A strong convergence theorem for countable families of nonlinear nonself mappings in Hilbert spaces and applications

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Abstract

In [17] Takahashi introduced the concept of demimetric mappings in Banach spaces and Alsulami and Takahashi [2] showed strong convergence theorems for demimetric mappings in Hilbert spaces. On the other hand, in [7] Kawasaki and Takahashi introduced the concept of widely more generalize hybrid mappings in Hilbert spaces. Such a mapping is not demimetric generally even if the set of fixed points of the mapping is nonempty. In this paper, we extend the class of demimetric mappings to a more broad class of mappings in Banach spaces and prove a strong convergence theorem applicable to the class of widely more generalized hybrid mappings in a Hilbert space. Using this result, we obtain strong convergence theorems which are connected to the class of widely more generalized hybrid mappings in a Hilbert spaces.

1 Introduction

Let E be a Banach space and let C be a nonempty subset of E. For a mapping T from C into E, we denote by $F(T)$ the set of all fixed points of T. Suppose that E is smooth. Then the duality mapping J on E is single-valued. Let $k \in (-\infty, 1)$. A mapping T from C into E with $F(T) \neq \emptyset$ is said to be k-demimetric [17] if

$$(1-k)\|x - Tx\|^2 \leq 2\langle x - q, J(x - Tx)\rangle$$

for any $x \in C$ and $q \in F(T)$. Let H be a real Hilbert space and let C be a nonempty, closed and convex subset of H. A mapping $T : C \to H$ is called nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C.$$

For $\alpha > 0$, a mapping $A : C \to H$ is called α-inverse strongly monotone if

$$\langle x - y, Ax - Ay\rangle \geq \alpha\|Ax - Ay\|^2, \quad \forall x, y \in C.$$

A mapping $U : C \to H$ is called demiclosed if a sequence $\{x_n\}$ in C satisfies that $x_n \rightharpoonup w$ and $x_n - Ux_n \to 0$, then $w = Uw$ holds. For example, if C is a nonempty, closed and convex subset of H and a nonself mapping $T : C \to H$ is nonexpansive, then T is demiclosed; see [3]. Let H be a Hilbert space and let G be a mapping from H into 2^H and let $D(G) = \{x \in H \mid Gx \neq \emptyset\}$. Then $D(G)$ is said to be the effective domain of G. A multi-valued mapping G is said to be monotone if $\langle x - y, u - v\rangle \geq 0$ for all $x, y \in D(G)$, $u \in Gx$ and $v \in Gy$. A monotone mapping is said to be maximal if its graph is not properly contained in the graph of any other monotone mapping. For a maximal monotone operator G on H and $r > 0$, we may define a single-valued operator $J_r = (I + rG)^{-1} \colon H \to D(G)$, which is called the resolvent of G for $r > 0$. Let G be a maximal monotone operator on H and let $G^{-1}0 = \{x \in H : 0 \in Gx\}$. It is known that the resolvent J_r is nonexpansive and $G^{-1}0 = F(J_r)$ for all $r > 0$; see [15].

Moreover Alsulami and Takahashi [2] showed the following strong convergence theorem.

Theorem 1.1 ([2]). *Let H be a real Hilbert space, let C be a nonempty, closed and convex subset of H, let $\{k_j\}_{j=1}^M \subset (-\infty, 1)$, let $\{T_j\}_{j=1}^M$ be a finite family of k_j-demimetric and demiclosed mappings from C into H, let $\{\mu_i\}_{i=1}^N \subset (0,\infty)$, let $\{B_i\}_{i=1}^N$ be a finite family of μ_i-inverse strongly monotone mappings from C into H, let G be a maximal monotone operator on H and let $J_r = (I + rG)^{-1}$ be the resolvent of G for $r > 0$. Suppose that $\left(\bigcap_{j=1}^M F(T_j)\right) \cap \left(\bigcap_{i=1}^N (B_i + G)^{-1}0\right) \neq \emptyset$. Let $x_1 \in C$ and let $\{x_n\}$ be a sequence generated by*

$$\begin{cases} z_n = \sum_{j=1}^M \xi_j((1-\lambda_n)I + \lambda_n T_j)x_n, \\ w_n = \sum_{i=1}^N \sigma_i J_{\eta_n}(I - \eta_n B_i)x_n, \\ y_n = \alpha_n x_n + \beta_n z_n + \gamma_n w_n, \\ C_n = \{z \in C \mid \|y_n - z\| \leq \|x_n - z\|\}, \\ Q_n = \{z \in C \mid \langle x_n - z, x_1 - x_n\rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1 \end{cases}$$

for any $n \in \mathbb{N}$, where $a, b, c \in (0,\infty)$, $\{\lambda_n\}, \{\eta_n\} \subset (0,\infty)$, $\{\xi_j\}_{j=1}^M, \{\sigma_i\}_{i=1}^N \subset (0,1)$ and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\} \subset (0,1)$ satisfy

$$a \leq \lambda_n \leq \min\{1 - k_j \mid j = 1,\dots,M\}, \quad b \leq \eta_n \leq 2\min\{\mu_i \mid i = 1,\dots,N\},$$

$$\sum_{j=1}^M \xi_j = \sum_{i=1}^N \sigma_i = 1 \quad \text{and} \quad c \leq \alpha_n, \beta_n, \gamma_n, \alpha_n + \beta_n + \gamma_n = 1.$$

Then $\{x_n\}$ is convergent to a point $z_0 \in \left(\bigcap_{j=1}^M F(T_j)\right) \cap \left(\bigcap_{i=1}^N (B_i + G)^{-1}0\right)$, where $z_0 = P_{\left(\bigcap_{j=1}^M F(T_j)\right) \cap \left(\bigcap_{i=1}^N (B_i+G)^{-1}0\right)} x_1$.

On the other hand, in [7] Kawasaki and Takahashi introduced the concept of widely more generalize hybrid mappings. Let H be a Hilbert space, let C be a nonempty subset of H and let $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta \in \mathbb{R}$. A mapping T from C into H is said to be $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta)$-widely more generalized hybrid if

$$\begin{aligned} &\alpha\|Tx - Ty\|^2 + \beta\|x - Ty\|^2 + \gamma\|Tx - y\|^2 + \delta\|x - y\|^2 \\ &\quad + \varepsilon\|x - Tx\|^2 + \zeta\|y - Ty\|^2 + \eta\|(x - Tx) - (y - Ty)\|^2 \leq 0 \end{aligned} \tag{1.1}$$

for all $x, y \in C$. Such a mapping is not demimetric generally even if $F(T) \neq \emptyset$.

In this paper, we extend the class of demimetric mappings to a more broad class of mappings which contains widely more generalize hybrid mappings in Banach spaces and prove a strong convergence theorem applicable to the class of widely more generalized hybrid mappings in a Hilbert space. Using this result, we obtain strong convergence theorems which are connected to the class of widely more generalized hybrid mappings in a Hilbert spaces.

2 Preliminaries

The following lemma is used in the proof of our main result.

Lemma 2.1 ([18]). *Let H be a Hilbert space and let C be a nonempty, closed and convex subset of H. Let $k \in (-\infty, 1)$ and let T be a k-demimetric mapping of C into H such that $F(T)$ is nonempty. Let λ be a real number with $0 < \lambda \leq 1-k$ and define $S = (1-\lambda)I + \lambda T$. Then S is a quasi-nonexpansive mapping of C into H.*

Let G be a maximal monotone mapping on H and let $J_r = (I + rG)^{-1}$ be the resolvent of G for $r > 0$. Then J_r is firmly nonexpansive, that is,

$$\|J_r x - J_r y\|^2 \leq \langle x - y, J_r x - J_r y \rangle$$

for any $x, y \in H$; for instance, see [15]. In this paper the following lemmas are used.

Lemma 2.2 ([2]). *Let H be a real Hilbert space, let C be a nonempty closed convex subset of H, let $\alpha > 0$, let B be an α-inverse strongly monotone mapping from C into H, let G be a maximal monotone operator on H and let J_r be the resolvent of G for $r > 0$. Suppose that $B^{-1}0 \cap G^{-1}0 \neq \emptyset$. Let $\lambda > 0$ and $z \in C$. Then the following are equivalent:*

(i) $z \in F(J_r(I - \lambda B))$;

(ii) $z \in (B + G)^{-1}0$;

(iii) $z \in B^{-1}0 \cap G^{-1}0$.

Lemma 2.3 ([13]). *Let H be a real Hilbert space, let G be a maximal monotone operator on H and let J_r be the resolvent of G for $r > 0$. Then the following holds:*

$$\|J_s x - J_t x\|^2 \leq \frac{s-t}{s} \langle J_s x - x, J_s x - J_t x \rangle$$

for any $s, t > 0$ and $x \in H$.

By Lemma 2.3 we obtain

$$\|J_s x - J_t x\| \leq \frac{|s-t|}{s} \|x - J_s x\| \tag{2.1}$$

for any $s, t > 0$ and $x \in H$.

Lemma 2.4 ([15]). *Let H be an inner product space and let $\{x_n\}$ be a bounded sequence in H. Suppose that $\{x_n\}$ is convergent to x weakly. Then the following inequality hold:*

$$\|x\| \leq \liminf_{n\to\infty} \|x_n\|.$$

3 Generalized demimetric mappings

Let E be a smooth Banach space and let C be a nonempty subset of E. A mapping T from C into E with $F(T) \neq \emptyset$ is said to be generalized demimetric if there exists $\theta \in \mathbb{R}$ such that

$$\|x - Tx\|^2 \leq \theta\langle x - q, J(x - Tx)\rangle$$

for all $x \in C$ and $q \in F(T)$, where J is the duality mapping on E. In particular, T is called θ-generalized demimetric.

Remark 3.1. Let $k \in (-\infty, 1)$. A k-demimetric mapping is $\frac{2}{1-k}$-generalized demimetric. Conversely, if $\theta > 0$, then a θ-generalized demimetric is $\left(1 - \frac{2}{\theta}\right)$-demimetric. If $\theta = 0$, then $T = I$. Conversely, I is θ-generalized demimetric for any $\theta \in \mathbb{R}$.

Let H be a Hilbert space, let C be a nonempty subset of H and let $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta \in \mathbb{R}$. Then a mapping T from C into H satisfying (1.1) is said to be $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta)$-widely more generalized hybrid, i.e.,

$$\begin{aligned} &\alpha\|Tx - Ty\|^2 + \beta\|x - Ty\|^2 + \gamma\|Tx - y\|^2 + \delta\|x - y\|^2 \\ &\quad + \varepsilon\|x - Tx\|^2 + \zeta\|y - Ty\|^2 + \eta\|(x - Tx) - (y - Ty)\|^2 \leq 0 \end{aligned} \tag{3.1}$$

for all $x, y \in C$.

Lemma 3.1. *Let H be a Hilbert space, let C be a nonempty subset of H and let T be an $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta)$-widely more generalized hybrid mapping from C into H with $F(T) \neq \emptyset$. Suppose that T satisfies one of the following conditions:*

(1) $\alpha + \beta + \gamma + \delta \geq 0$ *and* $\alpha + \gamma + \varepsilon + \eta > 0$;

(2) $\alpha + \beta + \gamma + \delta \geq 0$ *and* $\alpha + \beta + \zeta + \eta > 0$;

(3) $\alpha + \beta + \gamma + \delta \geq 0$ *and* $2\alpha + \beta + \gamma + \varepsilon + \zeta + 2\eta > 0$.

Then T is generalized demimetric.

The following three lemmas are crucial in the proof of our main result.

Lemma 3.2. *Let E be a smooth Banach space, let C be a nonempty and closed subset of E and let T be a θ-generalized demimetric mapping from C into E. Then $F(T)$ is closed.*

Lemma 3.3. *Let E be a smooth Banach space, let C be a nonempty and convex subset of E and let T be a θ-generalized demimetric mapping from C into E. Then $F(T)$ is convex.*

Lemma 3.4. *Let E be a smooth Banach space, let C be a nonempty subset of E, let T be a θ-generalized demimetric mapping from C into E and let $\kappa \in \mathbb{R}$. Then $(1 - \kappa)I + \kappa T$ is $\theta\kappa$-generalized demimetric from C into E.*

4 Main result

Now we can prove a strong convergence theorem for countable families of generalized demimetric mappings and inverse strongly monotone mappings in Hilbert spaces.

Theorem 4.1. *Let H be a Hilbert space, let C be a nonempty, closed and convex subset of H, let $\{\theta_j\}_{j=1}^{\infty} \subset \mathbb{R}\setminus\{0\}$, let $\{T_j\}_{j=1}^{\infty}$ be a countable family of θ_j-generalized demimetric and demiclosed mappings from C into H, let $\{\kappa_j\}_{j=1}^{\infty} \subset \mathbb{R}$ satisfying $\theta_j\kappa_j > 0$, let $\{\mu_i\}_{i=1}^{\infty} \subset (0,\infty)$, let $\{B_i\}_{i=1}^{\infty}$ be a countable family of μ_i-inverse strongly monotone mappings from C into H, let $\{G_i\}_{i=1}^{\infty}$ be a countable family of maximal monotone operators on H and let $J_{i,r} = (I + rG_i)^{-1}$ be the resolvent of G_i for $i \in \mathbb{N}$ and $r > 0$. Suppose that $\left(\bigcap_{j=1}^{\infty} F(T_j)\right) \cap \left(\bigcap_{i=1}^{\infty}(B_i + G_i)^{-1}0\right) \neq \emptyset$. Let $x_1 \in C$ and let $\{x_n\}$ be a sequence generated by*

$$\begin{cases} z_n = \sum_{j=1}^{\infty} \xi_j((1-\lambda_{j,n})I + \lambda_{j,n}T_j)x_n, \\ w_n = \sum_{i=1}^{\infty} \sigma_i J_{i,\eta_{i,n}}(I - \eta_{i,n}B_i)x_n, \\ y_n = \alpha_n x_n + \beta_n z_n + \gamma_n w_n, \\ C_n = \{z \in C \mid \|y_n - z\| \leq \|x_n - z\|\}, \\ Q_n = \{z \in C \mid \langle x_n - z, x_1 - x_n\rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x_1 \end{cases}$$

for any $n \in \mathbb{N}$, where $a, b, c \in (0,\infty)$, $\{\lambda_{j,n}\}, \{\eta_{i,n}\} \subset \mathbb{R}$, $\{\xi_j\}, \{\sigma_i\} \subset (0,1)$ and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\} \subset (0,1)$ satisfy

$$a \leq \frac{\lambda_{j,n}}{\kappa_j} \leq 2\inf\left\{\frac{1}{\theta_j\kappa_j} \;\middle|\; j \in \mathbb{N}\right\}, \quad b \leq \eta_{i,n} \leq 2\inf\{\mu_i \mid i \in \mathbb{N}\},$$

$$\sum_{j=1}^{\infty} \xi_j = \sum_{i=1}^{\infty} \sigma_i = 1, \quad c \leq \alpha_n, \beta_n, \gamma_n \;\; \text{and} \;\; \alpha_n + \beta_n + \gamma_n = 1.$$

Then $\{x_n\}$ is convergent to a point $z_0 \in \left(\bigcap_{j=1}^{\infty} F(T_j)\right) \cap \left(\bigcap_{i=1}^{\infty}(B_i + G_i)^{-1}0\right)$, where $z_0 = P_{\left(\bigcap_{j=1}^{\infty} F(T_j)\right) \cap \left(\bigcap_{i=1}^{\infty}(B_i+G_i)^{-1}0\right)} x_1$.

5 Application

In this section, using Theorem 4.1, we obtain a strong convergence theorem for countable families of widely more generalize hybrid mappings and inverse strongly monotone mappings in Hilbert spaces.

Lemma 5.1. *Let H be a Hilbert space, let C be a nonempty subset of H and let T be an $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta)$-widely more generalized hybrid mapping from C into H. Suppose that T satisfies one of the following conditions:*

(1) $\alpha + \beta + \gamma + \delta \geq 0$ *and* $\alpha + \gamma + \varepsilon + \eta > 0$*;*

(2) $\alpha+\beta+\gamma+\delta \geq 0$ *and* $\alpha+\beta+\zeta+\eta>0$;

(3) $\alpha+\beta+\gamma+\delta \geq 0$ *and* $2\alpha+\beta+\gamma+\varepsilon+\zeta+2\eta>0$.

Then T is demiclosed.

Theorem 5.1. *Let H be a Hilbert space, let C be a nonempty, closed and convex subset of H, let $\{T_j\}_{j=1}^{\infty}$ be a countable family of $(\alpha_j, \beta_j, \gamma_j, \delta_j, \varepsilon_j, \zeta_j, \eta_j)$-widely more generalized hybrid mappings from C into H. Suppose that T_j satisfies one of the following conditions:*

(1) $\alpha_j+\beta_j+\gamma_j+\delta_j \geq 0$, $\alpha_j+\gamma_j+\varepsilon_j+\eta_j>0$ *and* $\alpha_j+\gamma_j \neq 0$;

(2) $\alpha_j+\beta_j+\gamma_j+\delta_j \geq 0$, $\alpha_j+\beta_j+\zeta_j+\eta_j>0$ *and* $\alpha_j+\beta_j \neq 0$;

(3) $\alpha_j+\beta_j+\gamma_j+\delta_j \geq 0$, $2\alpha_j+\beta_j+\gamma_j+\varepsilon_j+\zeta_j+2\eta_j>0$ *and* $2\alpha_j+\beta_j+\gamma_j \neq 0$.

For (1), (2), (3), put

$$\theta_j=\frac{2(\alpha_j+\gamma_j)}{\alpha_j+\gamma_j+\varepsilon_j+\eta_j}, \quad \frac{2(\alpha_j+\beta_j)}{\alpha_j+\beta_j+\zeta_j+\eta_j}, \quad \frac{2(2\alpha_j+\beta_j+\gamma_j)}{2\alpha_j+\beta_j+\gamma_j+\varepsilon_j+\zeta_j+2\eta_j},$$

respectively. Let $\{\kappa_j\}_{j=1}^{\infty} \subset \mathbb{R}$ satisfying $\theta_j\kappa_j>0$, let $\{\mu_i\}_{i=1}^{\infty} \subset (0,\infty)$, let $\{B_i\}_{i=1}^{\infty}$ be a countable family of μ_i-inverse strongly monotone mappings from C into H, let $\{G_i\}_{i=1}^{\infty}$ be a countable family of maximal monotone operators on H and let $J_{i,r}=(I+rG_i)^{-1}$ be the resolvent of G_i for $i \in \mathbb{N}$ and $r>0$. Suppose that $\left(\bigcap_{j=1}^{\infty} F(T_j)\right) \cap (\bigcap_{i=1}^{\infty}(B_i+G_i)^{-1}0) \neq \emptyset$. Let $x_1 \in C$ and let $\{x_n\}$ be a sequence generated by

$$\begin{cases} z_n=\sum_{j=1}^{\infty}\xi_j((1-\lambda_{j,n})I+\lambda_{j,n}T_j)x_n, \\ w_n=\sum_{i=1}^{\infty}\sigma_i J_{i,\eta_{i,n}}(I-\eta_{i,n}B_i)x_n, \\ y_n=\alpha_n x_n+\beta_n z_n+\gamma_n w_n, \\ C_n=\{z \in C \mid \|y_n-z\| \leq \|x_n-z\|\}, \\ Q_n=\{z \in C \mid \langle x_n-z, x_1-x_n\rangle \geq 0\}, \\ x_{n+1}=P_{C_n\cap Q_n}x_1 \end{cases}$$

for any $n \in \mathbb{N}$, where $a,b,c \in (0,\infty)$, $\{\lambda_{j,n}\} \subset \mathbb{R}$, $\{\eta_{i,n}\} \subset (0,\infty)$, $\{\xi_j\},\{\sigma_i\} \subset (0,1)$ and $\{\alpha_n\},\{\beta_n\},\{\gamma_n\} \subset (0,1)$ satisfying

$$a \leq \frac{\lambda_{j,n}}{\kappa_j} \leq 2\inf\left\{\frac{1}{\theta_j\kappa_j} \,\middle|\, j \in \mathbb{N}\right\}, \quad b \leq \eta_{i,n} \leq 2\inf\{\mu_i \mid i \in \mathbb{N}\},$$

$$\sum_{j=1}^{\infty}\xi_j=\sum_{i=1}^{\infty}\sigma_i=1, \quad c \leq \alpha_n,\beta_n,\gamma_n \quad \text{and} \quad \alpha_n+\beta_n+\gamma_n=1.$$

Then $\{x_n\}$ is convergent to a point $z_0 \in \left(\bigcap_{j=1}^{\infty} F(T_j)\right) \cap (\bigcap_{i=1}^{\infty}(B_i+G_i)^{-1}0)$, where $z_0=P_{\left(\bigcap_{j=1}^{\infty} F(T_j)\right)\cap\left(\bigcap_{i=1}^{\infty}(B_i+G_i)^{-1}0\right)}x_1$.

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