Zbl 448.10040

Erdős, Paul; Sarközy, A.

On the number of prime factors of integers. (In English) Acta Sci. Math. 42, 237-246 (1980). [0001-6969]

Let $pi_i(x)$ be the number of integers $n \leq x$ such that $\Omega(n) = i$, where $\Omega(n)$ denotes the number of prime factors of n counted with multiplicity. Let δ be a constant satisfying $0 < \delta < 2$. Then the authors prove the following two results. First $2^i i^{-4} \pi_i(x) = 0(x \log x)$ uniformly for all $i \geq 1$. Next $(i-1)!(\log \log x)^{1-i} = 0\left(\frac{x}{\log x}\right)$ uniformly for all i satisfying $1 \leq i \leq (2-\delta) \log \log x$. They deduce some corollaries to these results. We may quote: for every $\varepsilon > 0$

$$\sum_{1 \le i \le z \log \log k} \pi_i(k) = 0(k(\log k)^{-\varphi(z) + \varepsilon})$$

and

$$\sum_{1 \le i \le z \log \log k} \pi_i(k^2) = 0(k^2(\log k)^{-\varphi(z)+\varepsilon}).$$

Here the 0-constant depends only on ε and z. $\varphi(x) = 1 * x \log x - x$, is defined for all x > 0 and z is defined as the unique real root of $\varphi(x + 1) = \varphi(x)$. It may be noted that z = 0.54...

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Classification:

11N37 Asymptotic results on arithmetic functions