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On the number of pairs of partitions of n without common subsums. (In English)

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Let $\Pi = \{n_1+n_2+\cdots+n_t = n;\ n_1 \geq n_2 \geq \cdots \geq n_t > 0\}$ be a generic partition of n where $t = t(\Pi)$ and the n_i's are integers. Each sum $n_{i_1} + \cdots + n_{i_j}$ $(i_1 < \cdots < i_j)$ is said to be a subsum of Π. Let $p(n)$ denote the number of partitions of n.

Let $R(n,a)$ be the number of partitions of n such that no subsum is equal to a. The asymptotic behaviour of $R(n,a)$ has been investigated by several authors, cf. *J. Dixmier* [Mém. Soc. Math. Fr., Nouv. Sér. 35, 1-70 (1988; Zbl 684.10047); Port. Math. 46, 137-154 (1989; Zbl 684.10048); Bull. Sci. Math., II. Sér. 113, 125-149, 505 (1989; Zbl 684.10049); Bull. Soc. Math. Belg., Ser. A 42, 477-500 (1990; Zbl 734.11051)]; *P. Erdős*, *J.-L. Nicolas* and *A. Sárközy* [Discrete Math. 75, 155-166 (1989; Zbl 673.05007); in "Analytic number theory", Prog. Math. 85, 205-234 (1990; Zbl 727.11038)].

In the paper under review the authors solve a problem of J. Dénes from 1967 by obtaining an asymptotic expansion for the number $G(n)$ of pairs of partitions of n which do not have nontrivial equal subsums:

$$G(n) = 2p(n)(1 + \alpha_1 n^{-1/2} + \alpha_2 n^{-1} + \cdots + \alpha_k n^{-k/2} + O(n^{-(k+1)/2})).$$

As to the $q(n)$ unequal partitions of n, the authors prove the following asymptotic estimate for the number $H(n)$ of pairs of unequal partitions of n without nontrivial equal subsums:

$$H(n) = cq(n)(1 + O(n^{-1/2}\log^2 n)), \qquad \text{where} \quad 13.83 \leq c \leq 14.29.$$

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11P82 Analytic theory of partitions
05A17 Partitions of integres (combinatorics)

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pairs of partitions without equal subsums; number of partitions; asymptotic expansion