The 14th Takagi Lectures

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ABSTRACT

A. Guionnet:

Random Matrices and Free Analysis

We describe the Schwinger–Dyson equation related with the free difference quotient, as encountered in the enumeration of planar maps, free probability, random matrices or particles in repulsive interaction. In these lecture notes, we shall discuss how this equation can uniquely define the system and lead to deep properties such as existence of transport maps between solutions, leading to isomorphisms between related algebras. This analysis can be extended to systems which approximately satisfy these equations, such as random matrices or Coulomb gas interacting particle systems, yielding topological expansions and universality for the fluctuations of the eigenvalues.

C. Manolescu:

Floer Theory and Its Topological Applications

We survey the different versions of Floer homology that can be associated to three-manifolds. We also discuss their applications, particularly to questions about surgery, homology cobordism, and four-manifolds with boundary. We then describe Floer stable homotopy types, the related Pin(2)-equivariant Seiberg–Witten Floer homology, and its application to the triangulation conjecture.

P. Scholze:

On Torsion in the Cohomology of Locally Symmetric Varieties

This note explains some of the author's work on understanding the torsion appearing in the cohomology of locally symmetric spaces such as arithmetic hyperbolic 3-manifolds.
The key technical tool was a theory of Shimura varieties with infinite level at p: As p-adic analytic spaces, they are perfectoid, and admit a new kind of period map, called the Hodge–Tate period map, towards the flag variety. Moreover, the (semisimple) automorphic vector bundles come via pullback along the Hodge–Tate period map from the flag variety.
In the case of the Siegel moduli space, the situation is fully analyzed in [13]. We explain the conjectural picture for a general Shimura variety.

A. Venkatesh:

Cohomology of Arithmetic Groups and Periods of Automorphic Forms

I will first give a brief introduction to the cohomology of arithmetic groups. I will not assume any prior exposure to the topic, although it will be helpful to be familiar with Hodge theory. I will emphasize one particularly interesting structure: a certain piece of their cohomology looks like the cohomology of a torus.
I will then propose an conjectural explanation for this structure, namely that there is a "hidden" action of certain motivic cohomology groups. I'll focus on consequences of this conjecture that can be understood without knowing details of motivic cohomology. In particular, the conjecture predicts numerical invariants attached to the cohomology (the "period matrix"), and I will discuss some verifications of these predictions (joint with K. Prasanna). As time permits, I will discuss how the conjecture is related to "derived" Hecke operators.