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Classification and rigidity in operator algebras arising from free groups

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Abstract

Higman has shown in 1939 that group algebras $\mathbb{C}\Gamma$ of torsion free orderable groups Γ can be isomorphic only if the groups are isomorphic. But letting $\mathbb{C}\Gamma$ act on the Hilbert space $\ell^2\Gamma$ by left convolution and then taking closure in the weak operator topology, gives rise to much larger algebras, denoted $L(\Gamma)$, that tend to forget the group Γ, for instance $L(\mathbb{Z}\wr\mathbb{Z}^n)$, $n \geq 1$ are all isomorphic (Connes 1976). The study of these algebras, now called *von Neumann algebras*, was initiated by Murray and von Neumann in 1936–1943. A famous problem going back to their work is whether the von Neumann algebras $L(\mathbb{F}_n)$, associated with the free groups on n generators, are non-isomorphic for different n's. While this is still open, its "group measure space" version, asking whether the crossed product von Neumann algebras $L^\infty(X) \rtimes \mathbb{F}_n$ arising from free ergodic probability measure preserving actions $\mathbb{F}_n \curvearrowright X$ are non-isomoprphic for $n = 2, 3, \ldots$, independently of the actions, has recently been settled by Stefaan Vaes and myself. I will comment on this result, as as well as on some related problems.