The 11th Takagi Lectures

November 17 (Sat)–18 (Sun), 2012
Graduate School of Mathematical Sciences
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ABSTRACT

P.F. Baum:
Non-Commutative Geometry and the Local Langlands Conjecture

Let G be a reductive p-adic group. Examples are $GL(n, F)$, $SL(n, F)$, etc where n can be any positive integer and F can be any finite extension of the field Q_p of p-adic numbers. The smooth dual of G is the set of (equivalence classes of) smooth representations of G. The representations are on vector spaces over the complex numbers. In a canonical way, the smooth dual of G is the disjoint union of countably many subsets known as the Bernstein components.

Results from non-commutative geometry—e.g. BC (Baum–Connes) conjecture, periodic cyclic homology of the Hecke algebra of G—indicate that a very simple geometric structure might be present in the smooth dual of G. The ABP (Aubert–Baum–Plymen) conjecture makes this precise by asserting that each Bernstein component in the smooth dual of G is a complex affine variety. These varieties are explicitly identified as certain extended quotients. For split G, (granted a mild restriction on the residual characteristic) the ABP conjecture has recently been proved for any Bernstein component in the principal series. A corollary is that the local Langlands conjecture is valid throughout the principal series. The above is joint work with Anne-Marie Aubert, Roger Plymen, and Maarten Solleveld.

Topics in these lectures:

#1. Review of the LL (Local Langlands) conjecture.

#2. Statement of the ABP conjecture.

#3. Outline of the proof that for any split reductive p-adic group G both ABP and LL are valid throughout the principal series of G. Class field theory, founded by Professor Teiji Takagi, is a basic point in all three topics.

A. Lubotzky:
Ramanujan Complexes and High Dimensional Expanders

Expander graphs, in general, and Ramanujan graphs, in particular, have been objects of intensive research in the last four decades. Many application came out, initially to computer science and combinatorics and more recently also to pure mathematics (number theory, geometry, group theory). In recent years, there has been an interest in generalizing this theory to higher dimensional simplical complexes. We plan to survey first the classical theory and then to describe the more recent developments. Some directions of current research will be presented as well as suggestions for future research.

R. Seiringer:
Hot Topics in Cold Gases—A Mathematical Physics Perspective

We present an overview of mathematical results on the low temperature properties of dilute quantum gases, which have been obtained in the past few years. The presentation includes a discussion of Bose–Einstein condensation, and focuses on the excitation spectrum for trapped gases and its relation to superfluidity, as well as the appearance of quantized vortices in rotating systems. All these properties are intensely being studied in current experiments on cold atomic gases. We will give a brief description of the mathematics involved in understanding these phenomena, starting from the underlying many-body Schrödinger equation.