**The Bender-Wu Analysis and the Voros Theory II.**

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We constructed in [AKT] a formal transformation that brings a Schrödinger equation

$$\left(\frac{d^2}{d\tilde{x}^2} - \eta^2 Q(\tilde{x})\right)\tilde{\psi} = 0 \quad (\eta : \text{a large parameter}) \tag{1}$$

with two simple turning points to the Weber equation

$$\left(\frac{d^2}{dx^2} - \eta^2(E(\eta) - \frac{1}{4}x^2)\right)\psi = 0, \tag{2}$$

where

$$E(\eta) = \sum_{j\geq 0} E_j \eta^{-j}. \tag{3}$$

There are three problems in making use of this result to obtain concrete information about the analytic structure of the WKB solutions $\psi_\pm$ of (1), or their Borel transforms $\psi_{\pm,B}$.

**Problem 1.** *Can we guarantee a sufficiently large domain of definition of the integral operator associated with the formal transformation?*

**Problem 2.** *Suppose first that $E(\eta)$ is a genuine constant $E_0$. Then how much do we know about the analytic properties of the Borel transformed WKB solutions of* (2)*?*

**Problem 3.** *What is the analytic meaning of WKB solutions of* (2) *if $E(\eta)$ is an infinite series?*

The purpose of this talk is to give satisfactory answers to these problems.

# References

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