FIXED POINT PROBLEMS IN CAT(1) SPACES

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Abstract. We state existence and convergence theorems for finding fixed points of spherically nonspreading mappings in CAT(1) spaces. These results can be applied to convex optimization in such spaces.

1. Introduction

The concepts of spherically nonspreading mappings and firmly spherically nonspreading mappings in CAT(1) spaces were originally obtained in [14]. The use of tangent sine perturbation for convex minimization problems made it possible for them to propose firmly spherically nonspreading resolvents of convex functions in such spaces.

As is well understood, convex minimization problems in Hilbert spaces is deeply related to fixed point problems for firmly nonexpansive mappings. In fact, if H is a real Hilbert space and $f\colon H \to (-\infty, \infty]$ is a proper lower semicontinuous convex function, then the resolvent J_f of f defined by

$$J_f x = \operatorname*{argmin}_{y\in H} \left\{ f(y) + \frac{1}{2}\|y-x\|^2 \right\}$$

for all $x \in H$ is a well defined and single valued firmly nonexpansive mapping of H into itself, i.e.,

$$\|J_f x - J_f y\|^2 \leq \langle J_f x - J_f y, x - y\rangle$$

for all $x, y \in H$ or equivalently

$$\|J_f x - J_f y\| \leq \big\|\lambda x + (1-\lambda)J_f x - \big(\lambda y + (1-\lambda)J_f y\big)\big\|$$

for all $x, y \in H$ and $\lambda \in [0,1]$. It is also known that the fixed point set $\mathcal{F}(J_f)$ coincides with the set $\operatorname{argmin}_H f$ of minimizers of f; see, for instance, [4, 25].

In 1990s, Jost [13] and Mayer [22] independently extended the notion of resolvents of convex functions in Hilbert spaces to more general complete CAT(0) spaces. In this case, the resolvent J_f of a proper lower semicontinuous convex function f of a complete CAT(0) space X into $(-\infty, \infty]$ is defined by

$$J_f x = \operatorname*{argmin}_{y\in H} \left\{ f(y) + \frac{1}{2} d(y,x)^2 \right\}$$

for all $x \in X$. It is also known that J_f is firmly nonexpansive, i.e.,

$$d(J_f x, J_f y) \leq d\big(\lambda x \oplus (1-\lambda)J_f x, \lambda y \oplus (1-\lambda)J_f y\big)$$

for all $x, y \in X$ and $\lambda \in [0,1]$ and $\mathcal{F}(J_f) = \operatorname{argmin}_X f$; see [1, 3].

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Our main interest here is on convex minimization problems in more general complete CAT(1) spaces. Since the geometric properties of such spaces are quite different from those in complete CAT(0) spaces, we need to discuss a suitable way to perturb convex functions in such spaces when such problems are to be solved. Considering these differences, we define a concept of resolvent of convex functions by using tangent sine perturbations.

Let (X, d) be a complete CAT(1) space such that

$$d(v, v') < \frac{\pi}{2}$$

for all $v, v' \in X$ and $f\colon X \to (-\infty, \infty]$ a proper lower semicontinuous convex function. Then the mapping R_f given by

$$R_f x = \operatorname*{argmin}_{y \in X} \{f(y) + \tan d(y, x) \sin d(y, x)\}$$

for all $x \in X$ is a well defined and single valued mapping of X into itself. It also holds that

$$\mathcal{F}(R_f) = \operatorname*{argmin}_X f$$

and R_f is firmly spherically nonspreading in the sense of [14], i.e.,

$$(\cos d(R_f x, x) + \cos d(R_f y, y)) \cos^2 d(R_f x, R_f y) \geq 2 \cos d(R_f x, y) \cos d(R_f y, x)$$

for all $x, y \in X$.

In this paper, we study the problem of finding fixed points of spherically nonspreading mappings and firmly spherically nonspreading mappings in complete CAT(1) spaces. We also study some basic properties of quasiconvex functions and convex functions in such spaces. Applications to the problem of finding minimizers of proper lower semicontinuous convex functions in the spaces are also included. The results we state here were originally obtained by Kimura and Kohsaka [14].

2. Preliminaries

We denote by $\mathbb{R}$ and $\mathbb{N}$ the sets of real numbers and positive integers, respectively. The convergence of a sequence $\{x_n\}$ in a metric space (X, d) is denoted by $x_n \to x$. The fixed point set $\mathcal{F}(T)$ of a mapping T of a nonempty set X into itself is defined by

$$\mathcal{F}(T) = \{x \in X : Tx = x\}.$$

We denote by $\operatorname{argmin}_X f$ or $\operatorname{argmin}_{y \in X} f(y)$ the set of minimizers of a function f of a nonempty set X into $(-\infty, \infty]$ defined by

$$\operatorname*{argmin}_X f = \bigl\{x \in X : f(x) = \inf f(X)\bigr\}.$$

If (X, d) is a metric space, then the diameter of X and the closed ball at a center p with a radius $r \geq 0$ are denoted by $\operatorname{diam}(X)$ and $B_r[p]$, respectively.

Let $\{x_n\}$ be a sequence in a metric space (X, d). Then the asymptotic center $\mathcal{A}(\{x_n\})$ of $\{x_n\}$ is defined by

$$\mathcal{A}\bigl(\{x_n\}\bigr) = \left\{u \in X : \limsup_n d(u, x_n) = \inf_{y \in X} \limsup_n d(y, x_n)\right\}.$$

It is obvious that $\mathcal{A}(\{x_n\}) = X$ for every unbounded sequence $\{x_n\}$ in X. Thus if $\mathcal{A}(\{x_n\}) = \{p\}$ for some $p \in X$, then $\{x_n\}$ is bounded. The sequence $\{x_n\}$ is also said to be Δ-convergent to $p \in X$ if

$$\mathcal{A}(\{x_{n_i}\}) = \{p\}$$

for each subsequence $\{x_{n_i}\}$ of $\{x_n\}$.

Let $\mathbb{R}^N$ be the N-dimensional Euclidean space with inner product $\langle \cdot\,, \cdot \rangle$ and induced norm $|\cdot|$ for $N \in \mathbb{N}$. Let H be a real Hilbert space with inner product $\langle \cdot\,, \cdot \rangle$ and induced norm $\|\cdot\|$. The unit sphere S_H of H is a complete metric space with the spherical metric d_{S_H} defined by

$$d_{S_H}(u, v) = \arccos \langle u, v \rangle$$

for all $u, v \in S_H$. This space is called a Hilbert sphere. Since

$$\|u - v\|^2 = 2\big(1 - \langle u, v \rangle\big)$$

for all $u, v \in S_H$, a sequence $\{x_n\}$ in S_H converges to $p \in S_H$ with respect to d_{S_H} if and only if it converges to p in norm. Thus the metric topology on (S_H, d_{S_H}) coincides with the relative topology on S_H induced by the norm topology on H. We denote by $(\mathbb{S}^2, d_{\mathbb{S}^2})$ the Hilbert sphere of $\mathbb{R}^3$.

Let (X, d) be a metric space. It is said to be uniquely π-geodesic if for each $x, y \in X$ with $d(x, y) < \pi$, there exists a unique mapping γ of $[0, l]$ into X such that

$$\gamma(0) = x, \quad \gamma(l) = y, \quad \text{and} \quad d\big(\gamma(s), \gamma(t)\big) = |s - t|$$

for all $s, t \in [0, l]$, where $l = d(x, y)$. The mapping γ is called a geodesic path from x to y. In this case, we can define a convex combination of x and y by

$$(1 - \lambda)x \oplus \lambda y = \gamma(\lambda l)$$

for all $\lambda \in [0, 1]$. It is well known that a Hilbert sphere (S_H, d_{S_H}) is uniquely π-geodesic. The unique geodesic γ from x to y is given by

$$\gamma(t) = (\cos t)x + (\sin t)u$$

for all $x, y \in S_H$ with $0 < d_{S_H}(x, y) < \pi$, where

$$u = \frac{y - \langle x, y \rangle x}{\|y - \langle x, y \rangle x\|}.$$

A subset C of a uniquely π-geodesic metric space (X, d) such that $d(v, v') < \pi$ for all $v, v' \in C$ is said to be convex if

$$(1 - \alpha)x \oplus \alpha y \in C$$

whenever $x, y \in C$ and $\alpha \in [0, 1]$.

A uniquely π-geodesic metric space (X, d) is called a CAT(1) space if

$$d\big((1 - \alpha)x \oplus \alpha y, (1 - \beta)x \oplus \beta z\big) \le d_{\mathbb{S}^2}\big((1 - \alpha)\bar{x} \oplus \alpha \bar{y}, (1 - \beta)\bar{x} \oplus \beta \bar{z}\big)$$

whenever $\alpha, \beta \in [0, 1]$, $x, y, z \in X$, $\bar{x}, \bar{y}, \bar{z} \in \mathbb{S}^2$, and

$$d(x, y) = d_{\mathbb{S}^2}(\bar{x}, \bar{y}), \quad d(y, z) = d_{\mathbb{S}^2}(\bar{y}, \bar{z}), \quad \text{and} \quad d(z, x) = d_{\mathbb{S}^2}(\bar{z}, \bar{x}).$$

Typical examples of complete CAT(1) spaces are nonempty closed convex subsets of real Hilbert spaces, Hilbert spheres, complete $\mathbb{R}$-trees, and complete CAT(0) spaces. A CAT(1) space (X, d) is said to be admissible if

$$d(v, v') < \frac{\pi}{2}$$

for all $v, v' \in X$. A sequence $\{x_n\}$ in a CAT(1) space (X, d) is said to be spherically bounded if

$$\inf_{y\in X} \limsup_n d(y, x_n) < \frac{\pi}{2}.$$

We know the following fundamental result.

Theorem 2.1 ([10, Proposition 4.1, Corollary 4.4])**.** *If $\{x_n\}$ is a spherically bounded sequence in a complete* CAT(1) *space (X, d), then $\mathcal{A}\bigl(\{x_n\}\bigr)$ consists of one point and $\{x_n\}$ has a Δ-convergent subsequence.*

3. Spherically nonspreading mappings

Let (X, d) be an admissible CAT(1) space and $T\colon X \to X$ a mapping. The mapping T is said to be firmly spherically nonspreading if

$$(\cos d(Tx, x) + \cos d(Ty, y)) \cos^2 d(Tx, Ty) \geq 2 \cos d(Tx, y) \cos d(Ty, x)$$

for all $x, y \in X$. It is also said to be spherically nonspreading if

$$\cos^2 d(Tx, Ty) \geq \cos d(Tx, y) \cos d(Ty, x)$$

for all $x, y \in X$. Since

$$2 \geq \cos d(Tx, x) + \cos d(Ty, y),$$

every firmly spherically nonspreading mapping is spherically nonspreading. If T is spherically nonspreading and its fixed point set $\mathcal{F}(T)$ is nonempty, then T is quasinonexpansive. In fact, if T is a spherically nonspreading mapping with a fixed point, $x \in X$, and $y \in \mathcal{F}(T)$, then we have

$$\cos^2 d(Tx, y) \geq \cos d(Tx, y) \cos d(y, x).$$

Since X is admissible, we know that $\cos d(Tx, y)$ is positive and hence

$$\cos d(Tx, y) \geq \cos d(y, x).$$

Since $d(Tx, y)$ and $d(y, x)$ belong to $[0, \pi/2)$, we obtain $d(y, Tx) \leq d(y, x)$.

The following is a fixed point theorem for spherically nonspreading mapping.

Theorem 3.1 ([14, Theorem 5.2])**.** *Let X be an admissible complete* CAT(1) *space and $T\colon X \to X$ a spherically nonspreading mapping. Then $\mathcal{F}(T)$ is nonempty if and only if there exists $x \in X$ such that*

$$\limsup_n d(Ty, T^n x) < \frac{\pi}{2}$$

for all $y \in X$.

The following follows from [24, Lemma 11].

Lemma 3.2 ([24, Lemma 11])**.** *Let $A\colon \mathbb{N} \times \mathbb{N} \to [0, \infty)$ be a bounded function such that $A(n, n) = 0$ for all $n \in \mathbb{N}$ and*

$$2A(m + 1, n + 1) \leq A(m + 1, n) + A(m, n + 1)$$

for all $m, n \in \mathbb{N}$. Then $\lim_n A(n, n + 1) = 0$.

Using Lemma 3.2, we can prove the following Δ-convergence theorem for spherically nonspreading mappings.

Theorem 3.3 ([14, Theorem 6.3])**.** *Let X be an admissible complete* CAT(1) *space and $T\colon X \to X$ a spherically nonspreading mapping such that $\mathcal{F}(T)$ is nonempty. Suppose that*

(1) $\sup_{m,n} d(T^m x, T^n x) < \pi/2$ *for all* $x \in X$;
(2) $\limsup_n d(Ty, y_n) < \pi/2$ *whenever* $\{y_n\}$ *is a sequence in* X *and* $y_n \to y \in X$.

Then $\{T^n x\}$ *is* Δ*-convergent to an element of* $\mathcal{F}(T)$ *for all* $x \in X$.

We can also obtain the following Δ-convergence theorem for firmly spherically nonspreading mappings.

Theorem 3.4 ([14, Theorem 6.5]). *Let* X *be an admissible complete* CAT(1) *space and* $T\colon X \to X$ *a firmly spherically nonspreading mapping such that* $\mathcal{F}(T)$ *is nonempty. Suppose that*

$$\limsup_n d(Ty, y_n) < \frac{\pi}{2}$$

whenever $\{y_n\}$ *is a sequence in* X *and* $y_n \to y \in X$. *Then* $\{T^n x\}$ *is* Δ*-convergent to an element of* $\mathcal{F}(T)$ *for all* $x \in X$.

4. Quasiconvex functions and convex functions

In this section, we apply our results to the problem of finding minimizers of proper lower semicontinuous convex functions in CAT(1) spaces.

We first recall some basic notions related to this problem. Let (X, d) be an admissible CAT(1) space and $f\colon X \to (-\infty, \infty]$ a function. The domain $\operatorname{dom} f$ of f is defined by

$$\operatorname{dom} f = \{x \in X : f(x) \in \mathbb{R}\}.$$

The function f is said to be

- proper if $\operatorname{dom} f$ is nonempty;
- lower semicontinuous if $\{x \in X : f(x) \le \lambda\}$ is closed for all $\lambda \in \mathbb{R}$;
- Δ-lower semicontinuous if $f(p) \le \liminf_n f(x_n)$ whenever $\{x_n\}$ is a sequence in X which is Δ-convergent to $p \in X$;
- quasiconvex if
$$f\big((1-\alpha)x \oplus \alpha y\big) \le \max\{f(x),\ f(y)\}$$
for all $x, y \in X$ and $\alpha \in (0, 1)$;
- convex if
$$f\big((1-\alpha)x \oplus \alpha y\big) \le (1-\alpha) f(x) + \alpha f(y)$$
for all $x, y \in X$ and $\alpha \in (0, 1)$.

A point $u \in X$ is called a minimizer of f if $f(u) = \inf f(X)$.

We know the following lemmas in CAT(1) spaces.

Lemma 4.1 ([14, Lemma 3.1]). *Let* X *be an admissible complete* CAT(1) *space and* $f\colon X \to (-\infty, \infty]$ *a proper lower semicontinuous quasiconvex function. Then* f *is* Δ*-lower semicontinuous.*

Lemma 4.2 ([14, Lemma 3.2]). *Let* X *be an admissible complete* CAT(1) *space,* $f\colon X \to (-\infty, \infty]$ *a proper lower semicontinuous quasiconvex function, and* $p \in X$. *Suppose that* $f(x_n) \to \infty$ *whenever* $\{x_n\}$ *is a sequence in* X *such that* $d(p, x_n) \to \pi/2$. *Then* $\operatorname{argmin}_X f$ *is nonempty. Further, if*

$$f\left(\frac{1}{2}x \oplus \frac{1}{2}y\right) < \max\{f(x), f(y)\}$$

whenever $x, y \in \operatorname{dom} f$ *and* $x \neq y$, *then* $\operatorname{argmin}_X f$ *is a singleton.*

Corollary 4.3 ([14, Corollary 3.3]). *If X is a complete* CAT(1) *space such that* $\operatorname{diam}(X) < \pi/2$*, then every proper lower semicontinuous quasiconvex function of X into $(-\infty, \infty]$ has a minimizer.*

Lemma 4.4 ([14, Lemma 3.4]). *Let X be an admissible complete* CAT(1) *space, K a nonempty closed convex subset of X, $f\colon X \to (-\infty, \infty]$ a proper lower semicontinuous quasiconvex function such that* $\operatorname{dom} f \cap K$ *is nonempty, and $p \in X$. Suppose that there exists $n_0 \in \mathbb{N}$ such that $x_n \notin K$ for all integer $n \geq n_0$ whenever $\{x_n\}$ is a sequence in X such that $d(p, x_n) \to \pi/2$. Then there exists $u \in K$ such that $f(u) = \inf f(K)$.*

Corollary 4.5 ([14, Corollary 3.5]). *Let X be an admissible complete* CAT(1) *space, $f\colon X \to (-\infty, \infty]$ a proper lower semicontinuous quasiconvex function, p an element of* $\operatorname{dom} f$*, and r an element of $[0, \pi/2)$. Then there exists $u \in B_r[p]$ such that $f(u) = \inf f(B_r[p])$.*

Theorem 4.6 ([14, Theorem 3.6]). *Every proper lower semicontinuous convex function of an admissible complete* CAT(1) *space into $(-\infty, \infty]$ is bounded below.*

5. Applications to convex minimizaiton problems

The following result is of fundamental importance in the definition of resolvents of convex functions in CAT(1) spaces.

Theorem 5.1 ([14, Theorem 4.2]). *Let X be an admissible complete* CAT(1) *space, $f\colon X \to (-\infty, \infty]$ a proper lower semicontinuous convex function, and x an element of X. Then there exists a unique $\hat{x} \in X$ such that*

$$f(\hat{x}) + \tan d(\hat{x}, x) \sin d(\hat{x}, x) = \inf_{y \in X} \bigl\{ f(y) + \tan d(y, x) \sin d(y, x) \bigr\}.$$

We next define the concept of resolvent of a convex function with a tangent sine perturbation.

Definition 5.2 ([14, Definition 4.3]). If X and f are the same as in Theorem 5.1, then the resolvent R_f of f is defined by

$$R_f x = \operatorname*{argmin}_{y \in X} \bigl\{ f(y) + \tan d(y, x) \sin d(y, x) \bigr\}$$

for all $x \in X$.

Theorem 5.3 ([14, Theorem 4.6]). *Let X be an admissible complete* CAT(1) *space, $f\colon X \to (-\infty, \infty]$ a proper lower semicontinuous convex function, and R_f the resolvent of f. Then $R_f\colon X \to X$ is a single valued, well defined, and firmly spherically nonspreading, and $\mathcal{F}(R_f) = \operatorname{argmin}_X f$.*

Using Theorems 3.1, 3.4, and 5.3, we can obtain the following result.

Theorem 5.4 ([14, Theorem 7.1]). *Let X be an admissible complete* CAT(1) *space, $f\colon X \to (-\infty, \infty]$ a proper lower semicontinuous convex function, and R_f the resolvent of f. Then the following hold.*

(1) $\operatorname{argmin}_X f$ *is nonempty if and only if there exists $x \in X$ such that*

$$\limsup_n d(R_f y, R_f^n x) < \frac{\pi}{2}$$

for all $y \in X$;

(2) *if* $\operatorname{argmin}_X f$ *is nonempty and*

$$\limsup_n d(R_f y, y_n) < \frac{\pi}{2}$$

whenever $\{y_n\}$ *is a sequence in* X *and* $y_n \to y \in X$*, then* $\{R_f^n x\}$ *is* Δ*-convergent to an element of* $\operatorname{argmin}_X f$ *for all* $x \in X$*.*

As a direct consequence of Theorem 5.4, we obtain the following corollary.

Corollary 5.5 ([14, Corollary 7.2])**.** *Let* X *be a complete* CAT(1) *space such that* $\operatorname{diam}(X) < \pi/2$*,* $f\colon X \to (-\infty, \infty]$ *a proper lower semicontinuous convex function, and* R_f *the resolvent of* f*. Then* $\operatorname{argmin}_X f$ *is nonempty and* $\{R_f^n x\}$ *is* Δ*-convergent to an element of* $\operatorname{argmin}_X f$ *for all* $x \in X$*.*

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