On Torsion in the Cohomology of Locally Symmetric Varieties

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Abstract

This note explains some of the author's work on understanding the torsion appearing in the cohomology of locally symmetric spaces such as arithmetic hyperbolic 3-manifolds.

The key technical tool was a theory of Shimura varieties with infinite level at p: As p-adic analytic spaces, they are perfectoid, and admit a new kind of period map, called the Hodge–Tate period map, towards the flag variety. Moreover, the (semisimple) automorphic vector bundles come via pullback along the Hodge–Tate period map from the flag variety.

In the case of the Siegel moduli space, the situation is fully analyzed in [13]. We explain the conjectural picture for a general Shimura variety.