ON A GENERAL CENTRE OF APPLIED FORCES

By

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Sir William Rowan Hamilton

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Sir W. R. Hamilton wishes a note to be preserved in the Proceedings of the Royal Irish Academy, that on recently reconsidering an application of Quaternions to the Statics of a Solid Body, some account of which was laid before the Academy many years ago (see the Proceedings for December 1845), he has been led to perceive the *theoretical* (and to suspect the *practical*) existence of a certain *Central Point* for *every system of applied forces*, not reducible to a *couple*, nor to *zero*: which *generally new point*, for the case of *parallel forces*, coincides with their well-known *centre*.

An *applied force* AB, acting at a point A, being said to have a *quaternion moment*, equal to the quaternion *product* $OA\,.\,AB$, with respect to any assumed point O, the *sum* of all *such* moments, or the quaternion, $\mathrm{Q} = \Sigma(OA\,.\,AB) = OA\,.\,AB + OA'\,.\,A'B' + \&\mathrm{c.}$, is called the *total quaternion moment* of the applied system with respect to the same point O.

This *total moment* Q *varies* generally with the *point* to which it is referred; and there is *one* point C, or *one position* of O, for which the condition

$$\mathrm{TQ} = a\ \textit{minimum},$$

is satisfied, with the exceptions (of *couple* and *equilibrium*) above alluded to.

It is *this point* C, which Sir W. R. H. proposes to call *generally* the *Centre of a System of Applied Forces*.

In the most general case of such a system, he finds it to be situated *on the Central Axis*, the *minimum* TQ representing then what was called by Poinsot the *Energy of the Central Couple*.

For the less general case of an *unique resultant force*, the quaternion Q reduces itself to *zero* at the new Central Point C, which is now situated *on the resultant*, and determines its *line of application*.