Fundamental study on internal exploration using electromagnetic induction in concrete structures

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Abstract

In this talk, we develop a non-destructive inspection method to detect the exact position of the reinforcing steel in reinforced concrete structures. It is a very important open problem. For the development of our method to probe the reinforcing steel, we apply an apparatus to measure thickness of concrete cover for reinforcing steel by electromagnetic induction method, whose measurements are well known to be inexact because of the errors and the noises the apparatus contains. We average the effect of errors in the apparatus by application of what is called 'the least square solutions', to detect the exact position of the reinforcing steel, which turns out to work very well.

1． Company Profile

First, I would like to explain about our company, West Nippon Expressway Engineering Shikoku Company Limited. Our company maintains highways and inspects various structures such as bridges and tunnels every year. Japanese highways are divided into three main areas: East Japan, Central Japan, and West Japan. We maintain about 478 km of highways in Shikoku in West Japan.

There are many structures on the highway, and it is very difficult to inspect all these structures one by one manually. Therefore, we have developed various new inspection technologies to efficiently inspect many existing structures. "J-System" in Figure 1 and "Eagle" in Figure 2 are one of the new inspection technologies we have developed. "J-System" is a new inspection technology that detects damage areas such as delamination and cracks in concrete by measuring the surface temperature of concrete using an infrared camera. This allows you to find any damage inside the concrete that may not be apparent. "Eagle" is equipped with a line sensor camera, and it is possible to acquire surface shape data of objects such as tunnels and pavements by a measurement method called light section method. By using these new inspection techniques, inspection can be performed more efficiently than before.

Figure 1 : J-System

Figure 2 : Eagle

2. Problem

There are numerous structures on highways made of reinforced concrete (RC for short). The service life of RC structures is said to be about 50 to 60 years, and maintenance of aging structures has become a problem in recent years. As aging progresses, concrete fragments fall and harm people and cars passing under the structure, and in the worst case, the structure itself collapses. Therefore, it is very important to know the condition of the structure by conducting inspections, and laws have been amended to require inspections to be performed approximately once every five years.

For the maintenance of RC structures, it is required to determine whether and where the restoration is necessary, by watching, by palpation, by micro-fracture inspections and by non-destructive inspections. It is, however, very difficult to precisely decide whether and where the restoration is necessary, for the time being. Though there exist various non-destructive inspection techniques for RC structures, there are few ones to give their precise interior information. It is required to develop an exact, safe and cheaply running non-destructive inspection technique to inspect the interior information of the structure.

3. Importance to exactly probe the rebar

In this section, we discuss how important it is to probe the exact position of the rebar near the surface of the structure by a non-destructive inspection. We enumerate the advantages to detect the exact position of the rebar near the surface of the structure.

(a) It enables us to check whether the structure has been constructed as it was designed, by a non-destructive inspection.

(b) When obtaining a concrete core for a microfracture inspection of the structure, we must not damage the rebar. The knowledge of the exact position of the rebar prevents us from damaging the rebar in such inspections.

(c) When some repairment is necessary, we can determine the precise chipping area for repairment in order not to damage the rebar.

(d) It helps us to check whether the rebar is fine by a non-destructive inspection.

(e) It enables us to develop a non-destructive inspection for the concrete cover by application of ultrasound, which is developed by Mita-Takiguchi [1].

By (e), we mean that if we know the exact position of the rebar, the it is possible to inspect the concrete cover, as well as the rebar itself, in a non-destructive way by the measurements of ultrasound (cf. [1]). Let us shortly review this technique.

Property 3.1 (cf. [1]). It is known that the ultrasonic primary waves take the route in the concrete structures where the travel time be the shortest.

Consider a section of the reinforce concrete structure in Figure 3. Let the velocity of the ultrasound in the rebar be V and the average velocity of the ultrasound in the concrete be v. In usual V ≫ v holds and the orbit of the ultrasonic primary wave between the points O and P is the spline ORP, and the orbit between the points O and Q is the spline OSQ.

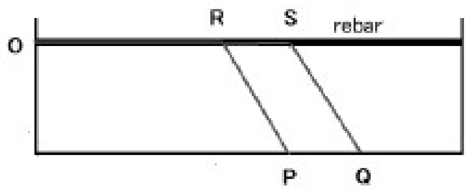

Figure 3 : Propagation of the ultrasound

Project the ultrasound from the point O and the travel time at the points P and Q. The gap of the travel time must be theoretically L/V, where L is the length of the segment RS. If the real gap of the travel time is much larger than L/V, then some part of the rebar in the segment RS gets corroded or some part of the concrete cover between the segment SQ is not sound. In either cases, it is necessary to repair the structure. For more detail of the non-destructive inspection of concrete cover, cf. [1]. We also note that the advantage (e) is closely related with the development of the ultrasonic imaging technique (cf. [1, 2, 3])

4. Apparatus to measure the thickness of concrete cover for reinforcement

In this section, we study properties of the apparatus to measure the thickness of concrete cover for reinforcement by electromagnetic induction. We will apply it as the main apparatus to detect the exact position of the rebar in this paper.

How to use this apparatus

The manual of this apparatus tell to scan it parallelly to the rebar for the measurement of the thickness of the concrete cover. By experience of practical use and by our preliminary experiments, we know the following properties of this apparatus.

Property 4.1. (Properties of the apparatus)

(i) The measurement is not exact.

(ii) It is impossible to measure the thickness of more than 9cm.

(iii) Although the measurement is not exact, it is stable.

(iv) This apparatus is not suitable to detect the endpoints of the rebar by its original directions for usage.

The property (i) is well known (cf. [4], for example). The property (ii) is natural since it can rarely be the case that the thickness of concrete cover is more than 9cm. We have verified Property (iii) by our preliminary experiment. By this property, we mean that inexactness of the measurement in this apparatus is stable, not random. For example, let us consider a cuboid RC structure whose section is given in Figure 4. In this structure, the rebar locates parallelly to the edge surfaces. By several preliminary experiments, we have ascertained that though the measurements of thickness for concrete cover are not exact, they are all the same, that is, the measurements of thickness from points *c, d, e, f,* are all the same.

By the property (iii), we recognized another big problem. It is the property (iv). In Figure 4, it is very difficult to determine the endpoint A (equivalently, the point *a*) by the non-destructive inspection with application of the apparatus with electromagnetic induction to measure thickness of concrete cover. Let us consider very simply. When we move the apparatus along the line going through the points *f, e, d, c* and *a,* it looks that the measured thickness is the same while the apparatus is between the points *f* and *a* and the measured thickness will increase after the apparatus goes through the point *a.*

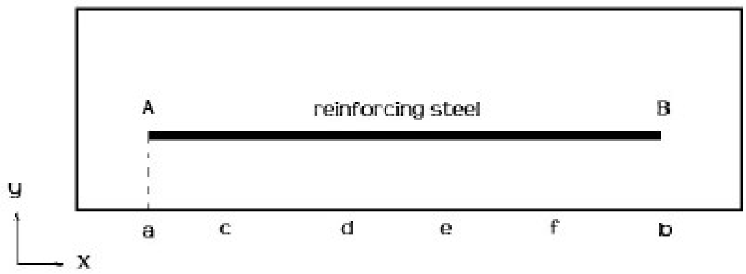

Figure 4 : Detecting the endpoints of the rebar

However, it does not. The measured thickness remains the same, for a while, after the apparatus goes through the point *a*, and the point where the measured thickness begins to increase varies in accordance with the real thickness of concrete cover. This property has also been verified in our preliminary experiments.

It is our main purpose to establish some good methods to compensate for the properties (i) and (iv), by virtue of the property (iii). A solution to this problems gives a new non-destructive inspection technique to exactly probe the rebar.

In general, it is very difficult to non-destructively inspect the exact position of the endpoints of the rebar. It is impossible by application of the ferroscan. In the next section, we shall give a solution to this problem. We also apply this solution to give a non-destructive inspection technique to give the exact position of the whole rebar.

5. Theoretical study to exactly probe the rebar

In this section, we shall establish a theory to detect the exact position of the rebar. Assume that a cuboid RC structure contains a rebar of the diameter d in its interior. It is natural to assume that the diameter of the rebar is a priori known. Let us assume that this cuboid is represented as

$$\{0 \leq x \leq L,\ 0 \leq y \leq l,\ 0 \leq z \leq l\} \tag{1}$$

for some positive numbers L and l. Confer Figure 5 for its image. (In Figure 5, we set the axes as shown.)

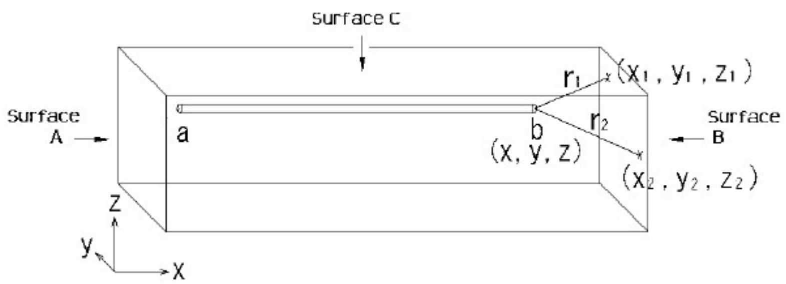

Figure 5: A cuboid test piece

First, we reconstruct the endpoints of the rebar. For this purpose, we utilize the apparatus to measure thickness of concrete cover in another way than its manual tells. The manual tells to scan the surface parallel to the rebar, i.e., the surface C in Figure 5, by the apparatus to measure the thickness of the concrete cover. We, however, mainly scan the surface A or B by this apparatus to measure the distance between the endpoint of the rebar (the point *b* in Figure 5) and the observation point where the apparatus exists (on the surface *B* in Figure 5).

For the reconstruction of the endpoint *b* in Figure 5, we take many observation points on the edge surface *B* and its around and measure the distances between the endpoint (the point *b* in Figure 5) and the observation points. We let the endpoint *b* = (x, y, z) and take *n* observation points (x_1, y_1, z_1) ,⋯, (xn, yn, zn), whose distances from the endpoint *b* are measured as r_1 ,⋯, rn, respectively.

By measurement, we obtain an overdetermined system of quadratic equations.

$$\begin{cases} (x-x_1)^2+(y-y_1)^2+(z-z_1)^2=r_1^2 \\ \cdots \\ (x-x_n)^2+(y-y_n)^2+(z-z_n)^2=r_n^2 \end{cases} \tag{2}$$

In the system (2), the unknowns are x, y and z. If n > 3 then the system (2) is overdetermined and it rarely admits a solution. If the measurements are exact, the system has the unique solution, however overdetermined it is. As was mentioned in Property 3.1 (i), our measurements contains errors and noises. Hence, there is almost no chance that the system (2) admits the unique solution. We equalize the effect of the errors and the noises by taking what is called "the least square solution" to the system (2), which yields the exact position of the endpoint *b* = (x, y, z).

Since it is not easy to give a least square solution to the system (2), we shall give the unique least square solution to the following equivalent system (3) of (2), which consists of n(n-1)/2 linear equations;

$$(x_i-x_j)x+(y_i-y_j)y+(z_i-z_j)z=\frac{1}{2}(x_i^2+y_i^2+z_i^2-x_j^2-y_j^2-z_j^2+r_j^2-r_i^2) \tag{3}$$

The system (3) is obtained by subtracting *i-th* equation in system (2) by the *j-th* one and dividing the both hand sides of the difference by 2.

By reformulation of the system (3), we have the system of n(n-1)/2 linear equations

$$a_k x+b_k y+c_k z=d_k, \tag{4}$$

where *k* = 1, 2 ,⋯, n(n-1)/2, *ak* is of the form *xi - xj*, *bk* is of the form *yi - yj*, *ck* is of the form *zi - zj* and *dk* is of the form $\frac{1}{2}(x_i^2+y_i^2+z_i^2-x_j^2-y_j^2-z_j^2+r_j^2-r_i^2)$. This system has the following matrix representation

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_{\frac{n(n-1)}{2}} & b_{\frac{n(n-1)}{2}} & c_{\frac{n(n-1)}{2}} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_{\frac{n(n-1)}{2}} \end{pmatrix}, \tag{5}$$

which we denote by

$$A\boldsymbol{x}=\boldsymbol{d}, \tag{6}$$

where A is the matrix in (5), $\mathbf{x} = {}^t(x, y, z)$ and $\mathbf{d} = {}^t(d_1, d_2, \cdots, d_{n(n-1)/2})$, where by ${}^tX$, we mean the transpose of the matrix X (we take the three-dimensional vector x for the 3×1 matrix). Let us review the basic theory for the least square solutions to a system of linear equations. Let us study the following system of linear equations.

Problem 5.1. Solve the following system of linear equations in $y_1, y_2, \cdots, y_n$.

$$\begin{cases} b_{11}y_1 + b_{12}y_2 + \cdots + b_{1n}y_n & = s_1, \\ b_{21}y_1 + b_{22}y_2 + \cdots + b_{2n}y_n & = s_2, \\ \cdots\cdots\cdots \\ b_{m1}y_1 + b_{m2}b_2 + \cdots + b_{mn}y_n & = s_m, \end{cases} \tag{7}$$

or equivalently

$$B\boldsymbol{y} = \boldsymbol{s}, \tag{8}$$

where B is the matrix in (7), $\mathbf{y} = {}^t(y_1, y_2, \cdots, y_n)$ and $\mathbf{s} = {}^t(s_1, s_2, \cdots, s_m)$.

For the least square solutions to the system (7), the following propositions are known to hold.

Proposition 5.1. Let $M_{mn}(\mathbb{R})$ be the set of $m \times n$ matrices whose components are real numbers. For $B \in M_{mn}(\mathbb{R})$, the following conditions are equivalent.

(i) $y \in \mathbb{R}^n$ is a least square solution to (7).

(ii) For any $z \in \mathbb{R}^n$, there holds

$$\langle B\boldsymbol{z}, B\boldsymbol{y} - \boldsymbol{s} \rangle_{\mathbb{R}^m} = 0 \tag{9}$$

where $\langle \cdot, \cdot \rangle_{\mathbb{R}^m}$ represents the inner product in $\mathbb{R}^m$.

(iii) There holds the following equation.

$${}^tBB\boldsymbol{y} - {}^tB\boldsymbol{s} = \mathbf{0}. \tag{10}$$

Proposition 5.2. If $\mathrm{rank}(B) = n$, for $B \in M_{mn}(\mathbb{R})$, then the least square solution to the system (7) is uniquely given by the unique solution to the following system;

$${}^tBB\boldsymbol{y} = {}^tB\boldsymbol{s}. \tag{11}$$

In fact, we can prove that $\mathrm{rank}(B) = \mathrm{rank}({}^tBB)$, though we omit its proof. The condition $\mathrm{rank}(B) = n$ is equivalent to the one that matrix ${}^tBB$ is regular, in other words, ${}^tBB$ has its inverse.

The least square solutions to the system (7) has strong relation with the practicalization of CT and development of the ultrasonic CT for which, confer [2, 3].

Let us turn to our overdetermined system (5) of linear equations. In practice, we can assume that $\mathrm{rank}(A) = 3$ for $n \geqq 3$, since the theoretical possibility that $\mathrm{rank}(A) < 3$ is

0. (If rank(A) < 3 then what we have to do is to take more data in order that rank(A) = 3.) Since rank(A) = 3, the least square solution to the system (5) is unique and it is given by the unique solution to the following system of the linear equations

$$ {}^tA A \boldsymbol{x} = {}^tA \boldsymbol{d}, \tag{12} $$

which is obtained by multiplying the transposed matrix

$$ {}^tA = \begin{pmatrix} a_1 & a_2 & \cdots & a_{\frac{n(n-1)}{2}} \\ b_1 & b_2 & \cdots & b_{\frac{n(n-1)}{2}} \\ c_1 & c_2 & \cdots & c_{\frac{n(n-1)}{2}} \end{pmatrix} \tag{13} $$

to the both hand sides in (5) or (6). We note that the matrix ${}^tAA$ is regular in this case. Therefore, we obtain the following theorem to determine the endpoints of the rebar.

Theorem 5.1. In practice, the uniquely determined least square solution to the system (5) is given as the unique solution to the system (12).

We remark that, in the papers [5, 6], they applied the least square solutions with some regularization parameters in order to obtain the rebar position. It seems that it is a little complicated to apply the least square solutions with some regularization parameters, it is simpler to apply the least square solutions with no parameter.

6. Verification of the theoretical study by experiments

In order to verify our theoretical study in the previous section, we made a test piece of reinforced concrete and experimented on them. We constructed three cuboid RC test pieces of the size 150 × 150 × 530mm^3 as shown in Figure 3. In these test pieces, the diameter of the rebar is 10mm and the coordinates of the endpoints were designed as

$$ a = (95, 95, 50), \quad b = (95, 95, 440) \tag{14} $$

on the construction. We note that the coordinates of the endpoints a,b represents the center of the section circle of the rebar, that is, in our test pieces, the rebar is designed to locate in the cylinder domain

$$ (y - 95)^2 + (z - 95)^2 \leq 25, \;\; 50 \leq x \leq 440 \tag{15} $$

We tried to detect the endpoints a,b of the test piece by application of Theorem 5.1. We took the observation points as shown in Figure 6 and measured the shortest distance between the observation points and the rebar, whose results are summed up in Table 1

Application of Theorem 5.1 with the data in Table 1, yields the coordinates of the endpoints a, b as

$$ a = (69, 100, 101), \quad b = (427, 100, 100). \tag{16} $$

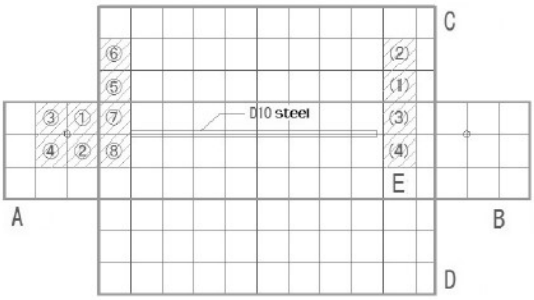

Figure 6: Observation points

Table 1: Measured distance from the rebar

Edge surface	Point	Coordinates	Distance(mm)
A	①	(0,125,125)	74
	②	(0,75,125)	75
	③	(0,125,75)	73
	④	(0,75,75)	75
	⑤	(25,150,125)	67
	⑥	(25,150,75)	67
	⑦	(25,125,150)	66
	⑧	(25,75,150)	67
B	(1)	(475,150,125)	68
	(2)	(475,150,75)	70
	(3)	(475,125,150)	68
	(4)	(475,75,150)	68

As we have discussed in the previous section, the y- and z-coordinates in (16) being reliable, the x-coordinates are not. In our test pieces, the average sonic velocity in the concrete part is about 5200m/s, which is faster than usual concrete (between 3600m/s and 5000m/s). In view of the above discussion, we can conclude that our concrete is denser and solider than usual one and it is expected that the measured thickness of concrete cover is longer than the real one, since, in our preliminary experiments, we ascertained that the denser and the solider the concrete part is, the thicker value for the concrete cover the apparatus with electromagnetic induction measures. Therefore, for the real position of a = (xa,ya,za) and b = (xb,yb,zb), it is expected that

$$y_a \approx 100,\ z_a \approx 101,\ y_b \approx 100,\ z_b \approx 100,\ x_a < 69,\ x_b > 427. \qquad (17)$$

In order to precisely determine the x-coordinates of the endpoints, we made a model in Figure 7 where the steel can be moved in the x-direction and the y- and z-coordinates of the steel is fixed as 100.

Figure 7: A model to probe the x-cordinates of the endpoints

We scanned the segment {(x,100,150); 0≦x≦530} on this model by the apparatus with electromagnetic induction and checked the relation between the endpoint of the steel and the position of the sensor of the apparatus, by which we can conclude the following.

Property 6.1. If the thickness of concrete cover is 45mm then the measured thickness of concrete cover begins to increase when the x-coordinates of top of the sensor goes farther than 35mm from xa.

By this calibration, we reconstructed the x-coordinates of the endpoints. We scanned the apparatus with electromagnetic induction on the segment {(x,100,100); $0 \leqq x \leqq 530$} and measured the points where the thickness of concrete cover began to increase with application of the above modification, whose results are summarized in Table 2.

Table 2: Reconstruction of the x-coordinates

Endpoint	y-coordinate	z-coordinate	Measurement surface	Scanning direction	Dist.	Avr. Dist.
a	100	101	Surface C	Surface B → A	51	50
				Surface A → B	53	
			Surface E	Surface B → A	51	
				Surface A → B	46	
b	100	100	Surface C	Surface A → B	87	84
				Surface B → A	87	
			Surface E	Surface A → B	79	
				Surface B → A	83	

By these measurements, the x-coordinates of the endpoints are determined as xa = 50 and xb = 446. Therefore, the result of our non-destructive inspection for the endpoints of the rebar is the following.

$$a = (50, 100, 101), \quad b = (446, 100, 100). \tag{18}$$

Scanning the segments {(x,100,150); $50 \leqq x \leqq 446$}and {(x,150,100); $50 \leqq x \leqq 446$} by the apparatus to measure the concrete cover for the rebar yielded that same thickness 53mm of concrete cover at any point on the both segments, which yields that the correction value of the thickness of the concrete cover is uniformly -8mm for this test piece. That is

$$\begin{aligned} \gamma_{t;z}(x) = \gamma_{t;y}(x) \equiv -8, \\ \gamma_{c;z}(x) = \gamma_{c;y}(x) \equiv -3 \end{aligned} \tag{19}$$

By the above argument, it was proved that the steel parallelled the surfaces C,D and E, by non-destructive inspection. After these theoretical study had finished, we cut-off the test piece in order to check whether the result (18) by our non-destructive inspection was correct. Figure 8 shows how to cut the test piece.

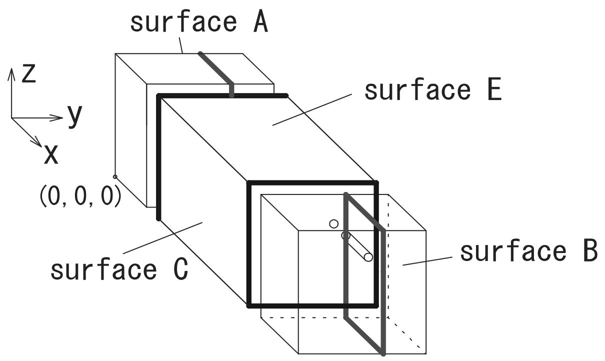

Figure 8: Cutting test piece

The actual positions of the endpoint measured on the cut surface are

$$a = (51, 100, 100), \quad b = (444, 100, 100). \tag{20}$$

Comparing (18) and (20), we can conclude that our reconstruction of the endpoints by non-destructive inspection is very precise. Furthermore our reconstruction (18) is much closer to the real position (20) than the designed one. Let us sum up the designed value, the reconstructed one and the real one in Table 3.

Table 3: Table of the results

Endpoint	coordinate	(i) Real value	(ii) Designed value	(iii) Reconstructed value	(i)-(ii)	(i)-(iii)
a	x	51	50	50	1	1
	y	100	95	100	5	0
	z	100	95	101	5	-1
b	x	444	440	446	4	-2
	y	100	95	100	5	0
	z	100	95	100	5	0

There being a number of researches for detection of the rebar, among which GPR (ground penetrating radar) techniques are known as a safe and cheaply running non-destructive inspection ones. In the non-destructive inspection with GPR techniques, they can detect the position of the rebars as well as the thickness of the concrete cover by various techniques with some accuracy. It is, however, not possible to exactly probe the endpoints of the rebar by GPR technique. We also claim that our technique can more accurately investigate the endpoints.

7. Summary

We make concluding remarks. The points in this paper are the followings.

(i) We have developed a non-destructive inspection technique to exactly probe the rebar by application of an apparatus to measure thickness of concrete cover for reinforcing steel by electromagnetic induction method (Theorem 5.1 and Property 6.1). Especially, we have established a method to exactly probe the endpoints of the rebar. It may be the first one.

(ii) In order to obtain the above conclusion (i), the idea of `the least square solutions' played an important role.

(iii) As a by-product of the above result (i), we have also developed a technique to measure the exact thickness of the concrete cover for reinforcing steel. Its error is less than 2%. We claim that it is more accurate than any other known technique.

(iv) Our theoretical studies to exactly probe the rebar (conclusion (i)) and to precisely measure the thickness of concrete cover for reinforcing steel (conclusion (iii)) have been verified to be good by our experiments (Section 6).

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