Zbl 222.10053
Erdős, Paul; Guy, R.K.
Distinct distances between lattice points. (In English)
Elemente Math. 25, 121-123 (1970).

Let k be the greatest number of points in real 2-space with integer coordinates between 1 and n and for which all mutual distances are distinct. By a simple counting argument, $k \leq n$. For $2 \leq n \leq 7$, $k = n$ is verified by a choice of points in the plane. From a result of *E. Landau* [Handbuch der Lehre von der Verteilung der Primzahlen (Leipzig, 1909), Bd. 2, p. 643] there is a positive constant c with $k < cn(\log n)^{-1/4}$. A simple combinatorial proof is given that for $\epsilon > 0$, if n is sufficiently large, then $k > n^{2/3-\epsilon}$. Results for dimensions 1 and 3 are mentioned.
Two problems are suggested: 1. Find the minimum number of points, determining distinct distances, so that no point may be added without duplicating a distance. 2. Given any n points in the plane (or d-space) how many can one select so that the distances are all distinct?

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