Zbl 399.10001
Erdős, Paul
Some unconventional problems in number theory. (In English)
Asterisque 61, 73-82 (1979).

Many interesting problems and clusters of problems, solved and unsolved, are listed here. For example, the author has proved [Bull. Am. Math. Soc. 54, 685-692 (1948; Zbl 032.01301)] that the set of integers having two divisors d_1 and d_2 satisfying $d_1 < d_2 < 2d_1$ does have a density, but it is still an open question as to whether that density is 1. It is also not yet known whether or not almost all integers n have two divisors satisfying $d_1 < d_2 < d_1[1 + (\varepsilon/3)^{1-\eta \log\log n}]$, in spite of a previous claim that this had been proved. As another example, if $p_1^{(n)} < \cdots < p_{v(n)}^{(n)}$ are the consecutive prime factors of n, then for almost all n the v-th prime factor of n satisfies $\log\log p_v^{(n)} = (1 + o(1))v$ or, more precisely, for every $\varepsilon > 0$, $\eta > 0$ there is a $c = c(\varepsilon, \eta)$ such that the density is greater than $1 - \eta$ for the set of integers n for which every $c < v \leq v(n)$, $v(1-\varepsilon) < \log\log p_v^{(n)} < (1+\varepsilon)v$. This is the only result for which a proof is provided in this paper. A final example: Let $P(n)$ be the greatest prime factor of n. Is it true that the density of the set of integers n satisfying $P(n+1) > P(n)$ is 1/2? Is it true that the density of the set of integers n for which $P(n+1) > P(n)n^\alpha$ exists for every α? The author warns that this problem is probably very difficult.

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Classification:
11-02 Research monographs (number theory)
11N05 Distribution of primes
11B83 Special sequences of integers and polynomials
00A07 Problem books

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greatest prime factor; divisor problems; consecutive prime factors; density