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**Zbl 422.10035**
**Erdős, Paul; Nicolas, Jean-Louis**
*Sur la fonction "nombre de facteurs premiers de n".*
*On the function "number of prime factors of n"* (In French)
**Sémin. Delange-Pisot-Poitou, 20e Année 1978/79, Théorie des nombres, Fasc. 2, Exp. 32, 19 p. (1980).**

[For the entire collection see Zbl 418.00004.]

The authors obtain significant results concerning $\omega(n)$ and $\Omega(n)$, which represent the number of distinct prime factors of $n$ and the number of total prime factor of $n$ respectively. Defining $\omega$-largely composite numbers as those $n$ for which $m \le n$ implies $\omega(m) \le \omega(n)$, they first prove that for some constants $0 < c_1 < c_2$

$$\exp(c_1 \log^{1/2} x) \le Q_\ell(x) \le \exp(c_2 \log^{1/2} x)$$

where $Q_\ell(x)$ is the number of $\omega$-largely composite numbers not exceeding $x$. The Brum-Titchmarsh inequality and *A.Selberg*'s result on primes in short intervals [Arch. Math. Naturvid. B 47, No. 6, 1-19 (1943; Zbl 028.34802)] are needed in the proof, and reasons for conjecturing that

$$\log Q_\ell(x) = (1 + o(1))\pi(2/3)^{1/2} \log^{1/2} x, \quad x \to \infty$$

are stated. Next an asymptotic formula for $f_c(x)$, the number of $n \le x$ for which $\omega(n)c \log x/\log\log x$, is derived, where $0 < c < 1$ is given. The result is $f_c(x) = x^{1-c+o(1)}$, the proof being elementary and elegant. The restriction $c < 1$ is natural, since $\omega(n) \le (1 + o(1)) \log n/\log\log n$ as $n \to \infty$. Properties of $\omega$-interesting numbers (defined as $n > 1$ which satisfy $\omega(m)/m < \omega(n)/n$ for $m > n$) are extensively discussed, and maximal and minimal order of $f(n) + f(n + 1)$ is investigated when $f(n)$ is $\sigma(n)$, $\varphi(n)$ and $\Omega(n)$. In the last case it is proved that, as $n \to \infty$,

$$\Omega(n) + \Omega(n + 1) \le (1 + o(1)) \log n/\log 2,$$

which is best possible, since $\Omega(n) \le \log n/\log 2$ is already attained when $n$ is a power of two. The corresponding problem for $\omega(n)$, i.e. determining

$$\limsup_{n\to\infty} (\omega(n) + \omega(n + 1)) \frac{\log\log n}{\log n},$$

remains yet to be solved.

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11N37 Asymptotic results on arithmetic functions
11N05 Distribution of primes

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