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# DYNAMIC LOCALIZATION OF A CHARGED PARTICLE UNDER THE INFLUENCE OF GENERALIZED ELECTRIC FIELD

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Abstract. Using technics of probability, it can be predicted the motion of a particle under the influence of an electric field. For different types of electric field, can be found different types of conditions, all involving Bessel functions.

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## 1. Introduction

The motion and the evolution in time of a charged particle on a linear chain of m sites are given by Schrodinger equation:

$$H|\psi(t)>= i\hbar\frac{d|\psi(t)>}{dt}$$

where $|\psi(t)>=\Sigma_m c_m(t)|m>$ is the wave function expressed as ca linear combination of Wannier states $|m>, c_m(t) =< m|\psi_m(t)>$ are the time-dependent amplitudes.

The Hamiltonian of this system is:

$$H(t) = V\Sigma_{m=-\infty}^{\infty}(|m>< m+1|+|m+1>< m|)-eE(t)a\Sigma_{m=-\infty}^{\infty}m(|m>< m|)$$

The Schrodinger equation becomes:

$$i\frac{dc_m}{dt} = -\mathcal{E}f(t)mc_m + V(c_{m+1} + c_{m-1})$$

where $\mathcal{E}f(t) = eaE$. We find the solution of equation for arbitrary $f(t)$ and use it to examine ,for different cases,two observables. The first observable is the probability propagator

$$\psi_m(t) = |c_m(t)|^2$$

which gives us the probability that the particle is at site m, at time t, given that it was at site 0 initially. The second observable is the mean-square displacement

$$< m^2 >= \Sigma_m m^2\psi_m(t)$$

## 2.Results and interpretations

Using the discrete Fourier transform, we get:

$$c^k(t) = \Sigma_m e^{-ikm} c_m(t)$$

The equation becomes :

$$\frac{\partial c^k}{\partial t} + 2iV coskc^k = \mathcal{E} f(t) \frac{\partial c^k}{\partial k}$$

Using the method of caracteristics, we get :

$$\frac{dt}{1} = \frac{dk}{-\mathcal{E} f(t)} = \frac{dc^k}{-2iV cos(k)c^k}$$

The solution is:

$$c^k(t) = c^{k+\mathcal{E}\eta(t)} e^{2iV[sin(k)V(t) - cos(k)U(t)]}$$

where:

$$\eta(t) = \int_0^t f(t')dt'$$

$$U(t) = \int_0^t cos\mathcal{E}(\eta(t) - \eta(t'))dt'$$

$$V(t) = \int_0^t sin\mathcal{E}(\eta(t) - \eta(t'))dt'$$

Using

$$e^{2iV sin(k)V(t)} = \Sigma_{n=-\infty}^{\infty} J_n(2VV(t))e^{ikn}$$

$$e^{-2iV cos(k)U(t)} = \Sigma_{n=-\infty}^{\infty} J_n(2VU(t))e^{ink - in\frac{\pi}{2}}$$

where $J_n(z)$ is the Bessel function of the first kind and order n, we get:

$$c_m(t) = \Sigma_r c_r(0) e^{ir\mathcal{E}\eta(t)} (-\frac{V(t) - iU(t)}{V(t) + iU(t)})^{\frac{1}{2}} J_{r-m}[2V(V^2(t) + U^2(t))^{\frac{1}{2}}]$$

We notice that:

$$U^2(t) + V^2(t) = u^2(t) + v^2(t)$$

where

$$u(t) = \int_0^t sin[\mathcal{E}\eta(t')]dt', v(t) = \int_0^t cos[\mathcal{E}\eta(t')]dt'$$

So

$$\psi_m(t) = |c_m|^2 = J_m^2(2V[v^2(t) + u^2(t)]^{\frac{1}{2}})$$

$$< m^2 >= 2V^2[v^2(t) + u^2(t)]$$

Case 1 $f(t) = sin\omega t$.Then

$$\eta(t) = \int_0^t f(t')dt' = \frac{1 - cos\omega t}{\omega}$$

$$u(t) = \int_0^t cos[\mathcal{E}\eta(t')]dt' = tcos\frac{\mathcal{E}}{\omega}J_0(\frac{\mathcal{E}}{\omega})$$

$$v(t) = \int_0^t sin[\mathcal{E}\eta(t')]dt' = tsin\frac{\mathcal{E}}{\omega}J_0(\frac{\mathcal{E}}{\omega})$$

Let be:

$$A(t) = u(t) - tcos\frac{\mathcal{E}}{\omega}J_0(\frac{\mathcal{E}}{\omega})$$

and

$$B(t) = v(t) - tsin\frac{\mathcal{E}}{\omega}J_0(\frac{\mathcal{E}}{\omega})$$

Then

$$\psi_m(t) = J_m^2[2V(u(t)^2 + v(t)^2)^{\frac{1}{2}}]$$

$$= J_m^2[2Vt(\frac{A^2(t)+B^2(t)}{t^2} + J_0^2(\frac{\mathcal{E}}{\omega}) + 2\frac{A(t)cos\frac{\mathcal{E}}{\omega}+B(t)sin\frac{\mathcal{E}}{\omega}}{t}J_0(\frac{\mathcal{E}}{\omega}))^{\frac{1}{2}}]$$

We notice that $\psi_m(t)$ is dominated by $t$, excepting the case $J_0(\frac{\mathcal{E}}{\omega}) = 0$. Also,

$$< m^2 >= 2V^2[u^2(t) + v^2(t)]$$

$$= 2V^2t^2[\frac{A^2(t)+B^2(t)}{t^2} + J_0^2(\frac{\mathcal{E}}{\omega}) + 2\frac{A(t)cos\frac{\mathcal{E}}{\omega}+B(t)sin\frac{\mathcal{E}}{\omega}}{t}J_0(\frac{\mathcal{E}}{\omega})]$$

The conclusion is that we have dynamical localization if and only if $\frac{\mathcal{E}}{\omega}$ is a zero of the Bessel function of the first kind and order 0.

Case 2 $f(t) = acos\omega t - b$. Then

$$\eta(t) = \int_0^t f(t')dt' = \frac{a}{\omega}sin\omega t - bt$$

$$u(t) = \int_0^t cos[\mathcal{E}\eta(t')]dt' = \int_0^t cos[\frac{\mathcal{E}a}{\omega} sin\omega t' - b\mathcal{E}t']dt'$$

If $\omega t' = \tau \Rightarrow$

$$u(t) = \frac{1}{\omega}\int_0^{\omega t} cos[\frac{\mathcal{E}a}{\omega} sin\tau - \frac{b\mathcal{E}}{\omega}\tau]$$

For $\omega t = 2n\pi \Rightarrow$

$$u(t) = \frac{t}{2n\pi}\int_0^{2n\pi} cos[\frac{\mathcal{E}a}{\omega} sin\tau - \frac{b\mathcal{E}}{\omega}\tau] = tJ_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}), \text{if } \frac{b\mathcal{E}}{\omega} \in \mathrm{N}$$

Also

$$v(t) = \int_0^t sin[\mathcal{E}\eta(t')]dt' = \int_0^t sin[\frac{a\mathcal{E}}{\omega} sin\omega t' - b\mathcal{E}t']dt'$$

If $\omega t' = \tau \Rightarrow$

$$v(t) = \frac{1}{\omega}\int_0^{\omega t} cos[\frac{\mathcal{E}a}{\omega} sin\tau - \frac{b\mathcal{E}}{\omega}\tau]$$

For $\omega t = 2n\pi \Rightarrow$

$$v(t) = \frac{t}{2n\pi}\int_0^{2n\pi} sin[\frac{\mathcal{E}a}{\omega} sin\tau - \frac{b\mathcal{E}}{\omega}\tau] = 0$$

Let

$$A_u(t) = u(t) - tJ_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}), A_v(t) = v(t)$$

Then

$$\psi_m(t) = J_m^2(2V[v^2(t) + u^2(t)]^{\frac{1}{2}})$$

$$= J_m^2(2V[A_v^2(t) + A_u^2(t) + t^2 J^2_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}) + 2[A_u(t) + A_v(t)]tJ_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega})]^{\frac{1}{2}})$$

$$= J_m^2(2Vt[\frac{A_v^2(t)+A_u^2(t)}{t^2} + J^2_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}) + 2\frac{A_u(t)+A_v(t)}{t}J_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega})]^{\frac{1}{2}})$$

and

$$< m^2 >= 2V^2[u^2(t) + v^2(t)]$$

$$= 2V^2t^2[\frac{A_v^2(t)+A_u^2(t)}{t^2} + J^2_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}) + 2\frac{A_u(t)+A_v(t)}{t}J_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega})]$$

We notice that $\psi_m(t)$ and $< m^2 >$ are both dominated by $t$ , except the case when $J_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}) = 0$.
So, we have dynamical localization if and only if $\frac{a\mathcal{E}}{\omega}$ is a zero of the Bessel function of the first kind and order $\frac{b\mathcal{E}}{\omega}$.


a) $a = 7.1$ b) $a = 8$

**Fig.1:** The probability propagator $\psi_0(t)$ for $\frac{\mathcal{E}}{\omega} = 1$ for $b = 1$ and different values of the parameter $a$.

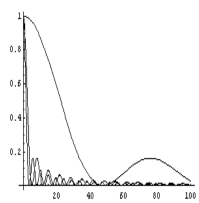


**Fig. 2:** The probability propagator $\psi_0(t)$ for $\frac{\mathcal{E}}{\omega} = 1$ for $b = 1$ and for $a = 5.5$, $a = 7.1$, $a = 8$, simultaneously.

All the curves are decaying, except the case when $J_{\frac{b\mathcal{E}}{\omega}}(\frac{a\mathcal{E}}{\omega}) = 0$ and this is the phenomenon of dynamic localization.

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