Proposition 4.4. ([12]) *Suppose that X is a G-like continuum for a graph G and Z is a Cantor set in X such that Z has the free tracing property by free chains* *and H is the unique minimal continuum H in X containing Z. Let $z \in Z$ and let $c(z, H)$ be the composant of z in the indecomposable continuum H. Then any subcontinuum A in $c(z, H)$ is an arc-like continuum.*

For hereditarily indecomposable continua, we have the following.

Corollary 4.5. ([12]) *Suppose that X is any G-like continuum for a graph G and H is a subcontinuum of X. Then the following* (1) *and* (2) *are equivalent.*
(1) *H is hereditarily indecomposable.*
(2) *For any subcontinuum K of H, there is a Cantor set Z_K in K such that Z_K has the free tracing property by free G-chains and K is the unique minimal continuum containing Z_K. In particular, Z_K is transversal for composants of K.*

The following is a characterization of pseudo-arc.

Corollary 4.6. ([12]) *Suppose that X is an arc-like continuum and H is a subcontinuum of X. Then the following* (1) *and* (2) *are equivalent.*
(1) *H is the pseudo-arc.*
(2) *For any subcontinuum K of H, there is a Cantor set Z_K in K such that Z_K has the free tracing property by free I-chains and K is the unique minimal continuum containing Z_K. In particular, Z_K is transversal for composants of K.*

In [23], Kuykendall studied irreducibility and indecomposability in inverse limits of continua. Also, we have the following.

Corollary 4.7. ([12]) *Let G be a graph and let $X = \varprojlim\{G_i, f_{i,i+1}\}$ be an inverse limit with onto bonding maps $f_{i,i+1}$, where $G_i = G$ for each $i \in \mathbb{N}$. Then the followings holds.*
(1) *There is an indecomposable subcontinuum in X if and only if there is a Cantor set Z in X such that for any order $z^1 \to z^2 \to \cdots \to z^m$ of any finite points $z^j = (z^j_i)_{i=1}^{\infty}$ $(j = 1, 2, ..., m)$ of Z and any $n \in \mathbb{N}$, there is $k \geq n$ and an edge of G_k which follows from the pattern*

$$[z^1_k \to z^2_k \to \cdots \to z^m_k].$$

(2) *Moreover, if G is a tree, there is an indecomposable subcontinuum in X if and only if there is a three points set Z in X such that for any order $z^1 \to z^2 \to z^3$ of Z and any $n \in \mathbb{N}$, there is $k \geq n$ and an edge of G_k which follows from the pattern $[z^1_k \to z^2_k \to z^3_k]$.*

5. Positive topological entropy

Let X be a compact metric space and $\mathcal{U}, \mathcal{V}$ be two covers of X. Put

$$\mathcal{U} \vee \mathcal{V} = \{U \cap V |\ U \in \mathcal{U}, V \in \mathcal{V}\}.$$

The quantity $N(\mathcal{U})$ denotes minimal cardinality of subcovers of $\mathcal{U}$. Let $f : X \to X$ be a map and let $\mathcal{U}$ be an open cover of X. Put

$$h(f, \mathcal{U}) = \lim_{n\to\infty} \frac{\log N(\mathcal{U} \vee f^{-1}(\mathcal{U}) \vee \ldots \vee f^{-n+1}(\mathcal{U}))}{n}.$$

The *topological entropy of f*, denoted by $h(f)$, is the supremum of $h(f, \mathcal{U})$ for all open covers $\mathcal{U}$ of X. Positive topological entropy of map is one of generally accepted definitions of chaos. We say that a set $I \subseteq \mathbb{N}$ has *positive density* if

$$\liminf_{n\to\infty} \frac{|I \cap \{1,2,...,n\}|}{n} > 0.$$

Let X be a compact metric space and $f : X \to X$ a map. Let $\mathcal{A}$ be a collection of subsets of X. We say that a set $I \subset \mathbb{N}$ is an *independence set* for $\mathcal{A}$ if for all finite sets $J \subseteq I$, and for all $(Y_j) \in \prod_{j\in J} \mathcal{A}$, we have that

$$\bigcap_{j\in J} f^{-j}(Y_j) \neq \emptyset.$$

We now recall the definition of IE-tuple which is related to independence set in $\mathbb{N}$ and (topological) entropy (see [20]). Let $(x_1, \ldots, x_n)$ be a sequence of points in X. We say that $(x_1, \ldots, x_n)$ is an *IE-tuple of f* if whenever $A_1, \ldots, A_n$ are open sets containing $x_1, \ldots, x_n$, respectively, we have that the collection $\mathcal{A} = \{A_1, \ldots, A_n\}$ has an independence set with positive density. In the case that $n = 2$, we use the term IE-pair. We use IE_k to denote the set of all IE-tuples of length k.

Let $f : X \to X$ be a map of a compact metric space X with metric d and let $\delta > 0$. A subset S of X is a *δ-scrambled set* of f if $|S| \geq 2$ and for any $x, y \in S$ with $x \neq y$, then one has

$$\liminf_{n\to\infty} d(f^n(x), f^n(y)) = 0 \text{ and } \limsup_{n\to\infty} d(f^n(x), f^n(y)) \geq \delta.$$

We say that $f : X \to X$ is *Li-Yorke chaotic* if there is an uncountable subset S of X such that for any $x, y \in S$ with $x \neq y$, then one has

$$\liminf_{n\to\infty} d(f^n(x), f^n(y)) = 0 \text{ and } \limsup_{n\to\infty} d(f^n(x), f^n(y)) > 0.$$

In [3], by use of ergodic theory method, Blanchard, Glasner, Kolyada and Maass proved the following theorem.

Theorem 5.1. ([3]) *If a map $f : X \to X$ of a compact metric space X has positive topological entropy, then there is an uncountable δ-scrambled subset of X for some $\delta > 0$ and hence the dynamics (X, f) is Li-Yorke chaotic.*

In [20], by use of local entropy theory (IE-tuples), Kerr and Li proved the following theorem.

Theorem 5.2. ([20, Theorem 3.18]) *Suppose that $f : X \to X$ is a positive topological entropy map of a compact metric space X, and $x_1, x_2, ..., x_m$ $(m \geq 2)$ are distinct points of X such that the tuple $(x_1, x_2, ..., x_m)$ is an IE-tuple of f. If A_i $(i = 1, 2, ..., m)$ is any neighborhood of x_i, then there are Cantor sets $Z_i \subset A_i$ such that the following conditions hold;*
(1) *any sequence $(z_1, z, ..., z_n)$ of points in the Cantor set $Z = \bigcup_i Z_i$ is an IE-tuple of f, and*
(2) *for all $k \in \mathbb{N}$, k distinct points $y_1, y_2, ..., y_k \in Z$ and any points $z_1, z_2, ..., z_k \in Z$, one has*

$$\liminf_{n\to\infty} \max\{d(f^n(y_i), z_i)| \ 1 \leq i \leq k\} = 0.$$

In particular, Z is a δ-scrambled set of f for some $\delta > 0$.

In [11] we have the following structure theorem for homeomorphisms on G-like continua.

Theorem 5.3. ([11]) *In the setting of Theorem 5.2 assume additionally that X is a G-like continuum for a graph G and $f : X \to X$ is a homeomorphism. Then the Cantor sets $Z_i \subset A_i$ $(i = 1, 2, ..., m)$ can be chosen so as to satisfy, in addition to the above conditions* (1) *and* (2), *also the following two ones;*
(3) $Z = \bigcup_{i=1}^{m} Z_i$ *has the free tracing property by free G-chains, and*
(4) *the unique minimal subcontinuum H of X containing Z is indecomposable and Z is transversal for composants of H.*

An onto map $f : X \to Y$ of continua is *monotone* if for any $y \in Y$, $f^{-1}(y)$ is connected.

Theorem 5.4. ([11]) *Let X be a G-like continuum, where G is a graph. If $f : X \to X$ is a monotone map with positive topological entropy, then there exists a Cantor set Z in X satisfying conditions (1) and (2) of Theorem 5.2 and embedded vertically to the composants of a certain indecomposable subcontinuum H of X. Moreover, H can be taken to be the unique minimal subcontinnum of X containing Z.*

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