# The geography of simply connected smooth 4-manifolds

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One of the main topological problems in smooth 4-manifolds is to classify simply connected smooth 4-manifolds. The classical invariants of a simply connected smooth 4-manifold are encoded by its intersection form $Q_X$, a unimodular symmetric bilinear pairing on $H_2(X : \mathbf{Z})$. For example, one can determine its rank $b_2(X) = b_2^+ + b_2^-$, its signature $\sigma(X) = b_2^+ - b_2^-$, and its Euler characteristic $e(X) = b_2(X) + 2$ from $Q_X$. M. Freedman showed that a simply connected smooth 4-manifold is determined up to homeomorphism by $Q_X$. The situation in the smooth category is strikingly different. Hence it is an important question to know which unimodular, symmetric, bilinear integral forms are realized as the intersection form of a simply connected smooth 4-manifold, and which simply connected smooth 4-manifolds admit more than one smooth structure. We call these *geography problems of simply connected smooth 4-manifolds.*

Gauge theory - Donaldson theory and Seiberg-Witten theory - has been very successful in the geography problems. For example, S. Donaldson proved that the intersection form of a simply connected, definite, smooth 4-manifold is diagonalizable, and it has been proved that most known simply connected irreducible smooth 4-manifolds with $b_2^+$ odd and large enough admit infinitely many distinct smooth structures. Recently, there has also been progress in the case of $b_2^+$ small, in particular $b_2^+ = 1$.

In this talk I'd like to present various constructions of simply connected smooth 4-manifolds which have not been known before. I also survey recent developments in this area.