Splitting curves in double covers and elliptic surfaces

Abstract: Let Σ be a smooth projective surface. Let $f: Z \to \Sigma$ be a double cover of Σ with branch locus B and let Z' be the Stein factorization of f. An irreducible curve on D is called a splitting curve with respect to f if f^*D is of the form

$$f^*D = D^+ + D^- + E,$$

where $D^+ \neq D^-$, $\sigma_f^* D^+ = D^-$, $f(D^+) = f(D^-) = D$ and $\operatorname{Supp}(E)$ is contained in the exceptional set of $\mu: Z \to Z'$. In this talk, we consider some properties in the case when $\Sigma = a$ rational ruled surface, Z = a double cover of Σ branched at two disjoint sections and D = a tri-sectons on Σ .