

## From Riemann and Kodaira to modern developments on complex manifolds<sup>★</sup>

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**Abstract.** We survey the theory of complex manifolds that are related to Riemann surface, Hodge theory, Chern class, Kodaira embedding and Hirzebruch–Riemann–Roch, and some modern development of uniformization theorems, Kähler–Einstein metric and the theory of Donaldson–Uhlenbeck–Yau on Hermitian Yang–Mills connections. We emphasize mathematical ideas related to physics. At the end, we identify possible future research directions and raise some important open questions.

*Keywords and phrases:* Kähler–Einstein metric, Donaldson–Uhlenbeck–Yau correspondence, mirror symmetry, Calabi–Yau manifold

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