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Moduli Spaces in Algebraic Geometry

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Abstract. Invariants of topological spaces of dimension three play a major role in many areas, in particular ...

Mathematics Subject Classification (2000): AMS-CLASSIFICATION.

Introduction by the Organisers

The workshop *Invariants of topological spaces of dimension three*, organised by Max Muster (München) and Bill E. Xample (New York) was held March 1st–March 6th, 2005. This meeting was well attended with over 30 participants with broad geographic representation from all continents. This workshop was a nice blend of researchers with various backgrounds ...

Workshop: Moduli Spaces in Algebraic Geometry

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Abstracts

Polarized K3 surfaces of gnenus 16

Shigeru Mukai

Let $\mathcal{T} = G(2,3;\mathbb{C}^4)$ be the EPS-moduli space of the twisted cubics in $\mathbb{P}^3$ constructed in [1]. $\mathcal{T}$ is the GIT-quotient of $\mathbb{C}^2\otimes\mathbb{C}^3\otimes\mathbb{C}^4$ by the action of $GL(2)\times GL(3)$ on the first and second factors. There exist two tautological vector bundles $\mathcal{E},\mathcal{F}$ of rank 3, 2 and the universal homomorphism $\mathcal{E}\otimes\mathbb{C}^4 \longrightarrow \mathcal{F}$ on $\mathcal{T}$. The vector bundle $\mathcal{E}$ embeds $\mathcal{T}$ into the 21-dimensional Grassmannian $G(S^2\mathbb{C}^4, 3)$.

Theorem (1) *A general complete intersection S with respect to the rank 10 vector bundle $\mathcal{E}^{\oplus 2}\oplus\mathcal{F}^{\oplus 2}$ in the EPS-moduli space $\mathcal{T}$ is a K3 surface, and $\det\mathcal{E}|_S$ is a polarization of genus 16, that is, degree 30.*

(2) *Moreover, a moduli-theoretically general polarized K3 surface (S,h) of genus 16 is obtained in this way.*

Let $\mathcal{F}_g$ be the moduli space of primitively (quasi-)polarized K3 surfaces (S,h) of degree $2g-2$, and $\mathcal{S}_g$ be the (quasi-)universal family over it. The theorem yields a dominant rational map $P^{36}\cdots\to\mathcal{F}_{16}$ from a $G(2,12)$-bundle P^{36} over the 16-dimensional Grassmannian $G(2,S^2\mathbb{C}^4)$ of pencils of quadrics to $\mathcal{F}_{16}$.

Corollary *The moduli space $\mathcal{F}_{16}$ is unirational.*

See [2] and [3] for the birational type of other $\mathcal{F}_g$'s.

Since $\mathcal{E}|_S$ is a stable semi-rigid vector bundle with Mukai vector $v=(3,h,5)$, the rational map factors through $\mathcal{S}_{16}$.

Conjecture The induced rational map $P^{36}//PGL(4)\cdots\longrightarrow\mathcal{S}_{16}$ between 21-dimensional varieties is birational.

References

[1] G. Ellingsrud, R. Piene, and S.A. Strømme, *On the variety of nets of quadrics defining twisted cubic curves.* In F. Ghione, C. Peskine and E. Sernesi, editors, *Space Curves*, Lecture Notes in Math. **1266** (1987), Springer-Verlag, pp. 84–96.

[2] V.A. Gritsenko, K. Hulek and G.A. Sankaran, *The Kodaira dimension of the moduli spaces of K3 surfaces*, Invent. math. **169** (2007), 519–567.

[3] S. Mukai, *Polarized K3 surfaces of genus thirteen*, In S. Mukai et. al, editors, *Moduli spaces and Arithmetic Geometry (Kyoto, 2004)*, Adv. Stud. Pure Math. **45** (2006), Math. Soc. Japan and Amer. Math. Soc., pp. 315–326.

Computing other invariants of topological spaces of dimension three

Fréderic Francqis Déchamps, Peter Mustermann

The computation of ...

Reporter: Daniel Greb