ON SOME PROPERTIES OF THE HILBERT SCHEME OF SMOOTH CURVES

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ABSTRACT

Let $H_{d,g,r}$ denotes the Hilbert scheme parametrizing smooth irreducible complex curves of degree d and genus g embedded in $\mathbb{P}^r$. Severi claimed in [Sev21] that $H_{d,g,r}$ is irreducible if $d \geq g + r$. As it has turned out in recent years, the conjecture is true for $r = 3$ and 4, while for $r \geq 6$ it is incorrect. I prove that $H_{g,g,3}$, $H_{g+3,g,4}$ and $H_{g+2,g,4}$ are irreducible, provided that $g \geq 13$, $g \geq 5$ and $g \geq 11$, respectively. This extends results obtained previously by Ein, [Ein86, Ein87] and by Keem and Kim, [KK92]. The technique can also be applied to extend the known irreducibility range of $H_{d,g,5}$ with respect to d.

References

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