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Erdős, Paul; Szemeredi, E.

On a problem of Graham. (In English)

Publ. Math., Debrecen 23, 123-127 (1976). [0033-3883]

The following conjecture is considered. Let p be a prime, and let a_1, \ldots, a_p be non-zero residues (mod p) such that, if $\sum \epsilon_i a_i$ ($\epsilon_i = 0$ or 1, not all $\epsilon_i = 0$) is a multiple of p, then $\sum \epsilon_i$ is uniquely determined. Then there are at most two distinct residues among the a_i . A proof of this conjecture, for sufficiently large p, is presented. It is remarked by the authors that the proof is surprisingly complicated. The lack of clarity in the exposition in no way helps the reader to overcome this difficulty. A preliminary theorem is proved, which 'easily implies Graham's conjecture in case each residue occurs with a multiplicity $< \eta_0 p'$. What this means is that the theorem implies that under the assumptions of Graham's conjecture, some residue must occur at least $\eta_0 p$ times. This is the first step in the proof of the conjecture. The remaining steps are difficult, using theorems of Dirichlet, Cauchy-Davenport and Erdős-Heilbronn. Finally, the reader is left wondering if Graham is R. L. Graham, and even if the first author is P. or E. Erdős.

I.Anderson

Classification: 11B13 Additive bases

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