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ON THE INFLUENCE OF VARIABLE VISCOSITY ON LAMINAR MAGNETOHYDRODYNAMIC THERMAL OSCILLATORY FLOW PAST A LIMITING SURFACE WITH VARIABLE SUCTION

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Abstract. This papers deals with the influence of variable viscosity on laminar magneto hydrodynamic thermal oscillatory flow past a limiting surface with variable suction. It considers two dimensional hydro magnetic flow of a viscous incompressible and electrically conducting fluid, past a limiting surface in the presence of a transverse magnetic field. The induced magnetic $H = [Hx^{'}, Hy^{'}, 0]$ and $x^{'} - axis$ is chosen along the surface in the direction of the flow and $y^{'} - axis$ is taken normal to the limiting surface.

Approximate solutions are obtained for the expression for velocity, induced magnetic and temperature when the magnetic Prandtl number $P_m = 1$, Prandtl number $P = 7$ and the magnetic parameter $M < 1$.

We observed that an increase in variable parameter leads to an increase in temperature and velocity; where as, an increase in the variable parameter leads to decrease in the induced magnetic field. These observations are presented in tables.

Key Words: Variable viscousity, laminar magneto hydrodynamic, thermal oscillatory flow, limiting surface, variable suction and magnetic field.

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1. Introduction

Technologically, oscillatory flow is always very important for its pratical applications, in the aerodynamics of a helicopter rotor, in a fluttering air foil and in many projects of bio-engineering.

Many research works has been done in area of oscillatory flow, among these are: lighthill (1954) discussed the effect of unsteady fluctuations of the free stream velocity on the flow of an incompressible fluid by assuming time-dependent. Stuart (1955) studied unsteady velocity field with unsteady temperature by assuming that there is no heat transfer between the plate and the fluid. The problem of oscillatory flow for variable suction was studied by Messiah (1966) and assumed the infinite limiting surface to be constant.

Soundalgekar (1976) improved Messiha's studies and discussed the effects of the oscillatory free stream on the flow past an impulsively constant velocity and assumed there is no heat transfer between the fluid and the plate. Georgantopolous (1979) carried out an improved work on Soundalgekar study by studying the velocity field in hydromagnetic oscillatory flow past an impulsively started porous limiting surface with variable suction.

It was observed that thermal convection and magnetic field has great influence on starts and plants. Pande (1977) studied hydromagnetic thermal boundary layer flow near an oscillatory limiting surface. Pande's studies were extended by Georgantopolous (1978) and studied the effects of the hydromagnetic free convection on the oscillatory flow of a viscous incompressible fluid past a porous limiting surface. It was also observed in the recent years that thickness of the hydromagnetic boundary layers are significantly affected by the porosity of the limiting surface and the presence of the applied magnetic field. Kafousias (1980) studied laminar hydromagnetic thermal boundary layer in fluctuating flow past a limiting surface with variable suction and assumed that there is no heat transfer between the plate and the fluid. The studies shows that the magnetic field is greatly influenced by the velocity.

Effects of variable fluid parameters on the flow are now attracting the attention of researchers. Gbadeyan and Dada (1998) studied the effect of variable fluid properties on radiative magneto hydrodynamic flow of a fluid in a vertical channel. Usman (1998) considered the effect of variable parameters on magnetolhydrodynamic two phase flows under optically thin limit radiative. Holzbecher (1998) examined the influence of variable viscosity on thermal convection in porous media. S.Chackrabory (2002) studied effect of variable viscosity on laminar convection flow of an eletrical conducting fluid in uniform magnetic field. Hassanien et al (2003) studied variable viscosity and thermal conductivity effect on heat transfer by natural convection form a cone and a wedge in porous media. Pantokratoras,A, (2005) presented new results

on forced and mixed convection boundary layer flow along a flat plate with variable viscosity and variable Prandtl number.
None of the research works carried on variable parameters on fluid flow do not consider the effect of these parameters on velocity, induced magnetic and temperature. Hence, this paper investigates the influence of variable viscosity on laminar magneto hydrodynamic thermal oscillatory flow past a limiting surface with variable suction.

2. Problem formulation

Many research studied carried out on oscillattory flow as discussed in the introductory part of this paper do not consider the effect of variable viscosity on the velocity, Induced magnetic and temperature.
Hence, the purpose of this paper is to investigate the influence of variable viscosity on the velocity on the velocity, induced magnetic and temperature of a two dimensional laminar magneto hydrodynamic thermal oscillatory flow past a limiting surface with variable suction.
Thus, we introduce $\nu = \nu_0(\lambda\theta + 1)$ in to energy and magnetic induced equations respectively, where λ is a variable parameter, θ is non dimensional temperature, hence, the relevant governing equations to the problem are as follows

$$\frac{\partial v^{'}}{\partial y^{'}} = 0 \tag{1}$$

$$\frac{\partial Hx^{'}}{\partial t^{'}} + V^{'}\frac{\partial Hx^{'}}{\partial y^{'}} = \frac{1}{\sigma\mu_0}\frac{\partial^2 Hx^{'}}{\partial y^{'2}} + Hy^{'}\frac{\partial u^{'}}{\partial y^{'}} \tag{2}$$

$$\frac{\partial u^{'}}{\partial t^{'}} + V^{'}\frac{\partial u^{'}}{\partial y^{'}} = \frac{1}{\rho}\frac{\partial p^{'}}{\partial x^{'}} + \nu_0(\lambda\theta+1)\frac{\partial^2 u^{'}}{\partial y^{'2}} + \frac{\mu_0}{\rho}Hy^{'}\frac{\partial Hx^{'}}{\partial y^{'}} \tag{3}$$

and

$$C_p(\frac{\partial T^{'}}{\partial t^{'}} + V^{'}\frac{\partial T^{'}}{\partial y^{'}}) = \frac{k}{\rho}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \nu_0(\lambda\theta+1)(\frac{\partial u^{'}}{\partial y^{'}})^2 + \frac{1}{\sigma\rho}(\frac{\partial Hx^{'}}{\partial y^{'}})^2 \tag{4}$$

The corresponding boundary conditions are:

$$\begin{aligned} & y^{'} = 0: \quad u^{'} = U_1^{'}, \quad Hx^{'} = 0 \\ & y^{'} \to \infty: \quad U^{'} \to U_1^{'}(t^{'}),\ Hx^{'} = 0 \\ & y^{'} = 0: \quad \frac{\partial T^{'}}{\partial y^{'}} = 0 \\ & y^{'} \to \infty: \quad T^{'} \to T_\infty^{'} \end{aligned} \tag{5}$$

Following Mesiha (1966) equation for variable velocity could be written as

$$V^{'} = V_0^{'}[1 + \epsilon A\ell^{iwt}] \tag{6}$$

Let $U^{'}(t^{'})$ be the free-stream velocity and by the boundary conditions (5), we have that

$$-\frac{1}{\rho}\frac{\partial p^{'}}{\partial x^{'}}=\frac{\partial U^{'}}{\partial t^{'}} \tag{7}$$

We now define the define the following non-dimensional parameters

$$y=\frac{y^{'}V_0^{'}}{\nu_0},\quad u=\frac{u^{'}}{U_0^{'}},\quad t=\frac{V_0^{'2}t^{'}}{4\nu_0},\quad U=\frac{U^{'}}{U_0^{'}}$$

$$\omega=\frac{4\nu_0\omega}{V_0^{'2}},\quad V=\frac{U_0^{'}}{U_0},\quad H=(\frac{\mu_0}{\rho})^{1/2}\frac{Hx^{'}}{U_0^{'}},$$

$$P_m=\sigma\mu_0\nu_0,\quad M=(\frac{\mu_0}{\rho})^{1/2}\frac{H_0}{V_0^{'}},\quad E=\frac{U_0^{'2}}{C_pT_\infty^{'}} \tag{8}$$

$$Pr=\frac{\nu_0}{K},\quad K=\frac{k}{\rho Cp},\quad \theta=\frac{T^{'}-T_\infty^{'}}{T_\infty^{'}}$$

We now non-dimensionless equations (1), (2), (3) and (4) using (6), (7), (8) and that $H_y=H_0=const$ (H_0 is the externally applied transverse magnetic field). From (8), we have

$$Hx^{'}=U_0^{'}(\frac{\rho}{\mu_0})^{1/2}H,\quad y^{'}=\frac{\nu_0 y}{V_0^{'}},\quad u^{'}=U_0^{'}u$$

$$t^{'}=\frac{4\nu_0 t}{V_0^{'2}},\quad \omega^{'}=\frac{V_0^{'2}\omega}{4\nu_0},\quad U^{'}=U_0^{'}U.$$

hence, we have

$$\frac{1}{4}\frac{\partial H}{\partial t}-[1+\epsilon A\ell^{iwt}]\frac{\partial H}{\partial y}=\frac{1}{P_m}\frac{\partial^2 H}{\partial y^2}+M\frac{\partial u}{\partial y} \tag{9}$$

$$\frac{1}{4}\frac{\partial u}{\partial t}-[1+\epsilon A\ell^{iwt}]\frac{\partial u}{\partial y}=\frac{1}{4}\frac{\partial U}{\partial t}+\lambda\theta\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial y^2}+M\frac{\partial H}{\partial y} \tag{10}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-[1+\epsilon A\ell^{iwt}]\frac{\partial \theta}{\partial y}=\frac{1}{P}\frac{\partial^2 \theta}{\partial y^2}+\lambda\theta E(\frac{\partial u}{\partial y})^2+E(\frac{\partial u}{\partial y})^2+E(\frac{\partial H}{\partial y})^2 \tag{11}$$

Using the given expressions for velocity, induced magnetic, temperature and free-stream velocity as defined below:

$$u(y,t)=q_1(y)+\epsilon\ell^{iwt}q_2(y) \tag{12}$$

$$H(y,t)=r_1(y)+\epsilon\ell^{iwt}r_2(y) \tag{13}$$

$$\theta(y,t) = \theta_0(y) + \epsilon\ell^{iwt}\theta_1(y) + \epsilon^2\ell^{2iwt}\theta_2(y) \tag{14}$$
$$U = 1 + \epsilon\ell^{iwt} \tag{15}$$

We reduce equation (9), (10) and (11) as follows;
From equation (9), substituting (12) and simplify gives

$$\frac{i\omega}{4}\epsilon\ell^{iwt}r_2 - r_1' - \epsilon\ell^{iwt}r_2'\epsilon A\ell^{iwt}r_1' - \epsilon^2 A\ell^{2iwt}r_1' = r_1'' + \epsilon\ell^{iwt}r_2'' + Mq_1' + M\epsilon\ell^{iwt}q_2' \tag{16}$$

Equating the coefficient of powers of ϵ, neglecting ϵ^2, we have,
For order zero, $\epsilon(0)$

$$r_1'' + r_1' = -Mq_1' \tag{17}$$

For order one, $\epsilon(1)$

$$r_2'' + r_2' - \frac{i\omega}{4}r_2 = -Ar_1' - Mq_2' \tag{18}$$

Substituting (12), (13), and (15), into (10) we have,

$$\frac{i\omega}{4}\epsilon\ell^{iwt}q_2 - q_1' + \epsilon A\ell^{iwt}q_2' + \epsilon A\ell^{iwt}q_1' - \epsilon^2 A\ell^{2iwt}q_2' = \frac{i\omega}{4}\epsilon\ell^{iwt} + \lambda\theta_0 q_1'' + \lambda\theta_0\epsilon\ell^{iwt}q_2''$$
$$+\lambda\theta_1\epsilon\ell^{iwt}q_1'' + \lambda\theta_1\epsilon^2\ell^{2iwt}q_2'' + \lambda\theta_2\epsilon^2\ell^{2iwt}q_1'' + \lambda\theta_2\epsilon^3\ell^{3iwt}q_2'' + q_1'' + \epsilon\ell^{iwt}q_2'' + Mr_1' + Mr_2'\epsilon\ell^{iwt} \tag{19}$$

Equating the coefficient of powers of ϵ and neglecting that of ϵ^2, we have,
For order zero, $\epsilon(0)$

$$\lambda\theta_0 q_1'' + q_1'' + q_1' = -r_1' \tag{20}$$

For order one, $\epsilon(1)$

$$\lambda\theta_0 q_2'' + q_2'' + q_2' - \frac{i\omega}{4}q_2 = -\frac{i\omega}{4} - \lambda\theta_1 q_1'' - Aq_1' - Mr_2' \tag{21}$$

Substitute (12), (13) and (14), into (11), we have

$$\frac{i\omega}{4}\epsilon\ell^{iwt}\theta_1 + \frac{i\omega}{2}\epsilon^2\ell^{2iwt}\theta_2 - \theta_0' - \epsilon\ell^{iwt}\theta_1' - \epsilon^2\ell^{2iwt}\theta_2' - \epsilon A\ell^{iwt}\theta_0' - \epsilon^2 A\ell^{2iwt}\theta_1' - \epsilon^3 A\ell^{3iwt}\theta_2'$$
$$= \frac{\theta_0''}{P} + \epsilon\ell^{iwt}\frac{\theta_1''}{P} + \epsilon^2\ell^{2iwt}\frac{\theta_2''}{P} + \lambda E\theta_0 q_1'^2 + 2\lambda E\theta_0\epsilon\ell^{iwt}q_1'q_2' + \lambda E\theta_0\epsilon^2\ell^{2iwt}q_2'^2$$
$$+\lambda E\theta_1\epsilon\ell^{iwt}q_1'^2 + 2\lambda E\theta_1\epsilon^2\ell^{2iwt}q_1'q_2' + \lambda E\theta_1\epsilon^3\ell^{3iwt}q_2'^2 + \lambda E\theta_2\epsilon^2\ell^{2iwt}q_1'^2$$
$$+2\lambda E\theta_2\epsilon^3\ell^{3iwt}q_1'q_2' + \lambda E\theta_2\epsilon^4\ell^{4iwt}q_2'^2 + Eq_1'^2 + 2E\epsilon\ell^{iwt}q_1'q_2' + E\epsilon^2\ell^{2iwt}q_2'^2 + Er_1'^2$$

$$+2E\epsilon\ell^{iwt}r_1^{'}r_2^{'}+E\epsilon^2\ell^{2iwt}r_2^{'2} \tag{22}$$

Equating the coefficient of powers of ϵ and ϵ^2, neglecting ϵ^3, we have
For order zero $\epsilon(0)$

$$\frac{1}{P}\theta_0^{''}+\theta_0^{'}+\lambda E\theta_0 q_1^{'2}=-E(q_1^{'2}+r_1^{'2}) \tag{23}$$

For order one $\epsilon(1)$

$$\frac{1}{P}\theta_1^{''}+\theta_1^{'}(\lambda Eq_1^{'2}-\frac{i\omega}{4})\theta_1=-A\theta^{'}-2\lambda E\theta_0 q_1^{'}q_2^{'}-2E(q_1^{'}q_2^{'}+r_1^{'}r_2^{'} \tag{24}$$

For order two $\epsilon(2)$

$$\frac{1}{P}\theta_2^{''}+\theta_2^{'}(\lambda E_2 q_1^{'2}-\frac{i\omega}{2})\theta_2=-A\theta_1^{'}-2\lambda E\theta_1 q_1^{'}q_2^{'}+\lambda E\theta_0 q_2^{'}-E(q_2^{'2}+r_2^{'2}) \tag{25}$$

The equations for the problem therefore, are (17), (18), (20), (23)-(25)
where,
q_1 is mean velocity
q_2 is oscillatory part of the velocity
r_1 is mean induced magnetic field
r_2 is oscillatory part of the induced magnetic field
λ is variable parameter
θ_0 is mean temperature
E is Ecket number, P is Prandtl number

3. Numerical Solution

In this section, we provide solution to the set of formulated equations to the problem. The equations are solved using Runge-Kutta method for solving non-linear ordinary differential equations.
Runge-Kutta method is a method of numerically integrating ordinary differential equations by using a trial step at the midpoint of an interval to cancel out lower-order error term. The second-order formula is sometimes known as RK2, and the fourth-order formula is sometime known as RK4. The method is reasonably simple and is a good for numerical solution of differential equations.

$$r_1=x_1,\quad r_2=x_2,\quad q_1=x_3,\quad q_2=x_4 \tag{26}$$
$$\theta_0=x_5,\quad \theta_1=x_6,\quad \theta_2=x_7$$

Then,

$$r_1^{'}=x_8,\quad r_2^{'}=x_9,\quad q_1^{'}=x_{10},\quad q_2^{'}=x_{11} \tag{27}$$
$$\theta_0^{'}=x_{12},\quad \theta_1^{'}=x_{13},\quad \theta_2^{'}=x_{14}$$

From equation (3.23), we have

$r_1^{''} = -(r_1^{'} + Mq_1^{'})$ (28)
From equation (3.2.4), we have
$r_2^{''} = -(Ar_1^{'} + Mq_2^{'} + r_2^{'} - \frac{i\omega}{4} r_2)$ (29)
From equation (3.2.8), we have
$q_1^{''} = -(q_1^{'} - Mq_2^{'})/(1 + \lambda\theta_0)$ (30)
From equation (3.2.9), we have
$q_1^{''} = -(\frac{i\omega}{4} + \lambda\theta_1 q_1^{''} + Aq_1^{'} + Mr_2^{'} + q_2^{'} - \frac{i\omega}{4} q_2)/(1 + \lambda\theta_0)$ (31)
From equation (23), we have
$\theta_0^{''} = -p[E(q_1^{'2} + r_1^{'2}) + \lambda E q_1^{'2}\theta_0 + \theta_1^{'}]$ (32)
From equation (24), we have
$\theta_1^{''} = -p[A\theta_0^{'} + 2\lambda E q_1^{'} q_2^{'} \theta_0 + 2E(q_1^{'} q_2^{'} + r_1^{'} r_2^{'}) + (\lambda E q_1^{'2} - \frac{i\omega}{4})\theta_1 + \theta_1^{'}]$ (33)
From equation (25), we have
$\theta_1^{''} = -p[A\theta_1^{'} + 2\lambda E q_1^{'} q_2^{'} \theta_1 + 2\lambda E r_1^{'2}\theta_0 + E(q_1^{'2} + r_1^{'2}) + (\lambda E q_1^{'2} - \frac{i\omega}{4})\theta_2 + \theta_2^{'}]$ (34)
Thus, in terms of x_1, equations (17), (18), (20), (21), (23)-(25), respectively, becomes
$r_1^{''} = -(x_8 + Mx_{10})$ (35)
$r_2^{''} = -(Ax_8 + Mx_{11} + x_9 - \frac{i\omega}{4} x_2)$ (36)
$q_1^{''} = -(x_{10} + Mx_8)/(1 + \lambda x_5)$ (37)
$q_2^{''} = \frac{-(\frac{i\omega}{4} + \lambda x_6[-(x_{10} + Mx_8)/(1+\lambda x_5)] + Ax_{10} + Mx_9 + x_{11} - \frac{i\omega}{4} x_4)}{(1+\lambda x_5)}$ (38)
$\theta_0^{''} = -p[E(x_{10}^2 + x_9^2) + \lambda E x_{10}^2 x_5 + x_{12}$ (39)
$\theta_1^{''} = -p[Ax_{12} + 2\lambda E x_{10} x_{11} x_5 + 2E(x_{10} x_{11} + x_8 x_9) + (\lambda E x_{10}^2 - \frac{i\omega}{4})x_6 + x_{13}]$ (40)
$\theta_2^{''} = -p[Ax_{13} + 2\lambda E x_{10} x_{11} x_6 + 2\lambda E x_8^2 x_5 + 2E(x_{10}^2 + x_8^2) + (\lambda E x_{10}^2 - \frac{i\omega}{2})x_7 + x_{14}]$ (41)
We now solve these, using the boundary conditions
$y = 0: \quad r_1 = v, \quad r_2 = 0, \quad q_1 = 0, \quad q_2 = 0$
$y \to \infty: \quad r_1 \to 1, \quad r_2 \to 1, \quad q_1 \to 0, \quad q_2 \to 0$
and (42)
$y = 0: \quad \theta_\tau^{'} = 0;$
$y \to 0: \quad \theta_\tau \to 0;$
$where \quad \tau = 0, 1, 2;$
Now
$U = x_1[i] + \epsilon \cos \omega t x_2[i]$ (43)
$H = x_3[i] + \epsilon \cos \omega t x_4(i)$ (44)
$\theta = x_5[i] + \epsilon \cos \omega t x_6[i] + \epsilon^2 \cos 2\omega t x_7[i]$ (45)
Applying the RK2, using the Computer (Pascal programme language) to solve equations (35)-(41), using (42)-(45), the following results were obtained for temperature, velocity and induced magnetic field.

4. Numerical Results

The tables at the appendix shows the results for various values of temperature, velocity and induced magnetic fields.

5. Discussion of results

For the purpose of discussing the results, some numerical calculations are carried out for different values of the magnetic parameter M, the limiting surface velocity V, the free-stream oscillatory frequency ω, the variable parameter λ, the suction parameter A and the Eckert number E.
In all these cases, the Prandtl number P is taken as constant= 7.
Tables 1, 2, and 3 shows that the values of Fluid temperature (θ) for different values of λ, E, M, V, A and ω. From these values, we observe that:

(a) Fluid temperature (θ) increases as the variable parameter λ increases (see all cases of tables 4.1, 4.2 and 4.3).

(b) Fluid temperature is greater when the limiting surface velocity moves in direction opposite to the flow i.e ($V < 0$) than when the limiting surface velocity is stationary i.e ($V = 0$) or moves in the direction of the flow i.e ($V > 0$) (see cases 1,2,3,4 and 5 of tables 1,2 and 3).

(c) Fluid temperature increases as the frequency of oscillatory increases when the limiting surface velocity moves in opposite direction to the flow i.e ($V = 0$) (see cases 1 and 2 of tables 1,2 and 3).

(d) Fluid temperature is not significantly affected by increase in suction parameter, A(see cases 3 and 7 of tables 1,2 and 3).

(e) Fluid temperature is not significantly affected by increase in magnetic parameter, M (see cases 3 and 6 of tables 1,2 and 3).
Tables 4,5 and 6 shows the values of limiting surface temperature $\theta(0)$ for different values of dimensionless parameters. From these tables, we conclude that:

(a) The surface temperature increases as variable parameter λ increases, but, higher values of variable parameter do not significantly affect the limiting surface temperature. (see all cases of tables 4,5 and 6)

(b) The limiting surface temperature is greater when the limiting surface velocity moves in the directon opposite to the flow i.e($V < 0$) than when the limiting surface velocity is stationary i.e ($V = 0$), or moves in the direction of the flow i.e ($V > 0$) (see cases 1,2,3 and 4 of tables 4,5 and 6).

(c) The limiting surface temperature decreases as the Eckert number e, dereases (see cases 2 and 5 of tables 4,5 and 6).
Tables 7,8 and 9 shows the values mean steady temperature (θ_0) for different values λ, E, M, V, A and ω, from these tables, we see that:

(a) Mean steady temperature increases as the variable parameter λ, increases (see all cases of tables 7, 8 and 9).

(b) Mean steady temperature is greater when the limiting surface velocity moves in the direction opposite to the flow i.e ($V < 0$) than when the limiting surface velocity is stationary i.e ($V = 0$), or moves in the direction of the flow i.e ($V > 0$) (see cases 1, 2, 3 and 4 of tables 7, 8 and 9)

(c) Mean steady temperature decreases as the magnetic parameter M increases (see cases 2 and 5 of tables 7, 8 and 9).
Tables 10, 11 and 12 shows the values of transient velocity u for different values of λ, M, V, A and ω. From these tables we conclude that:

(a) The transient velocity u, moves in the reserved type of flow when the limiting surface velocity moves in the reserved type of flow i.e ($V < 0$). (see cases 4, 5, 6, 7 and 8 of tables 10, 11 and 12)

(b) The transient velocity u, is greater when the limiting surface velocity moves in the direction of the flow i.e ($V > 0$), than when the limiting surface velocity is stationary i.e ($V = 0$) (see cases 1, 2, and 3 of tables 10, 11 and 12).

(c) The transient velocity u, is not significantly affected by increase in oscillatory frequency ω. (see cases 1, and 2, 4 and 5 of tables 10,11 and 12).

(d) The transient velocity u, is not significantly affected by increase in suction parameter A. (see cases 7 and 8 of tables 10, 11 and 12).

(e) The transient velocity u, increases when the variable parameter λ increases and the limiting surface velocity moves in direction opposite to the flow i.e ($V < 0$) or when the limiting surface velocity is stationary i.e $V = 0$. But, the transient velocity is not significantly affected by increase in variable parameter when limiting surface velocity moves in the direction of the flow i.e ($V > 0$).
Tables 13, 14 and 15 shows the values of induced magnetic field for different values of λ, M, V, A and ω. From these values, we conclude that:

(a) The induced magnetic field, decreases when the variable parameter λ increases, but, higher values of the variable parameter do not have any signifant effect on the induced magnetic field (see tables 13, 14 and 15).

(b) The induced magnetic field is greater when the limiting surface velocity moves in the direction opposite to the flow i.e ($V < 0$) than when the limiting surface velocity is stationary i.e ($V = 0$) or moves in the direction of the flow i.e ($V = 0$) (see cases 1, 2, 3, 4, 5 and 6, 7, 8 of tables 13, 14 and 15).

(c) The induced magnetic increases with a turning point when the frequency oscillatory ω increases.

(d) The induced magnetic decreases when the magnetic parameter M decreases (see cases 1 and 5 of tables 13, 14 and 15).

(e) The induced magnetic increases with a turning when the suction parameter A increases (see cases 2 and 3 of tables 13, 14 and 15).

6. Conclusion

The paper deals with the influence of variable viscosity on laminar magneto hydrodynamic thermal oscillatory flow past a limiting surface with variable suction. The results show that:

(a) The fluid temperature increases as variable parameter λ increases.

(b) The limiting surface temperature increases as variable parameter λ, increases, but, further, increase in the variable parameter does not have significant effect on limiting surface temperature.

(c) The mean steady temperature increases with increase in variable parameter λ.

(d) The velocity u increases when the variable parameter λ, increases and the limiting surface velocity moves in the direction opposite to the flow.

(e) The induced magnetic field decreases when the variable parameter λ, increase, but further increase in variable parameter do not have significant effect on induced magnetic field.

Thus, the variable parameter has greater influence on the velocity, induced magnetic and temperature, when the limiting surface velocity moves in opposite direction of the flow.

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Appendix

Fluid temperature θ for different values of λ, E, M, V, A, ω

	E	M	V	A	ω
1.	0.01	0.4	-1	0.3	5
2.	0.01	0.4	-1	0.3	20
3.	0.01	0.4	-0.5	0.3	20
4.	0.01	0.4	0.0	0.3	20
5.	0.01	0.4	0.5	0.3	20
6.	0.01	0.8	-0.5	0.3	20
7.	0.01	0.4	-0.5	0.6	20

TABLE 1 Results of Fluid temperature θ at $\lambda = 0$

θ

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
0	0.0171	0.0181	0.0096	0.0043	0.0011	0.0096	0.0096
0.1	0.016	0.0169	0.009	0.004	0.001	0.009	0.009
0.2	0.0139	0.0147	0.0078	0.0035	0.0009	0.0078	0.0078
0.3	0.0116	0.0125	0.0065	0.0029	0.0007	0.0065	0.0065
0.4	0.0096	0.0105	0.0054	0.0024	0.0006	0.0054	0.0054
0.5	0.0079	0.0088	0.0044	0.002	0.0005	0.0044	0.0044
0.6	0.0065	0.0063	0.0036	0.0016	0.0004	0.0036	0.0036
0.7	0.0053	0.0055	0.003	0.0013	0.0003	0.003	0.003
0.8	0.0044	0.0049	0.0025	0.0011	0.0003	0.0025	0.0025
0.9	0.0037	0.0043	0.0021	0.0009	0.0002	0.0021	0.0021
1.0	0.0031	0.0039	0.0017	0.0008	0.0002	0.0017	0.0017
1.1	0.0026	0.0034	0.0014	0.0006	0.0002	0.0014	0.0014
1.2	0.0022	0.0032	0.0012	0.0005	0.0001	0.0012	0.0012
1.3	0.0018	0.0029	0.001	0.0005	0.0001	0.001	0.001
1.4	0.0016	0.0026	0.0009	0.0004	0.0001	0.0009	0.0009
1.5	0.0014	0.0024	0.0008	0.0003	0.0001	0.0008	0.0008
1.6	0.0012	0.0022	0.0007	0.0003	0.0001	0.0007	0.0007
1.7	0.0010	0.0018	0.0006	0.0003	0.0001	0.0006	0.0006
1.8	0.0009	0.0014	0.0005	0.0002	0.0001	0.0005	0.0005
1.9	0.0008	0.0012	0.0004	0.0002	0	0.0004	0.0004
2.0	0.0007	0.0012	0.0004	0.0002	0	0.0004	0.0004

TABLE 2 Results of Fluid temperature θ at $\lambda = 0.5$

θ

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
0	0.1994	0.2003	0.1127	0.0501	0.0125	0.1127	0.1127
0.1	0.1977	0.1985	0.1116	0.0496	0.0124	0.1116	0.1116
0.2	0.1954	0.1962	0.1103	0.049	0.0122	0.1103	0.1103
0.3	0.1925	0.1934	0.1086	0.0482	0.0121	0.1086	0.1086
0.4	0.1890	0.1899	0.1066	0.0473	0.0118	0.1066	0.1066
0.5	0.1848	0.1857	0.1042	0.0462	0.0115	0.1042	0.1042
0.6	0.1799	0.1809	0.1013	0.0449	0.0112	0.1013	0.1013
0.7	0.1642	0.1752	0.098	0.0434	0.0108	0.098	0.098
0.8	0.1675	0.1688	0.0942	0.0417	0.0104	0.0942	0.0942
0.9	0.1599	0.1611	0.0899	0.0397	0.0099	0.0899	0.0899
1.0	0.1513	0.1525	0.0849	0.0375	0.0093	0.0849	0.0849
1.1	0.1415	0.1428	0.0793	0.0349	0.0087	0.0793	0.0793
1.2	0.1304	0.1318	0.073	0.0321	0.008	0.073	0.073
1.3	0.1179	0.1195	0.0658	0.0288	0.0071	0.0658	0.0658
1.4	0.1038	0.1057	0.0578	0.0252	0.0062	0.0578	0.0578
1.5	0.0881	0.0902	0.0489	0.0212	0.0052	0.0489	0.0489
1.6	0.0705	0.0729	0.0389	0.0167	0.0041	0.0389	0.0389
1.7	0.0509	0.0536	0.0279	0.0117	0.0028	0.0279	0.0279
1.8	0.0291	0.0322	0.0155	0.0061	0.0014	0.0155	0.0155
1.9	0.0049	0.0084	0.0018	−0.0001	−0.0001	0.0018	0.0018
2.0	−0.0221	−0.018	−0.0133	−0.0069	−0.0019	−0.0133	−0.0133

TABLE 3 Results of Fluid temperature θ at $\lambda = 1$

θ

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
0	0.1995	0.2004	0.1127	0.0501	0.0125	0.1127	0.1127
0.1	0.1978	0.1987	0.1117	0.0496	0.0124	0.1117	0.1117
0.2	0.1957	0.1966	0.1104	0.049	0.0122	0.1104	0.1104
0.3	0.1931	0.194	0.1088	0.0483	0.0121	0.1088	0.1088
0.4	0.1899	0.1908	0.1069	0.0474	0.0118	0.1069	0.1069
0.5	0.1861	0.187	0.1046	0.0463	0.0115	0.1046	0.1046
0.6	0.1815	0.1825	0.1018	0.045	0.0112	0.1018	0.1018
0.7	0.1763	0.1772	0.0987	0.0435	0.0108	0.0987	0.0987
0.8	0.1701	0.1712	0.095	0.0418	0.0104	0.095	0.095
0.9	0.1631	0.1642	0.0908	0.0399	0.0099	0.0908	0.0908
1.0	0.1551	0.1562	0.086	0.0377	0.0093	0.086	0.086
1.1	0.1459	0.1472	0.0806	0.0352	0.0087	0.0806	0.0806
1.2	0.1356	0.137	0.0745	0.0323	0.008	0.0745	0.0745
1.3	0.1239	0.1255	0.0676	0.0292	0.0072	0.0676	0.0676
1.4	0.1108	0.1125	0.0599	0.0256	0.0063	0.0599	0.0599
1.5	0.0961	0.098	0.0513	0.0216	0.0052	0.0513	0.0513
1.6	0.0796	0.0817	0.0416	0.0172	0.0041	0.0416	0.0416
1.7	0.0611	0.0636	0.0308	0.0122	0.0029	0.0308	0.0308
1.8	0.0406	0.0434	0.0189	0.0067	0.0015	0.0189	0.0189
1.9	0.0177	0.0209	0.0056	0.0006	0.0001	0.0056	0.0056
2.0	−0.078	−0.0042	−0.0092	−0.0061	−0.0018	−0.0092	−0.0092

Limiting surface temperature $\theta(0)$ for different values of λ, E, M, V, A

	E	M	V	A
1.	0.03	0.4	-1.0	0.3
2.	0.03	0.4	-0.5	0.3
3.	0.03	0.4	0.0	0.3
4.	0.03	0.4	0.5	0.3
5.	0.03	0.8	-0.5	0.3
6.	0.01	0.4	-0.5	0.3
7.	0.03	0.4	-0.5	0.6

TABLE 4 Results of Limiting surface temperature $\theta(0)$ at $\lambda = 0$
$\theta(0)$
ω

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
1	0.055209	0.0308	0.0131	0.003215	0.0286	0.009645	0.027
2	0.054974	0.029475	0.001286	0.003275	0.027731	0.009825	0.025352
5	0.054729	0.030352	0.01349	0.003372	0.028688	0.010117	0.02706
10	0.053959	0.030785	0.013682	0.003421	0.02931	0.010262	0.027691
15	0.0524	0.030923	0.013743	0.003436	0.029552	0.010308	0.027908
20	0.051442	0.031055	0.013802	0.003451	0.029738	0.010352	0.02816

TABLE 5 Results of Limiting surface temperature $\theta(0)$ at $\lambda = 0.5$

$\theta(0)$

ω

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
1	0.6039	0.3397	0.151	0.0377	0.3436	0.1132	0.3391
2	0.5911	0.3325	0.1478	0.0369	0.3234	0.1108	0.323
5	0.5983	0.3365	0.1496	0.0374	0.3347	0.1122	0.3299
10	0.5991	0.3333	0.1481	0.037	0.3258	0.1111	0.3318
15	0.5999	0.3329	0.148	0.037	0.3248	0.111	0.3294
20	0.6008	0.338	0.1502	0.0376	0.3387	0.1127	0.3339

TABLE 6 Results of Limiting surface temperature $\theta(0)$ at $\lambda = 1$

$\theta(0)$

ω

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
1	0.6039	0.3397	0.151	0.0377	0.3436	0.1132	0.3391
2	0.5911	0.3325	0.1478	0.0369	0.3234	0.1108	0.323
5	0.5983	0.3365	0.1496	0.0374	0.3347	0.1122	0.3299
10	0.5991	0.3333	0.1481	0.037	0.3258	0.1111	0.3318
15	0.5999	0.3329	0.148	0.037	0.3248	0.111	0.3294
20	0.6008	0.338	0.1502	0.0376	0.3387	0.1127	0.3339

Mean steady temperature θ_0 for different values of λ, E, M, V, A, ω

	E	M	V	A	ω
1.	0.01	0.4	-1.0	0.3	20
2.	0.01	0.4	-0.5	0.3	20
3.	0.01	0.4	0.0	0.3	20
4.	0.01	0.4	0.5	0.3	20
5.	0.01	0.8	-0.5	0.3	20

TABLE 7 Results of Mean steady temperature θ_0 at $\lambda = 0$

θ_0
Y

	Case 1	Case 2	Case 3	Case 4	Case 5
0	0.1728	0.0972	0.0432	0.0108	0.0885
0.1	0.1617	0.091	0.0404	0.0101	0.0807
0.2	0.1401	0.0788	0.035	0.0088	0.0665
0.3	0.1173	0.066	0.0293	0.0073	0.0525
0.4	0.0969	0.0545	0.0242	0.0061	0.0409
0.5	0.0796	0.0447	0.0199	0.005	0.0318
0.6	0.0653	0.0368	0.0163	0.0041	0.025
0.7	0.0538	0.0303	0.0135	0.0034	0.02
0.8	0.0445	0.025	0.0111	0.0028	0.0163
0.9	0.037	0.0208	0.0093	0.0023	0.0136
1.0	0.031	0.0174	0.0077	0.0019	0.0117
1.1	0.026	0.0146	0.0065	0.0016	0.0102
1.2	0.022	0.0124	0.0055	0.0014	0.0091
1.3	0.0187	0.0105	0.0047	0.0012	0.0082
1.4	0.016	0.009	0.004	0.001	0.0076
1.5	0.0137	0.0077	0.0034	0.0009	0.007
1.6	0.0118	0.0066	0.0029	0.0007	0.0066
1.7	0.0102	0.0057	0.0025	0.0006	0.0062
1.8	0.088	0.005	0.0022	0.0006	0.0059
1.9	0.0077	0.0043	0.0019	0.0005	0.0056
2.0	0.0067	0.0038	0.0017	0.0004	0.0053

TABLE 8 Results of Mean steady temperature θ_0 at $\lambda = 0.5$

θ_0

Y

	Case 1	Case 2	Case 3	Case 4	Case 5
0	0.2	0.2	0.1125	0.05	0.0125
0.1	0.1982	0.1982	0.1115	0.0495	0.0124
0.2	0.1959	0.1959	0.1101	0.0489	0.0122
0.3	0.193	0.193	0.1084	0.0481	0.012
0.4	0.1895	0.1895	0.1064	0.0472	0.0118
0.5	0.1853	0.1853	0.1039	0.0461	0.0115
0.6	0.1803	0.1803	0.101	0.0448	0.0112
0.7	0.1744	0.1744	0.0976	0.0432	0.0108
0.8	0.1677	0.1677	0.0937	0.0415	0.0103
0.9	0.16	0.16	0.0892	0.0394	0.0098
1.0	0.1512	0.1512	0.0841	0.0371	0.0093
1.1	0.1411	0.1411	0.0783	0.0345	0.0086
1.2	0.1298	0.1298	0.0718	0.0315	0.0078
1.3	0.117	0.117	0.0644	0.0282	0.007
1.4	0.1026	0.1026	0.056	0.0244	0.0061
1.5	0.0864	0.0864	0.0467	0.0202	0.005
1.6	0.0683	0.0683	0.0363	0.0155	0.0038
1.7	0.0481	0.0481	0.0246	0.0102	0.0025
1.8	0.0256	0.0256	0.0117	0.0044	0.001
1.9	0.0005	0.0005	−0.0028	−0.0022	−0.0006
2.0	−0.0275	−0.0275	−0.0188	−0.0094	−0.0025

TABLE 9 Results of Mean steady temperature θ_0 at $\lambda = 1$

θ_0

Y

	Case 1	Case 2	Case 3	Case 4	Case 5
0	0.2	0.2	0.1125	0.05	0.0125
0.1	0.1984	0.1984	0.1115	0.0495	0.0124
0.2	0.1963	0.1963	0.1102	0.0489	0.0122
0.3	0.1936	0.1936	0.1086	0.0482	0.012
0.4	0.1904	0.1904	0.1066	0.0473	0.0118
0.5	0.1865	0.1865	0.1043	0.0462	0.0115
0.6	0.1819	0.1819	0.1015	0.0449	0.0112
0.7	0.1765	0.1765	0.0983	0.0434	0.0108
0.8	0.1703	0.1703	0.0945	0.0416	0.0103
0.9	0.1631	0.1631	0.0902	0.0396	0.0098
1.0	0.1549	0.1549	0.0853	0.0373	0.0098
1.1	0.1456	0.1456	0.0797	0.0347	0.0086
1.2	0.135	0.135	0.0734	0.0318	0.0078
1.3	0.1231	0.1231	0.0662	0.0285	0.007
1.4	0.1096	0.1096	0.0582	0.0248	0.0061
1.5	0.0945	0.0945	0.0491	0.0206	0.005
1.6	0.0775	0.0775	0.0391	0.016	0.0038
1.7	0.0585	0.0585	0.0278	0.0108	0.0025
1.8	0.0373	0.0373	0.0152	0.005	0.001
1.9	0.0137	0.0137	0.0011	−0.0014	−0.0006
2.0	−0.0127	−0.0127	−0.0145	−0.0086	−0.0025

Velocity u for different values of λ, A, M, V, ω

	A	M	V	ω
1.	0.3	0.4	0.5	5
2.	0.3	0.4	0.5	20
3.	0.3	0.4	0	20
4.	0.3	0.4	-0.5	5
5.	0.3	0.4	-0.5	20
6.	0.3	0.8	-0.5	20
7.	0.3	0.4	-1	20
8.	0.6	0.4	-1	20

TABLE 10 Results of Velocity u at $\lambda = 0$
U
Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
0	0.5	0.5	0	−0.5	−0.5	−0.5	−1	−1
0.1	0.5047	0.5047	0.0094	−0.4858	−0.4858	−0.4861	−0.981	−0.981
0.2	0.5094	0.5094	0.0188	−0.4718	−0.4718	−0.4725	−0.9621	−0.9621
0.3	0.514	0.514	0.028	−0.4579	−0.4579	−0.459	−0.9434	−0.9435
0.4	0.5186	0.5186	0.0372	−0.4441	−0.4442	−0.4458	−0.9249	−0.925
0.5	0.5231	0.5231	0.0462	−0.4305	−0.4306	−0.4327	−0.9066	−0.9067
0.6	0.5275	0.5275	0.0551	−0.4171	−0.4171	−0.4199	−0.8885	−0.8886
0.7	0.532	0.532	0.064	−0.4038	−0.4038	−0.4072	−0.8705	−0.8707
0.8	0.5363	0.5363	0.0727	−0.3906	−0.3907	−0.3948	−0.8527	−0.853
0.9	0.5406	0.5406	0.0813	−0.4776	−0.3777	−0.3825	−0.8351	−0.8354
1.0	0.5449	0.5449	0.0899	−0.3647	−0.3648	−0.3705	−0.8177	−0.8181
1.1	0.5491	0.5491	0.0983	−0.352	−0.3521	−0.3586	−0.8005	−0.8009
1.2	0.5533	0.5533	0.1067	−0.3394	−0.3395	−0.3469	−0.7834	−0.7839
1.3	0.5574	0.5574	0.115	−0.3269	−0.327	−0.3354	−0.7666	−0.7671
1.4	0.5615	0.5615	0.1231	−0.3146	−0.3147	−0.3241	−0.7499	−0.7505
1.5	0.5655	0.5655	0.1312	−0.3024	−0.3026	−0.313	−0.7334	−0.7341
1.6	0.5695	0.5695	0.1392	−0.2904	−0.2906	−0.3021	−0.7171	−0.7179
1.7	0.5735	0.5735	0.1471	−0.2785	−0.2787	−0.2914	−0.701	−0.7018
1.8	0.5774	0.5774	0.1549	−0.2667	−0.2669	−0.2808	−0.6851	−0.686
1.9	0.5812	0.5812	0.1626	−0.2551	−0.2553	−0.2705	−0.6695	−0.6704
2.0	0.585	0.585	0.1702	−0.2436	−0.2439	−0.2604	−0.654	−0.6551

TABLE 11 Results of Velocity u **at** $\lambda = 0.5$

U

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
0	0.5	0.5	0	−0.5	−0.5	−0.5	−1	−1
0.1	0.5047	0.5047	0.0095	−0.4858	−0.4858	−0.4861	−0.981	−0.981
0.2	0.5094	0.5094	0.0188	−0.4717	−0.4717	−0.4724	−0.9621	−0.9621
0.3	0.514	0.514	0.0281	−0.4577	−0.4578	−0.4588	−0.9434	−0.9435
0.4	0.5186	0.5186	0.0372	−0.4439	−0.444	−0.4455	−0.9249	−0.925
0.5	0.5231	0.5231	0.0463	−0.4303	−0.4303	−0.4324	−0.9066	−0.9067
0.6	0.5275	0.5275	0.0552	−0.4168	−0.4168	−0.4194	−0.8885	−0.8886
0.7	0.532	0.532	0.0641	−0.4034	−0.4034	−0.4067	−0.8705	−0.8707
0.8	0.5363	0.5363	0.0729	−0.3901	−0.3902	−0.3942	−0.8527	−0.853
0.9	0.5407	0.5406	0.0815	−0.377	−0.3771	−0.3818	−0.8351	−0.854
1.0	0.5449	0.5449	0.0901	−0.3641	−0.3642	−0.3696	−0.8177	−0.8181
1.1	0.5491	0.5491	0.0986	−0.3513	−0.3544	−0.3577	−0.8005	−0.8009
1.2	0.5533	0.5533	0.1069	−0.3386	−0.3387	−0.3459	−0.7834	−0.7839
1.3	0.5575	0.5574	0.1152	−0.3261	−0.3262	−0.3343	−0.7666	−0.7671
1.4	0.5615	0.5615	0.1234	−0.3137	−0.3138	−0.3229	−0.7499	−0.7505
1.5	0.5656	0.5655	0.1315	−0.3014	−0.3016	−0.3118	−0.7334	−0.7341
1.6	0.5696	0.5695	0.1395	−0.2893	−0.2895	−0.3008	−0.7171	−0.7179
1.7	0.5735	0.5735	0.1475	−0.2773	−0.2775	−0.29	−0.701	−0.7018
1.8	0.5774	0.5773	0.1553	−0.2655	−0.2657	−0.2794	−0.6851	−0.686
1.9	0.5813	0.5812	0.163	−0.2538	−0.254	−0.2691	−0.6695	−0.6704
2.0	0.5851	0.585	0.1706	−0.2423	−0.2425	−0.2589	−0.654	−0.6551

TABLE 12 Results of Velocity u at $\lambda = 1$

U

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
0	0.5	0.5	0.5	−0.5	−0.5	−0.5	−1	−1
0.1	0.5047	0.5047	0.0095	−0.4857	−0.4857	−0.486	−0.9808	−0.9808
0.2	0.5094	0.5094	0.0189	−0.4715	−0.4715	−0.4722	−0.9618	−0.9618
0.3	0.514	0.514	0.0281	−0.4575	−0.4575	−0.4585	−0.943	−0.943
0.4	0.5186	0.5186	0.0373	−0.4436	−0.4436	−0.445	−0.9243	−0.9243
0.5	0.5231	0.5231	0.0464	−0.4298	−0.4299	−0.4318	−0.9057	−0.9058
0.6	0.5276	0.5276	0.0554	−0.4162	−0.4162	−0.4187	−0.8873	−0.8875
0.7	0.532	0.532	0.0643	−0.4027	−0.427	−0.4058	−0.8691	−0.8693
0.8	0.5364	0.5364	0.0701	−0.3893	−0.3894	−0.393	−0.8511	−0.8513
0.9	0.5407	0.5407	0.0818	−0.3761	−0.3761	−0.3805	−0.8332	−0.8334
1.0	0.545	0.5449	0.0905	−0.363	−0.363	−0.3682	−0.8154	−0.8158
1.1	0.5492	0.5492	0.099	−0.35	−0.3501	−0.356	−0.7979	−0.7982
1.2	0.5534	0.5533	0.1074	−0.3372	−0.3373	−0.3441	−0.7805	−0.7809
1.3	0.5575	0.5575	0.1158	−0.3245	−0.3246	−0.3323	−0.7633	−0.7638
1.4	0.5616	0.5616	0.124	−0.3119	−0.312	−0.3208	−0.7462	−0.7468
1.5	0.5657	0.5656	0.1321	−0.2995	−0.2997	−0.3094	−0.7294	−0.73
1.6	0.5697	0.5696	0.1402	−0.2872	−0.2874	−0.2983	−0.7127	−0.7134
1.7	0.5736	0.5735	0.1482	−0.2751	−0.2753	−0.2874	−0.6963	−0.697
1.8	0.5775	0.5774	0.156	−0.2632	−0.2634	−0.2768	−0.68	−0.6808
1.9	0.5814	0.5813	0.1638	−0.2514	−0.2516	−0.2664	−0.664	−0.6649
2.0	0.5852	0.5851	0.1715	−0.2397	−0.2399	−0.2562	−0.6482	−0.6492

Values of Induced Magnetic field H for different values of λ, A, M, V, ω

	A	M	V	ω
1.	0.3	0.8	-0.5	20
2.	0.3	0.4	-1	20
3.	0.6	0.4	-1	20
4.	0.3	0.4	-0.5	5
5.	0.3	0.4	-0.5	20
6.	0.3	0.8	0	20
7.	0.3	0.4	0.5	5
8.	0.3	0.4	0.5	20

TABLE 13 Results of Induced Magnetic field (H) **at** $\lambda = 0$

H

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
0	0	0	0	0	0	0	0	0
0.1	0.1157	0.077	0.0816	0.578	0.571	0.35	0.193	0.19
0.2	0.2065	0.1372	0.1432	0.1029	0.103	0.686	0.343	0.343
0.3	0.2777	0.1837	0.1891	0.1378	0.1394	0.0918	0.0459	0.0465
0.4	0.334	0.2197	0.224	0.1648	0.1098	0.0549	0.0549	0.0559
0.5	0.3789	0.2474	0.2505	0.1855	0.1889	0.1237	0.0618	0.063
0.6	0.4144	0.2682	0.2705	0.2012	0.2046	0.1341	0.0671	0.0682
0.7	0.4423	0.2833	0.2849	0.2125	0.2155	0.1417	0.0708	0.0718
0.8	0.4637	0.2935	0.2945	0.2201	0.2225	0.1468	0.0734	0.0742
0.9	0.4797	0.2997	0.3002	0.2247	0.2263	0.1498	0.0749	0.0754
1.0	0.4913	0.3024	0.3026	0.2268	0.2276	0.1512	0.0756	0.0759
1.1	0.4991	0.3024	0.3022	0.2268	0.2269	0.1512	0.0756	0.0756
1.2	0.5039	0.3	0.2996	0.225	0.2245	0.15	0.75	0.0748
1.3	0.5062	0.2958	0.2952	0.2219	0.2208	0.1479	0.074	0.0736
1.4	0.5064	0.2901	0.2894	0.2176	0.216	0.1451	0.0725	0.072
1.5	0.505	0.2832	0.2824	0.2124	0.2105	0.1416	0.0708	0.0702
1.6	0.5021	0.2755	0.2745	0.2066	0.2044	0.1377	0.0689	0.0681
1.7	0.4981	0.267	0.266	0.2003	0.1978	0.1335	0.0668	0.659
1.8	0.4933	0.258	0.2569	0.1935	0.191	0.129	0.0645	0.0637
1.9	0.4877	0.2487	0.2476	0.1866	0.1839	0.1244	0.0622	0.0613
2.0	0.4815	0.2392	0.2381	0.1094	0.1767	0.1196	0.0598	0.0589

TABLE 14 Results of Induced Magnetic field (H) at $\lambda = 0.5$

H

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
0	0	0	0	0	0	0	0	0
0.1	0.0114	0.0076	0.0076	0.0057	0.0057	0.0038	0.0019	0.0019
0.2	0.0227	0.0151	0.0151	0.0114	0.0114	0.0076	0.0038	0.0038
0.3	0.0339	0.0226	0.0226	0.0169	0.017	0.0113	0.0056	0.0057
0.4	0.045	0.0299	0.03	0.0225	0.0225	0.015	0.0075	0.0075
0.5	0.0559	0.0372	0.0373	0.028	0.028	0.0186	0.0093	0.0093
0.6	0.0666	0.0445	0.0445	0.0334	0.0334	0.0223	0.0111	0.0111
0.7	0.0776	0.0516	0.0517	0.0388	0.0388	0.0259	0.0129	0.0129
0.8	0.0882	0.0587	0.0588	0.0441	0.0441	0.0294	0.0147	0.0147
0.9	0.0988	0.0657	0.0658	0.0494	0.0494	0.0329	0.0165	0.0165
1.0	0.1092	0.0727	0.0728	0.0546	0.0546	0.0364	0.0182	0.0182
1.1	0.1196	0.0795	0.0797	0.0598	0.0598	0.0399	0.0199	0.0199
1.2	0.1298	0.0864	0.0865	0.0649	0.065	0.0433	0.216	0.0217
1.3	0.14	0.0931	0.0933	0.07	0.07	0.0467	0.0233	0.0233
1.4	0.15	0.0998	0.1	0.075	0.0751	0.05	0.025	0.025
1.5	0.16	0.1064	0.1066	0.08	0.08	0.0533	0.0267	0.0267
1.6	0.1698	0.113	0.1132	0.0849	0.085	0.0566	0.0283	0.0283
1.7	0.1796	0.1195	0.1997	0.0898	0.0899	0.0599	0.0299	0.03
1.8	0.1898	0.1259	0.1262	0.0946	0.0947	0.0631	0.0315	0.0316
1.9	0.1989	0.1322	0.1326	0.0994	0.0995	0.0663	0.0331	0.0332
2.0	0.2083	0.1385	0.1389	0.1042	0.1043	0.0695	0.0347	0.0348

TABLE

15 Results of Induced Magnetic field (H) at $\lambda = 1$

H

Y

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
0	0	0	0	0	0	0	0	0
0.1	0.0114	0.0076	0.0076	0.0057	0.0057	0.0038	0.0019	0.0019
0.2	0.0227	0.0151	0.0151	0.0114	0.0114	0.0076	0.0038	0.0038
0.3	0.0339	0.0226	0.0226	0.0169	0.017	0.0113	0.0056	0.0057
0.4	0.045	0.0299	0.03	0.0225	0.0225	0.015	0.0075	0.0075
0.5	0.0559	0.0372	0.0373	0.028	0.028	0.0186	0.0093	0.0093
0.6	0.0666	0.0445	0.0445	0.0334	0.0334	0.0223	0.0111	0.0111
0.7	0.0776	0.0516	0.0517	0.0388	0.0388	0.0259	0.0129	0.0129
0.8	0.0882	0.0587	0.0588	0.0441	0.0441	0.0294	0.0147	0.0147
0.9	0.0988	0.0657	0.0658	0.0494	0.0494	0.0329	0.0165	0.0165
1.0	0.1092	0.0727	0.0728	0.0546	0.0546	0.0364	0.0182	0.0182
1.1	0.1196	0.0795	0.0797	0.0598	0.0598	0.0399	0.0199	0.0199
1.2	0.1298	0.0864	0.0865	0.0649	0.065	0.0433	0.216	0.0217
1.3	0.14	0.0931	0.0933	0.07	0.07	0.0467	0.0233	0.0233
1.4	0.15	0.0998	0.1	0.075	0.0751	0.05	0.025	0.025
1.5	0.16	0.1064	0.1066	0.08	0.08	0.0533	0.0267	0.0267
1.6	0.1698	0.113	0.1132	0.0849	0.085	0.0566	0.0283	0.0283
1.7	0.1796	0.1195	0.1997	0.0898	0.0899	0.0599	0.0299	0.03
1.8	0.1898	0.1259	0.1262	0.0946	0.0947	0.0631	0.0315	0.0316
1.9	0.1989	0.1322	0.1326	0.0994	0.0995	0.0663	0.0331	0.0332
2.0	0.2083	0.1385	0.1389	0.1042	0.1043	0.0695	0.0347	0.0348

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