Extended Aluthge Transforms and Applications

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Abstract

Given a bounded linear operator T with canonical polar decomposition $T = V|T|$, the Aluthge transform of T is the operator $\Delta(T) := \sqrt{|T|}V\sqrt{|T|}$.

For P an arbitrary positive operator such that $VP = T$, we define the *extended* Aluthge transform of T associated with P by $\Delta_P(T) := \sqrt{P}V\sqrt{P}$.

First, we establish some basic properties of Δ_P; second, we study the fixed points of the extended Aluthge transform; third, we consider the case when T is an idempotent; next, we discuss whether Δ_P leaves invariant the class of complex symmetric operators.

We also study how Δ_P transforms the numerical radius and numerical range.

As a key application, we prove that the spherical Aluthge transform of a commuting pair of operators corresponds to the extended Aluthge transform of a 2×2 operator matrix built from the pair; thus, the theory of extended Aluthge transforms yields results for spherical Aluthge transforms.

(This is a joint work with Raul E. Curto)