Japan. J. Math. 11, 33–68 (2016)
DOI: 10.1007/s11537-016-1489-1

JAPANESE JOURNAL OF MATHEMATICS
SINCE 1924

# Free analysis and random matrices*

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Received: 2 December 2015 / Revised: 12 February 2016 / Accepted: 11 March 2016
Published online: 16 May 2016


Communicated by: Yasuyuki Kawahigashi

**Abstract.** We describe the Schwinger–Dyson equation related with the free difference quotient. Such an equation appears in different fields such as combinatorics (via the problem of the enumeration of planar maps), operator algebra (via the definition of a natural integration by parts in free probability), in classical probability (via random matrices or particles in repulsive interaction). In these lecture notes, we shall discuss when this equation uniquely defines the system and in such a case how it leads to deep properties of the solution. This analysis can be extended to systems which approximately satisfy these equations, such as random matrices or Coulomb gas interacting particle systems.

*Keywords and phrases:* random matrices, non-commutative measure, Schwinger–Dyson equation

*Mathematics Subject Classification (2010):* 15A52 (primary), 46L50, 46L54 (secondary)

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* This article is based on the 14th Takagi Lectures that the author delivered at University of Tokyo on November 15 and 16, 2014.

** This work was partially supported by the NSF and Simons foundation.

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