# Kyoto Dynamics Days 3

Date: March 3-5, 2004
Place: Department of Mathematics, Kyoto University,
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## - Program -

**March 3rd (Wednesday)**

2:00-4:00 Yakov B. Pesin (Pennsylvania State Univ., USA & RIMS)

Correlation dimension, data analisys and thermodynamical formalis <Special Lecture for students>

**March 4th (Thursday)**

10:30-11:30 Yakov B. Pesin (Pennsylvania State Univ., USA, and RIMS)

A non-Markov approach to SRB-measures

1:30-2:30 Stefano Luzzatto (Imperial College, UK)

The likelihood of chaos in the quadratic family

2:45-3:45 Wei-Xiao Shen (Univ. Warwick, UK & RIMS)

Invariant measures exist without a growth condition

4:00-5:00 Marian Mrozek (Jagiellonian Univ., Poland)

Index pairs algorithms

**March 5th (Friday)**

10:30-11:30 Sergei Yu. Pilyugin (St. Petersburg Univ., Russia)

Some genericity problems in shadowing theory

1:30-2:30 Bill Kalies (Florida South Univ., USA)

Variational Methods for Second-Order Lagrangian Systems

2:45-3:45 Oliver Junge (Univ. Paderborn, Germany)

Numerical tools for the analysis of transport phenomena in astrodynamics

4:00-5:00 Konstantin Mischaikow (Georgia Tech., USA)

A constructive proof of Conley's decomposition theorem

# - Abstracts -

**Yakov Pesin (Pennsylvania State Univ., USA & RIMS)**
**Title: Correlation dimension, data analisys and thermodynamical formalism**
**Abstract:** I will discuss the notion of the correlation dimension for dynamical systems from various points of view including data analisys of numerical information of the system, general dimension theory, and ergodic theory of smooth hyperbolic dynamical systems (thermodynamical formalism of Sinai, Ruelle and Bowen). I will also discuss some interesting open problems in dimension theory of dynamical systems.

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**Yakov Pesin (Pennsylvania State Univ., USA & RIMS)**
**Title: A non-Markov approach to SRB-measures**
**Abstract:** The classical way of constructing SRB measures due to Sinai, Ruelle, and Bowen uses Markov partitions in an essential way. I will describe an approach to SRB-measures which does not use Markov partitions and hence, can be helpful in situations where these partitions do not exist. Examples include partially hyperbolic attractors, attractors with singularities (e.g. Lorenz attractor), etc.

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**Stefano Luzzatto (Imperial College, UK)**
**Title: The likelihood of chaos in the quadratic family**
**Abstract:** Numerical experiments in the 70's indicated that extremely complicated, possibly non-periodic, dynamical phenomena occur in the relatively simple quadratic family of one-dimensional maps. In a groundbreaking paper of Jakobson (CMP 1981) this was confirmed by showing that chaotic dynamics, or more technically, maps with absolutely continuous invariant measures, occurred with positive probability in the family. This is in remarkable contrast to later results of Graczyk.Swiatek and Lyubich that there is an "open and dense" set of parameters for which the dynamics is periodic, indicating in particular that the set of "chaotic" parameters is similar to a positive measure Cantor set.

Notwithstanding extensive work on the subject nobody as yet has any idea as to the relative measures of these two sets, even though it is known that together they account for almost every parameter. I will describe work in progress with Kokubu, Mischaikov, Oka, Pilarczyk, Takahasi on a strategy for addressing this question. This requires a combination of rigorous numerical methods with subtle geometric/ probabilisttic arguments.

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**Wei-Xiao Shen (Univ. Warwick, UK & RIMS)**
**Title: Invariant measures exist without a growth condition**
**Abstract:** This is a joint work with Henk Bruin and Sebastian van Strien. Given an S-unimodal map $f$ with critical order $\ell$, we show that there exists a constant $C=C(\ell)$ such that if $|Df^n(f(c))|\ge C$ holds for all $n$ sufficiently large, then $f$ has an absolutely continuous invariant measure, where $c$ denotes the critical point. This result shows that the well-known Collet-Eckmann and Nowicki-van Strien conditions can be weakened considerably.

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**Marian Mrozek (Jagiellonian Univ., Poland)**
**Title: Index Pairs Algorithms**
**Abstract:** On one hand index pairs constitute the technical machinery needed in the construction of the Conley index. On the other hand they are the principial tool used in the algorithmic computation of the index. In the literature one can find several definitions of index pairs. Not all of them are good for the algorithmic computation of

the Conley index. In the lecture we discuss various definitions of index pairs from the algorithmic point of view. We also introduce the concept of weak index pair and show why it is especially convenient in the computation of the Conley index of numerically unstable dynamical systems.

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**Sergei Yu. Pilyugin (St. Petersburg State University, Russia)**
**Title: Some genericity problems in shadowing theory**
Abstract: The following topics will be discussed:

- $C^0$-genericity of the shadowing property;
- $C^1$-open sets of systems without the shadowing property;
- genericity of the weak shadowing property and its relation to the problem of stability of limit sets of domains;
- problem of genericity for inverse shadowing properties.

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**Bill Kalies (Florida South Univ., USA)**
**Title: Variational Methods for Second-Order Lagrangian Systems**
**Abstract:** A second-order Lagrangian system (in one space dimension) is defined by the Euler-Lagrange equation of an action functional with Lagrangian L(u,u',u''). Such systems arise in a variety of applications including phase transitions, pattern formation, and optics. The goal of the work described in this talk is twofold. First, we want to develop variational techniques which require only mild assumptions on the Lagrangian. Specifically, no transversality or similar non-degeneracy conditions will be imposed. Second, we would like to characterize the bounded solutions. In particular we investigate the existence and multitude of periodic solutions, patterned solutions, heteroclinics and homoclinics, and chaotic solutions.

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**Oliver Junge (Univ. Paderborn, Germany)**
**Title: Numerical tools for the analysis of transport phenomena in astrodynamics**
**Abstract:** In astrodynamics, statements about the probability by which asteroids or other bodies eventually transit from one region within the solar system into another one are of high interest. Due to the complicated nature of the dynamical behaviour of the underlying differential equations an answer to this question requires a numerical approach. In this talk we show how both, geometric methods exploiting the structure of certain invariant manifolds, and set oriented methods that yield a discretization of the natural transfer operator, yield reliable answers for quite large time scales. As an application we compute transport rates for objects near Jupiter."

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**Konstantin Mischaikow (Georgia Tech., USA)**
**Title: A constructive proof of Conley's decomposition theorem**
**Abstract:** We give a new definition of the chain recurrent set of a continuous map or semi-flow using finite spatial discretizations. This approach allows for an algorithmic construction of index filtrations for Morse decompositions which approximate the chain recurrent set arbitrarily closely as well as discrete approximations of Conley's Lyapunov function. This is a natural framework in which to develop computational techniques for the analysis of qualitative dynamics of maps and flows including rigorous computer-assisted proofs.