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A NOTE ON DIFFERENTIAL SUPERORDINATIONS USING SĂLĂGEAN AND RUSCHEWEYH OPERATORS

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Abstract. In the present paper we define a new operator using the Sălăgean and Ruscheweyh operators. Denote by SR^m the Hadamard product of the Sălăgean operator S^m and the Ruscheweyh operator R^m, given by $SR^m : \mathcal{A}_n \to \mathcal{A}_n$, $SR^m f(z) = (S^m * R^m) f(z)$ and $\mathcal{A}_n = \{f \in \mathcal{H}(U),\ f(z) = z + a_{n+1}z^{n+1} + \dots,\ z \in U\}$ is the class of normalized analytic functions. We study some differential superordinations regarding the operator SR^m.

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1. Introduction and definitions

Denote by U the unit disc of the complex plane $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{H}(U)$ the space of holomorphic functions in U.

Let

$$\mathcal{A}_n = \{f \in \mathcal{H}(U),\ f(z) = z + a_{n+1}z^{n+1} + \dots,\ z \in U\}$$

and

$$\mathcal{H}[a, n] = \{f \in \mathcal{H}(U),\ f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \dots,\ z \in U\}$$

for $a \in \mathbb{C}$ and $n \in \mathbb{N}$.

If f and g are analytic functions in U, we say that f is superordinate to g, written $g \prec f$, if there is a function w analytic in U, with $w(0) = 0$, $|w(z)| < 1$, for all $z \in U$ such that $g(z) = f(w(z))$ for all $z \in U$. If f is univalent, then $g \prec f$ if and only if $f(0) = g(0)$ and $g(U) \subseteq f(U)$.

Let $\psi : \mathbb{C}^2 \times U \to \mathbb{C}$ and h analytic in U. If p and $\psi(p(z), zp'(z); z)$ are univalent in U and satisfies the (first-order) differential superordination

$$h(z) \prec \psi(p(z), zp'(z); z), \qquad \text{for } z \in U, \tag{1}$$

then p is called a solution of the differential superordination. The analytic function q is called a subordinant of the solutions of the differential superordination, or more simply a subordinant, if $q \prec p$ for all p satisfying (1).

An univalent subordinant $\widetilde{q}$ that satisfies $q \prec \widetilde{q}$ for all subordinants q of (1) is said to be the best subordinant of (1). The best subordinant is unique up to a rotation of U.

Definition 1. *(Sălăgean [4]) For $f \in \mathcal{A}_n$, $n \in \mathbb{N}$, $m \in \mathbb{N} \cup \{0\}$, the operator S^m is defined by $S^m : \mathcal{A}_n \to \mathcal{A}_n$,*

$$\begin{aligned} S^0 f(z) &= f(z) \\ S^1 f(z) &= z f'(z) \\ &\dots \\ S^{m+1} f(z) &= z \left(S^m f(z)\right)', \quad z \in U. \end{aligned}$$

Remark 1. If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $S^m f(z) = z + \sum_{j=n+1}^{\infty} j^m a_j z^j$, $z \in U$.

Definition 2. *(Ruscheweyh [3]) For $f \in \mathcal{A}_n$, $n \in \mathbb{N}$, $m \in \mathbb{N} \cup \{0\}$, the operator R^m is defined by $R^m : \mathcal{A}_n \to \mathcal{A}_n$,*

$$\begin{aligned} R^0 f(z) &= f(z) \\ R^1 f(z) &= z f'(z) \\ &\dots \\ (m+1) R^{m+1} f(z) &= z \left(R^m f(z)\right)' + m R^m f(z), \quad z \in U. \end{aligned}$$

Remark 2. If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $R^m f(z) = z + \sum_{j=n+1}^{\infty} C_{m+j-1}^{m} a_j z^j$, $z \in U$.

Definition 3. [2] *We denote by Q the set of functions that are analytic and injective on $\overline{U} \backslash E(f)$, where $E(f) = \{\zeta \in \partial U : \lim_{z \to \zeta} f(z) = \infty\}$, and $f'(\zeta) \neq 0$ for $\zeta \in \partial U \backslash E(f)$. The subclass of Q for which $f(0) = a$ is denoted by $Q(a)$.*

We will use the following lemmas.

Lemma 1. *(Miller and Mocanu [2]) Let h be a convex function with $h(0) = a$, and let $\gamma \in \mathbb{C} \backslash \{0\}$ be a complex number with $Re\ \gamma \geq 0$. If $p \in \mathcal{H}[a, n] \cap Q$, $p(z) + \frac{1}{\gamma} z p'(z)$ is univalent in U and*

$$h(z) \prec p(z) + \frac{1}{\gamma} z p'(z), \quad for \;\; z \in U,$$

then

$$q(z) \prec p(z), \quad \textit{for} \;\; z \in U,$$

where $q(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t)t^{\gamma/n-1}dt$, *for* $z \in U$. *The function* q *is convex and is the best subordinant.*

Lemma 2. *(Miller and Mocanu [2]) Let* q *be a convex function in* U *and let* $h(z) = q(z) + \frac{1}{\gamma} zq'(z)$, *for* $z \in U$, *where* $Re\ \gamma \geq 0$. *If* $p \in \mathcal{H}[a,n] \cap Q$, $p(z) + \frac{1}{\gamma}zp'(z)$ *is univalent in* U *and*

$$q(z) + \frac{1}{\gamma}zq'(z) \prec p(z) + \frac{1}{\gamma}zp'(z), \quad \textit{for} \;\; z \in U,$$

then

$$q(z) \prec p(z), \quad \textit{for} \;\; z \in U,$$

where $q(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t)t^{\gamma/n-1}dt$, *for* $z \in U$. *The function* q *is the best subordinant.*

2. Main results

Definition 4. [1] *Let* $m \in \mathbb{N}$. *Denote by* SR^m *the operator given by the Hadamard product (the convolution product) of the Sălăgean operator* S^m *and the Ruscheweyh operator* R^m, $SR^m : \mathcal{A}_n \to \mathcal{A}_n$,

$$SR^m f(z) = (S^m * R^m) f(z).$$

Remark 3. If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $SR^m f(z) = z + \sum_{j=n+1}^{\infty} C_{m+j-1}^m j^m a_j^2 z^j$.

Theorem 1. *Let* h *be a convex function,* $h(0) = 1$. *Let* $m \in \mathbb{N} \cup \{0\}$, $f \in \mathcal{A}_n$ *and suppose that* $\frac{1}{z}SR^{m+1}f(z) + \frac{m}{m+1}z(SR^m f(z))''$ *is univalent and* $(SR^m f(z))' \in \mathcal{H}[1,n] \cap Q$. *If*

$$h(z) \prec \frac{1}{z}SR^{m+1}f(z) + \frac{m}{m+1}z(SR^m f(z))'', \quad \textit{for} \;\; z \in U, \tag{2}$$

then

$$q(z) \prec (SR^m f(z))', \quad \textit{for} \;\; z \in U,$$

where $q(z) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t)t^{\frac{1}{n}-1}dt$. *The function* q *is convex and it is the best subordinant.*

Proof. With notation $p(z) = (SR^m f(z))' = 1 + \sum_{j=n+1}^{\infty} C_{m+j-1}^m j^{m+1} a_j^2 z^{j-1}$

and $p\left(0\right)=1$, we obtain for $f(z)=z+\sum_{j=n+1}^{\infty}a_jz^j$,
$p\left(z\right)+zp'\left(z\right)=\frac{1}{z}SR^{m+1}f\left(z\right)+z\frac{m}{m+1}\left(SR^mf\left(z\right)\right)''$. Evidently $p\in\mathcal{H}[1,n]$.
Then (2) becomes

$$h(z)\prec p(z)+zp'(z),\quad \text{for}\ \ z\in U.$$

By using Lemma 1, we have

$$q(z)\prec p(z),\quad \text{for}\ \ z\in U,\quad \text{i.e.}\quad q(z)\prec\left(SR^mf\left(z\right)\right)',\quad \text{for}\ \ z\in U,$$

where $q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. The function q is convex and it is the best subordinant.

Corollary No. 1 *Let $h(z)=\frac{1+(2\beta-1)z}{1+z}$ be a convex function in U, where $0\leq\beta<1$. Let $m\in\mathbb{N}\cup\{0\}$, $f\in\mathcal{A}_n$ and suppose that $\frac{1}{z}SR^{m+1}f\left(z\right)+\frac{m}{m+1}z\left(SR^mf\left(z\right)\right)''$ is univalent and $\left(SR^mf\left(z\right)\right)'\in\mathcal{H}\left[1,n\right]\cap Q$. If*

$$h(z)\prec\frac{1}{z}SR^{m+1}f\left(z\right)+\frac{m}{m+1}z\left(SR^mf\left(z\right)\right)'',\quad \textit{for}\ \ z\in U, \tag{3}$$

then

$$q(z)\prec\left(SR^mf\left(z\right)\right)',\quad \textit{for}\ \ z\in U,$$

where q is given by $q(z)=2\beta-1+\frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z\frac{t^{\frac{1}{n}-1}}{1+t}dt$, for $z\in U$. The function q is convex and it is the best subordinant.

Proof. Following the same steps as in the proof of Theorem 1 and considering $p(z)=\left(SR^mf\left(z\right)\right)'$, the differential superordination (3) becomes

$$h(z)=\frac{1+(2\beta-1)z}{1+z}\prec p(z)+zp'(z),\quad \text{for}\ \ z\in U.$$

By using Lemma 1 for $\gamma=1$, we have $q(z)\prec p(z)$, i.e.,

$$q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h\left(t\right)t^{\frac{1}{n}-1}dt=\frac{1}{nz^{\frac{1}{n}}}\int_0^z t^{\frac{1}{n}-1}\frac{1+\left(2\beta-1\right)t}{1+t}dt$$

$$=2\beta-1+\frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z\frac{t^{\frac{1}{n}-1}}{1+t}dt\prec\left(SR^mf\left(z\right)\right)',\quad \text{for}\ \ z\in U.$$

The function q is convex and it is the best subordinant.

Theorem No. 2 *Let q be convex in U and let h be defined by $h(z) = q(z) + zq'(z)$. If $m \in \mathbb{N} \cup \{0\}$, $f \in \mathcal{A}_n$, suppose that $\frac{1}{z}SR^{m+1}f(z) + \frac{m}{m+1}z(SR^m f(z))''$ is univalent, $(SR^m f(z))' \in \mathcal{H}[1,n] \cap Q$ and satisfies the differential superordination*

$$h(z) = q(z) + zq'(z) \prec \frac{1}{z}SR^{m+1}f(z) + \frac{m}{m+1}z(SR^m f(z))'', \quad \text{for } z \in U, \tag{4}$$

then

$$q(z) \prec (SR^m f(z))', \quad \text{for } z \in U,$$

where $q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. The function q is the best subordinant.

Proof. Let $p(z) = (SR^m f(z))' = 1 + \sum_{j=n+1}^{\infty} C_{m+j-1}^m j^{m+1} a_j^2 z^{j-1}$.

Differentiating, we obtain $p(z) + zp'(z) = \frac{1}{z}SR^{m+1}f(z) + z\frac{m}{m+1}(SR^m f(z))''$, for $z \in U$ and (4) becomes

$$q(z) + zq'(z) \prec p(z) + zp'(z), \quad \text{for } z \in U.$$

Using Lemma 2, we have

$$q(z) \prec p(z), \quad \text{for } z \in U, \text{ i.e. } q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt \prec (SR^m f(z))', \quad \text{for } z \in U,$$

and q is the best subordinant.

Theorem 3. *Let h be a convex function, $h(0) = 1$. Let $m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and suppose that $(SR^m f(z))'$ is univalent and $\frac{SR^m f(z)}{z} \in \mathcal{H}[1,n] \cap Q$. If*

$$h(z) \prec (SR^m f(z))', \quad \text{for } z \in U, \tag{5}$$

then

$$q(z) \prec \frac{SR^m f(z)}{z}, \quad \text{for } z \in U,$$

where $q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. The function q is convex and it is the best subordinant.

Proof. Consider $p(z) = \frac{SR^m f(z)}{z} = \frac{z + \sum_{j=n+1}^{\infty} C_{m+j-1}^m j^m a_j^2 z^j}{z} =$

$1 + \sum_{j=n+1}^{\infty} C_{m+j-1}^m j^m a_j^2 z^{j-1}$. Evidently $p \in \mathcal{H}[1,n]$.
Differentiating, we obtain $p(z) + zp'(z) = (SR^m f(z))'$.
Then (5) becomes

$$h(z) \prec p(z) + zp'(z), \quad \text{for } z \in U.$$

By using Lemma 1, we have

$$q(z) \prec p(z), \quad \text{for} \quad z \in U, \quad \text{i.e.} \quad q(z) \prec \frac{SR^m f(z)}{z}, \quad \text{for} \quad z \in U,$$

where $q(z) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t) t^{\frac{1}{n}-1} dt$. The function q is convex and it is the best subordinant.

Corollary 2. *Let $h(z) = \frac{1+(2\beta-1)z}{1+z}$ be a convex function in U, where $0 \leq \beta < 1$. Let $m \in \mathbb{N} \cup \{0\}$, $f \in \mathcal{A}_n$ and suppose that $(SR^m f(z))'$ is univalent and $\frac{SR^m f(z)}{z} \in \mathcal{H}[1,n] \cap Q$. If*

$$h(z) \prec (SR^m f(z))', \quad \text{for} \quad z \in U, \tag{6}$$

then

$$q(z) \prec \frac{SR^m f(z)}{z}, \quad \text{for} \quad z \in U,$$

where q is given by $q(z) = 2\beta - 1 + \frac{2(1-\beta)}{nz^{\frac{1}{n}}} \int_0^z \frac{t^{\frac{1}{n}-1}}{1+t} dt$, for $z \in U$. The function q is convex and it is the best subordinant.

Proof. Following the same steps as in the proof of Theorem 3 and considering $p(z) = \frac{SR^m f(z)}{z}$, the differential superordination (6) becomes

$$h(z) = \frac{1 + (2\beta - 1)z}{1 + z} \prec p(z) + zp'(z), \quad \text{for} \quad z \in U.$$

By using Lemma 1 for $\gamma = 1$, we have $q(z) \prec p(z)$, i.e.,

$$q(z) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t) t^{\frac{1}{n}-1} dt = \frac{1}{nz^{\frac{1}{n}}} \int_0^z t^{\frac{1}{n}-1} \frac{1 + (2\beta - 1)t}{1 + t} dt$$

$$= 2\beta - 1 + \frac{2(1-\beta)}{nz^{\frac{1}{n}}} \int_0^z \frac{t^{\frac{1}{n}-1}}{1+t} dt \prec \frac{SR^m f(z)}{z}, \quad \text{for} \quad z \in U.$$

The function q is convex and it is the best subordinant.

Theorem 4. *Let q be convex in U and let h be defined by $h(z) = q(z) + zq'(z)$. If $m \in \mathbb{N} \cup \{0\}$, $f \in \mathcal{A}_n$, suppose that $(SR^m f(z))'$ is univalent, $\frac{SR^m f(z)}{z} \in \mathcal{H}[1,n] \cap Q$ and satisfies the differential superordination*

$$h(z) = q(z) + zq'(z) \prec (SR^m f(z))', \quad \text{for} \quad z \in U, \tag{7}$$

then

$$q(z) \prec \frac{SR^m f(z)}{z}, \quad \text{for} \quad z \in U,$$

where $q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. *The function q is the best subordinant.*

Proof. Let $p\left(z\right)=\frac{SR^m f(z)}{z}=\frac{z+\sum_{j=n+1}^{\infty}C_{m+j-1}^{m}j^m a_j^2 z^j}{z}=$

$1+\sum_{j=n+1}^{\infty}C_{m+j-1}^{m}j^m a_j^2 z^{j-1}$. Evidently $p\in\mathcal{H}[1,n]$.

Differentiating, we obtain $p(z)+zp'(z)=\left(SR^m f\left(z\right)\right)'$, for $z\in U$ and (7) becomes

$$q(z)+zq'(z)\prec p(z)+zp'\left(z\right),\quad \text{for}\quad z\in U.$$

Using Lemma 2, we have

$$q(z)\prec p(z),\quad \text{for}\quad z\in U,\quad \text{i.e.}\quad q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt\prec\frac{SR^m f\left(z\right)}{z},\quad \text{for}\quad z\in U,$$

and q is the best subordinant.

Theorem 5. *Let h be a convex function,* $h(0)=1$. *Let* $m\in\mathbb{N}\cup\{0\}$, $f\in\mathcal{A}_n$ *and suppose that* $\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)'$ *is univalent and* $\frac{SR^{m+1}f(z)}{SR^m f(z)}\in\mathcal{H}\left[1,n\right]\cap Q$. *If*

$$h(z)\prec\left(\frac{zSR^{m+1}f\left(z\right)}{SR^m f\left(z\right)}\right)',\qquad \textit{for}\quad z\in U,\tag{8}$$

then

$$q(z)\prec\frac{SR^{m+1}f\left(z\right)}{SR^m f\left(z\right)},\qquad \textit{for}\quad z\in U,$$

where $q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. *The function q is convex and it is the best subordinant.*

Proof. Consider $p\left(z\right)=\frac{SR^{m+1}f(z)}{SR^m f(z)}=\frac{z+\sum_{j=n+1}^{\infty}C_{m+j}^{m+1}j^{m+1}a_j^2 z^j}{z+\sum_{j=n+1}^{\infty}C_{m+j-1}^{m}j^m a_j^2 z^j}=$

$\frac{1+\sum_{j=n+1}^{\infty}C_{m+j}^{m+1}j^{m+1}a_j^2 z^{j-1}}{1+\sum_{j=n+1}^{\infty}C_{m+j-1}^{m}j^m a_j^2 z^{j-1}}$. Evidently $p\in\mathcal{H}[1,n]$.

We have $p'\left(z\right)=\frac{\left(SR^{m+1}f(z)\right)'}{SR^m f(z)}-p\left(z\right)\cdot\frac{(SR^m f(z))'}{SR^m f(z)}$ and $p\left(z\right)+zp'\left(z\right)=\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)'$.

Then (8) becomes

$$h(z)\prec p(z)+zp'(z),\quad \text{for}\quad z\in U.$$

By using Lemma 1, we have

$$q(z)\prec p(z),\quad \text{for}\quad z\in U,\quad \text{i.e.}\quad q(z)\prec\frac{SR^{m+1}f\left(z\right)}{SR^m f\left(z\right)},\quad \text{for}\quad z\in U,$$

where $q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. The function q is convex and it is the best subordinant.

Corollary 3. *Let $h(z)=\frac{1+(2\beta-1)z}{1+z}$ be a convex function in U, where $0\le\beta<1$. Let $m\in\mathbb{N}\cup\{0\}$, $f\in\mathcal{A}_n$ and suppose that $\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)'$ is univalent, $\frac{SR^{m+1}f(z)}{SR^m f(z)}\in\mathcal{H}[1,n]\cap Q$. If*

$$h(z)\prec\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)',\quad \textit{for}\;\; z\in U, \tag{9}$$

then

$$q(z)\prec\frac{SR^{m+1}f(z)}{SR^m f(z)},\quad \textit{for}\;\; z\in U,$$

where q is given by $q(z)=2\beta-1+\frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z\frac{t^{\frac{1}{n}-1}}{1+t}dt$, for $z\in U$. The function q is convex and it is the best subordinant.

Proof. Following the same steps as in the proof of Theorem 5 and considering $p(z)=\frac{SR^m f(z)}{z}$, the differential superordination (9) becomes

$$h(z)=\frac{1+(2\beta-1)z}{1+z}\prec p(z)+zp'(z),\quad \text{for}\;\; z\in U.$$

By using Lemma 1 for $\gamma=1$, we have $q(z)\prec p(z)$, i.e.,

$$q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)\,t^{\frac{1}{n}-1}dt=\frac{1}{nz^{\frac{1}{n}}}\int_0^z t^{\frac{1}{n}-1}\frac{1+(2\beta-1)\,t}{1+t}dt$$

$$=2\beta-1+\frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z\frac{t^{\frac{1}{n}-1}}{1+t}dt\prec\frac{SR^{m+1}f(z)}{SR^m f(z)},\quad \text{for}\;\; z\in U.$$

The function q is convex and it is the best subordinant.

Theorem 6. *Let q be convex in U and let h be defined by $h(z)=q(z)+zq'(z)$. If $m\in\mathbb{N}\cup\{0\}$, $f\in\mathcal{A}_n$, suppose that $\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)'$ is univalent, $\frac{SR^{m+1}f(z)}{SR^m f(z)}\in\mathcal{H}[1,n]\cap Q$ and satisfies the differential superordination*

$$h(z)=q(z)+zq'(z)\prec\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)',\quad \textit{for}\;\; z\in U, \tag{10}$$

then

$$q(z)\prec\frac{SR^{m+1}f(z)}{SR^m f(z)},\quad \textit{for}\;\; z\in U,$$

where $q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. *The function* q *is the best subordinant.*

Proof. Let $p(z)=p(z)=\frac{SR^{m+1}f(z)}{SR^m f(z)}=\frac{z+\sum_{j=n+1}^{\infty}C_{m+j}^{m+1}j^{m+1}a_j^2z^j}{z+\sum_{j=n+1}^{\infty}C_{m+j-1}^{m}j^m a_j^2 z^j}=$

$\frac{1+\sum_{j=n+1}^{\infty}C_{m+j}^{m+1}j^{m+1}a_j^2z^{j-1}}{1+\sum_{j=n+1}^{\infty}C_{m+j-1}^{m}j^m a_j^2 z^{j-1}}$. Evidently $p\in\mathcal{H}[1,n]$.

Differentiating, we obtain $p(z)+zp'(z)=\left(\frac{zSR^{m+1}f(z)}{SR^m f(z)}\right)'$, for $z\in U$ and (10) becomes

$$q(z)+zq'(z)\prec p(z)+zp'(z), \quad \text{for} \quad z\in U.$$

Using Lemma 2, we have

$$q(z)\prec p(z), \quad \text{for} \quad z\in U, \text{ i.e. } q(z)=\frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt\prec\frac{SR^{m+1}f(z)}{SR^m f(z)}, \quad \text{for} \quad z\in U,$$

and q is the best subordinant.

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