

The seventh Takagi Lectures

November 22, 2009 (Sun) 11:30–12:30

November 23, 2009 (Mon) 12:30–13:30

Graduate School of Mathematical Sciences

The University of Tokyo

Arithmetic applications of the Langlands program

Michael Harris

(Université Paris 7)

Abstract

The functoriality conjecture is at the heart of the Langlands program and will undoubtedly remain as a challenge to number theorists for many decades to come. Shortly after formulating his program, however, Langlands proposed to test it in two interdependent settings. The first was the framework of *Shimura varieties*, already understood by Shimura as a natural setting for a non-abelian generalization of the Shimura–Taniyama theory of complex multiplication. The second was the phenomenon of endoscopy, which can be seen alternatively as a classification of the *obstacles* to the stabilization of the trace formula or as an *opportunity* to prove the functoriality conjecture in some of the most interesting cases. After three decades of research, much of it by Langlands and his associates, these two closely related experiments are coming to a successful close, at least for classical groups, thanks in large part to the recent proof of the so-called *Fundamental Lemma* by Waldspurger, Laumon, and especially Ngô.

My primary interest in these lectures is to give an account of these developments insofar as they are relevant to the Galois groups of number fields. Algebraic number theorists are just beginning to take stock of the new information provided by the successful resolution of the problem of endoscopy and the analysis of the most important classes of Shimura varieties. I will devote special attention to the application in this setting of the methods pioneered by Wiles, and developed further by Taylor, Kisin, and others, in his proof of statements in non-abelian class field theory that imply Fermat's Last Theorem.