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TRANS-SASAKIAN MANIFOLD ADMITTING QUARTER-SYMMETRIC NON-METRIC CONNECTION

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Abstract. In this paper we shall introduce a quarter-symmetric non-metric connection in a trans-Sasakian manifold and prove its existence. We shall discuss some properties of quarter-symmetric non-metric connection on trans-Sasakian manifold. Also we shall compare the quarter-symmetric non-metric connection with the Levi-Civita connection in trans-Sasakian manifold.

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1. Introduction

In 1985, J.A. Oubina introduced a new class of almost contact manifold namely trans-Sasakian manifold [6]. Many geometers in [1], [4], [8] have studied the structure of trans-Sasakian manifold and obtained many results on it. In 1975, Golab introduced quarter-symmetric metric connection in Riemannian manifold [3] and S. Mukhopadhyay et. al. studied some properties on quarter-symmetic metric connection on Riemannian manifold [5].

Let M be an almost contact metric manifold of dimension $n(=2m+1)$ with an almost contact metric structure (ϕ, ξ, η, g) where ϕ is $(1,1)$ tensor field, ξ is contravariant vector field, η is a 1-form and g is a associated Riemannian metric such that,

$$\phi^2 = -I + \eta \otimes \xi, \tag{1}$$

$$\eta(\xi) = 1, \qquad \phi\xi = 0, \qquad \eta \circ \phi = 0, \tag{2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{3}$$

$$g(X, \phi Y) = -g(\phi X, Y), \tag{4}$$

and

$$g(X, \xi) = \eta(X), \tag{5}$$

$\forall \quad X, \quad Y \in \chi(M)$, then M is called a *trans-Sasakian manifold of type* (α, β) provided,

$$(\nabla_X \phi)(Y) = \alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \eta(Y)\phi X\}, \tag{6}$$

holds, for smooth functions α and β on M [8].

On a trans-Sasakian manifold,it can be shown that [8],

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi), \tag{7}$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y), \tag{8}$$

$$F(X,Y) = -F(Y,X), \tag{9}$$

where $F(X,Y) = g(\phi X, Y)$, is fundamental 2-form.

Now we shall prove the following property:

Property 1. Let M be a trans-Sasakian manifold with the structure (ϕ, ξ, η, g), then

$$(\nabla_X F)(\xi, Y) = (\nabla_X \eta)\phi Y, \tag{10}$$

$$(\nabla_X F)(\phi Y, \phi Z) = 0, \tag{11}$$

$$(\nabla_X F)(Y, \phi Z) = -\eta(Y)(\nabla_X \eta)(Z), \tag{12}$$

$$(\nabla_X F)(Y,Z) = -\{\alpha\{g(X,Y)\eta(Z) - \eta(Y)g(X,Z)\}$$
$$+\beta\{g(\phi X, Y)\eta(Z) - \eta(Y)g(\phi X, Z)\}\}, \tag{13}$$

$$(\nabla_X F)(\phi Y, Z) = -\eta(Z)(\nabla_X \eta)(Y), \tag{14}$$

$$(\nabla_X F)(Y,Z) + (\nabla_Y F)(Z,X) + (\nabla_Z F)(X,Y)$$
$$= 2\beta\{g(\phi X, Y)\eta(Z) + g(\phi Y, Z)\eta(X) + g(\phi Z, X)\eta(Y)\}. \tag{15}$$

Proof. From the definition of $F(X,Y)$,
it is clear that $F(\xi, Y) = 0$.
Here, $(\nabla_X F)(\xi, Y) = \nabla_X F(\xi, Y) - F(\nabla_X \xi, Y) - F(\xi, \nabla_X Y)$,
$= -F(-\alpha\phi X + \beta(X - \eta(X)\xi), Y)$, using (7)
$= -\{\alpha g(\phi X, \phi Y) + \beta g(\phi X, Y)\}$.
Replacing Y by ϕY in (8) we get,
$(\nabla_X \eta)\phi Y = -\alpha g(\phi X, \phi Y) + \beta g(\phi X, \phi^2 Y)$,
$= -\{\alpha g(\phi X, \phi Y) + \beta g(\phi X, Y)\}$, using(1).
It proves (10).

From (1), (2), (5) and (6) we get
$(\nabla_X F)(\phi Y, \phi Z) = -\nabla_X g(Y, \phi Z) + \alpha\eta(Y)g(\phi X, \phi Z) - \beta\eta(Y)g(X, \phi Z)$
$\quad +g(\nabla_X Y, \phi Z) + g(Y, \nabla_X(\phi Z)) - g(\eta(Y)\xi, \nabla_X(\phi Z))$.
Again using (3) and (8) in above equation we find,
$(\nabla_X F)(\phi Y, \phi Z) = 0$.
Using (2) and (6), we can find
$(\nabla_X F)(Y, \phi Z) = (\nabla_X g)(\phi X, \phi Y) - \alpha\eta(Y)g(X, \phi Z) - \beta\eta(Y)g(\phi X, \phi Z)$
$\quad = -\eta(Y)(\nabla_X \eta)Z$.
Using (3) and (6) we get
$(\nabla_X F)(Y, Z) = -\{\alpha\{g(X, Y)\eta(Z) - \eta(Y)g(X, Z)\}$
$\quad +\beta\{g(\phi X, Y)\eta(Z) - \eta(Y)g(\phi X, Z)\}\}$.
Also replacing Y by ϕY in (13) it can be easily shown
$(\nabla_X F)(\phi Y, Z) = -\eta(Z)(\nabla_X \eta)(Y)$.
Replacing X, Y and Z in cyclic order in (13) and then adding three equations we can show equation (15).

2. Quarter-Symmetric Non-Metric Connection on Trans-Sasakian Manifold

Let M be a trans-sasakian manifold with Levi-Civita connection ∇ and $X, Y \in \chi(M)$. We define a linear connection D on M by

$$D_X Y = \nabla_X Y + \eta(Y)\phi(X) \tag{16}$$

where η is $1-$form and ϕ is a tensor field of type $(1, 1)$. D is said to be quarter symmetric connection if $\bar{T}$, the torsion tensor with respect to the connection D, satisfies

$$\bar{T}(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y. \tag{17}$$

D is said to be non-metric connection if $(Dg) \neq 0$. Using (16) we have

$$(D_X g)(Y, Z) = -\{\eta(Y)g(\phi X, Z) + \eta(Z)g(\phi X, Y)\}. \tag{18}$$

A linear connection D is said to be quarter-symetric non-metric connection if it satisfies (16), (17) and (18).

3. Existence of a Quarter-Symmetric Non-Metric Connection D in a Trans-Sasakian Manifold

In this setion we shall show the existance of the quarter-symmetric non-metric connection D on a trans-Sasakian manifold M. Next we shall prove some theorems on quarter-symmetric non-metric connection on trans-Sasakian manifold.

Theorem 2. *Let X, Y, Z be any vectors fields on a trans-Sasakian manifold M with an almost structure (ϕ, ξ, η, g). Let us define a connection D by*

$$\begin{aligned} 2g(D_X Y, Z) = {} & Xg(Y,Z) + Yg(Z,X) - Zg(X,Y) \\ & + g([X,Y],Z) - g([Y,Z],X) + g([Z,X],Y) \\ & + g(\eta(Y)\phi X - \eta(X)\phi Y, Z) + g(\eta(X)\phi Z \\ & - \eta(Z)\phi X, Y) + g(\eta(Y)\phi Z + \eta(Z)\phi Y, X). \end{aligned} \tag{19}$$

Then D is a quarter-symmetric non-metric connection on M.

Proof. It can be verified that $D : (X,Y) \to D_X Y$ satisfies the following equations:

$$D_X(Y+Z) = D_X Y + D_X Z, \tag{20}$$

$$D_{X+Y} Z = D_X Z + D_Y Z, \tag{21}$$

$$D_{fX} Y = f D_X Y, \tag{22}$$

$$D_X(fY) = f(D_X Y) + (Xf)Y, \tag{23}$$

for all $X, Y, Z \in \chi(M)$ and for all f, all differentiable function on M.

From (20), (21), (22) and (23) we can conclude that D is a linear connection on M. From (19) we have,

$$D_X Y - D_Y X - [X,Y] = \eta(Y)\phi X - \eta(X)\phi Y$$

or,

$$\bar{T}(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y. \tag{24}$$

Again from (19) we get,

$$\begin{aligned} & 2g(D_X Y, Z) + 2g(D_X Z, Y) \\ & \quad = 2Xg(Y,Z) + 2\eta(Y)g(\phi X, Z) + 2\eta(Z)g(\phi X, Y). \end{aligned}$$

$$(D_X g)(Y,Z) = -\{\eta(Y)g(\phi X, Z) + \eta(Z)g(\phi X, Y)\}. \tag{25}$$

This shows that D is a quarter-symmetric non-metric connection on M.

Theorem 3. *Let D be a linear connection on a trans-Sasakian manifold M, given by*

$$D_X Y = \nabla_X Y + H(X,Y), \tag{26}$$

where $H(X,Y)$ is a $(1,2)$ tensor field and ∇ is Levi-Civita connection, satisfying (18). Then $H(X,Y) = \eta(Y)\phi(X)$.

Proof. Using (26) in the definition of torsion tensor, we get

$$\bar{T}(X,Y) = H(X,Y) - H(Y,X). \tag{27}$$

From (26), we have

$$g(H(X,Y),Z) + g(H(X,Z),Y) = -(D_X g)(Y,Z). \tag{28}$$

From (18), (26), (27) and (28) we have

$$\begin{aligned} &g(\bar{T}(X,Y),Z) + g(\bar{T}(Z,Y),X) + g(\bar{T}(Z,X),Y) \\ &= 2g(H(X,Y),Z) - (D_Z g)(X,Y) + (D_Y g)(X,Z) + (D_X g)(Y,Z). \end{aligned}$$

We get from above equation,

$$\begin{aligned} g(H(X,Y),Z) &= \tfrac{1}{2}[g(\bar{T}(X,Y),Z) + g(\bar{T}(Z,Y),X) \\ &\quad + g(\bar{T}(Z,X),Y)] + [\eta(Y)g(\phi X,Z) + \eta(X)g(\phi Y,Z)]. \end{aligned}$$

Thus, we get

$$\begin{aligned} H(X,Y) &= \tfrac{1}{2}[\bar{T}(X,Y) + \tilde{T}(X,Y) + \tilde{T}(Y,X)] \\ &\quad + [\eta(Y)\phi X + \eta(X)\phi Y], \end{aligned}$$

where $\tilde{T}$ is a tensor field of type (1,2) defined by

$$g(\tilde{T}(X,Y),Z) = g(\bar{T}(Z,X),Y).$$

Thus $H(X,Y) = \eta(Y)\phi X$.
Hence $D_X Y = \nabla_X Y + \eta(Y)\phi X$.

Theorem 4. *Under the quarter-symmetric non-metric connection,*

$$(D_X g)(Y,Z) + (D_Y g)(Z,X) + (D_Z g)(X,Y) = 0, \tag{29}$$

$$\begin{aligned} &g(\bar{T}(X,Y),Z) + g(\bar{T}(Y,Z),X) + g(\bar{T}(Z,X),Y) \\ &= 2[\eta(Y)g(\phi X,Z) + \eta(Z)g(\phi Y,X) + \eta(X)g(\phi Z,Y)], \end{aligned} \tag{30}$$

Proof. By (4) and (18), we have

$$\begin{aligned} &(D_X g)(Y,Z) + (D_Y g)(Z,X) + (D_Z g)(X,Y) \\ &= -[\eta(Y)\{g(\phi X,Z) + g(X,\phi Z)\} + \eta(X)\{g(\phi Y,Z) \\ &\quad + g(Y,\phi Z)\} + \eta(Z)\{g(\phi X,Y) + g(X,\phi Y)\}] = 0. \end{aligned}$$

From (17) we have

$$\begin{aligned} &g(\bar{T}(X,Y),Z) + g(\bar{T}(Y,Z),X) + g(\bar{T}(Z,X),Y) \\ &= \eta(Y)g(\phi X,Z) - \eta(X)g(\phi Y,Z) + \eta(Z)g(\phi Y,X) \\ &\quad - \eta(Y)g(\phi Z,X) + \eta(X)g(\phi Z,Y) - \eta(Z)g(\phi X,Y) \\ &= 2[\eta(Y)g(\phi X,Z) + \eta(Z)g(\phi Y,X) + \eta(X)g(\phi Z,Y)]. \end{aligned}$$

Theorem 5. *Let M be a trans-Sasakian manifold with the quarter-symmetric non-metric connection, then*

$$(D_X\phi)Y = (\nabla_X\phi)Y - X\eta(Y) + \eta(X)\eta(Y)\xi, \tag{31}$$

$$D_X\xi = \nabla_X\xi + \phi X, \tag{32}$$

$$(D_X\eta)Y = (\nabla_X\eta)Y, \tag{33}$$

$$(D_XF)(Y,Z) = (\nabla_XF)(Y,Z). \tag{34}$$

Proof. Using (16), we have

$$\begin{aligned}(D_X\phi)Y &= (\nabla_X\phi)Y - \phi(\nabla_XY) - \phi(\eta(Y)\phi X)\\ &= (\nabla_X\phi)Y - X\eta(Y) + \eta(X)\eta(Y)\xi.\end{aligned}$$

Putting ξ in place of Y in (16), we get

$$D_X\xi = \nabla_X\xi + \phi X.$$

Using (2) and (16), we get

$$(D_X\eta)Y = \nabla_X\eta(Y) - \eta(\nabla_XY) = (\nabla_X\eta)Y.$$

4. Curvature Tensor and Ricci Tensor on a Trans-Sasakian Manifold with respect to Quarter-Symmetric Non-Metric Connection

Let $\bar{R}$ and R be the curvature tensors with respect to the quarter-symmetric non-metric connection D and the Levi-Civita connection ∇ respectively on a trans-Sasakian manifold M. In this section we shall find the relation between $\bar{R}$ and R. Also we shall find the relation between $\bar{S}$ and S, $\bar{r}$ and r, where $\bar{S}$ and $\bar{r}$ are the Ricci tensor and scaler curvature with respect to quarter-symmetric non-metric connection D on M respectively, S and r are Ricci tensor and scaler curvature with respect to Levi-Civita connection ∇ on M respectively. After this we shall prove some theorems on curvature tensor, Ricci tensor and Einstein manifold.

Theorem 6. *Let X, Y and Z be vector fields on a trans-Sasakian manifold M and $\bar{R}$ and R be the curvature tensors with respect to the quarter-symmetric non-metric connection D and with respect to the Levi-Civita connection ∇ on M respectively. Then*

$$\begin{aligned}\bar{R}(X,Y)Z = R(X,Y)Z &+ \alpha[\eta(X)Y - \eta(Y)X]\eta(Z) + 2\beta\eta(Z)g(\phi X,Y)\xi\\ &+\alpha\{g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y\} + \beta\{g(X,Z)\phi Y - g(Y,Z)\phi X\}.\end{aligned}$$

Proof. We define the curvature tensor $\bar{R}$ with respect to quarter-symmetric non-metric connection on M by
$\bar{R}(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z.$
Using (8) and (16) we get
$\bar{R}(X,Y)Z = R(X,Y)Z + \eta(Z)[(\nabla_X\phi)(Y) - (\nabla_Y\phi)(X)] + \{\alpha g(\phi Y, Z)$
$\quad -\beta g(\phi Y, \phi Z)\}\phi X - \{\alpha g(\phi X, Z) - \beta g(\phi X, \phi Z)\}\phi Y.$
Then using (3), (4) and (6) we get

$$\begin{aligned}\bar{R}(X,Y)Z &= R(X,Y)Z + \alpha[\eta(X)Y - \eta(Y)X]\eta(Z)\\ &\quad +2\beta\eta(Z)g(\phi X, Y)\xi + \alpha\{g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y\}\\ &\quad +\beta\{g(X,Z)\phi Y - g(Y,Z)\phi X\}.\end{aligned} \tag{35}$$

Theorem 7. *If $\bar{S}$ and $\bar{r}$ are the Ricci tensor and scaler curvature with respect to quarter-symmetric non-metric connection D on M respectively, S and r are Ricci tensor and scaler curvature with respect to Levi-Civita connection ∇ on M respectively, then*
$\bar{S}(X,Y) = S(X,Y) - (n-1)\alpha\eta(Y)\eta(X) + \alpha g(\phi X, \phi Y) + \beta g(\phi Y, X)$
and $\bar{r} = r$.

Proof. By (35) we get

$$\begin{aligned}g(\bar{R}(X,Y)Z,W) &= g(R(X,Y)Z,W) + \alpha\eta(Z)[\eta(X)g(Y,W)\\ &-\eta(Y)g(X,W)] + 2\beta\eta(Z)g(\phi X, Y)g(\xi, W) + \alpha\{g(\phi Y, Z)g(\phi X, W)\\ &-g(\phi X, Z)g(\phi Y, W)\} + \beta\{g(X,Z)g(\phi Y, W) - g(Y,Z)g(\phi X, W)\}.\end{aligned}$$

By contraction we get

$$\bar{S}(X,Y) = S(X,Y) - (n-1)\alpha\eta(Y)\eta(X) + \alpha g(\phi X, \phi Y) + \beta g(\phi Y, X). \tag{36}$$

Again by contraction, from (36) we get

$$\bar{r} = r. \tag{37}$$

Theorem 8. *In a trans-Sasakian manifold with the quarter-symmetric non-metric connection*

$$\begin{aligned}&\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y\\ &\quad = 2\alpha\{g(\phi X, Y)\phi Z + g(\phi Y, Z)\phi X + g(\phi Z, X)\phi Y\}\\ &\quad +2\beta\{g(\phi X, Y)\eta(Z) + g(\phi Y, Z)\eta(X) + g(\phi Z, X)\eta(Y)\}\xi.\end{aligned}$$

Proof. Using first Binachi identity with respect to Levi-Civita connection ∇ and (35) we get the result.

Theorem 9. *In a trans-Sasakian manifold with quarter-symmetric non-metric connection D the Ricci tensor is symmetric if and only if $\beta = 0$, i.e., M is $\alpha-$Sasakian manifold.*

Proof. By (36) and using (4) we get,

$$\bar{S}(X,Y) - \bar{S}(Y,X) = 2\beta g(X, \phi Y).$$

Clearly, $\bar{S}(X,Y)$ is symmetric if and only if $2\beta g(X, \phi Y) = 0$, i.e., $\beta = 0$.

Theorem 10. *In a trans-Sasakian manifold with a quarter-symmetric non-metric connection D, the Ricci tensor $\bar{S}$ is skew-symmetic if and only if the Ricci tensor of the Levi-Civita connection ∇ is $S(X,Y) = (n-1)\alpha\eta(Y)\eta(X) - \alpha g(\phi X, \phi Y)$.*

Proof. From (36) we get,

$$\begin{aligned}\bar{S}(X,Y) + \bar{S}(Y,X) = 2S(X,Y) - 2(n-1)\alpha\eta(Y)\eta(X) \\ +2\alpha g(\phi X, \phi Y) + \beta\{g(\phi Y, X) + g(\phi X, Y)\}.\end{aligned}$$

Using (4), we have,

$$\bar{S}(X,Y) + \bar{S}(Y,X) = 2S(X,Y) - 2(n-1)\alpha\eta(Y)\eta(X) + 2\alpha g(\phi X, \phi Y). \quad (38)$$

If $\bar{S}(X,Y)$ is skew-symmetric then the L.H.S. of above equation vanishes and we get

$$S(X,Y) = (n-1)\alpha\eta(Y)\eta(X) - \alpha g(\phi X, \phi Y). \quad (39)$$

Using (39) in (38) we get,

$$\bar{S}(X,Y) + \bar{S}(Y,X) = 0.$$

Thus Ricci tensor of D is skew-symmetric.

Theorem 11. *In a trans-Sasakian manifold with the quarter-symmetric non-metric connection D, if $\alpha\{n\eta(X)\eta(Y) - g(X,Y)\} = \beta g(X, \phi Y)$ then the Einstein manifold for quarter-symmetric non-metric connection D is equal to the Einstein manifold for the Riemannian connection.*

Proof. We define Einstein manifold with respect to quarter-symmetric non-metric connection D by

$$\bar{S}(X,Y) = \frac{\bar{r}}{n} g(X,Y), \quad (40)$$

X, $Y \in \chi(M)$. From (36), (37) and (40) we get,
$\bar{S}(X,Y) - \frac{\bar{r}}{n}g(X,Y) = S(X,Y) - \frac{r}{n}g(X,Y)$
$-\alpha\{n\eta(X)\eta(Y) - g(X,Y)\} + \beta g(X,\phi Y)$.
If $\alpha\{n\eta(X)\eta(Y) - g(X,Y)\} - \beta g(X,\phi Y) = 0$, then $\bar{S}(X,Y) - \frac{\bar{r}}{n}g(X,Y) = S(X,Y) - \frac{r}{n}g(X,Y)$.
Hence the theorem is proved.

5. Example of a trans-Sasakian manifold with quarter-symmetric non-metric connection

In this section we shall show a three dimensional trans-Sasakian manifold with quarter-symmetric non-metric connection.

Example 1. We consider the three-dimensional real manifold
$M = \{(x,y,z) \in R^3, z \neq 0,\}$ with the basis $\{e_1, e_2, e_3\}$, where $e_1 = z\frac{\partial}{\partial x}$, $e_2 = z\frac{\partial}{\partial y}$, $e_3 = z\frac{\partial}{\partial z}$ [2].
Let g be the Riemannian metric defined by
$g(e_i, e_j) = 1$, if $i = j$,
$= 0$, if $i \neq j$.
The $1-$form η can be defined by $\eta(X) = g(X, e_3)$, where $X \in \chi(M)$. Then clearly $\eta(e_1) = \eta(e_2) = 0$ and $\eta(e_3) = 1$. Let ϕ be the $(1,1)$ tensor field defined by $\phi(e_1) = -e_2$, $\phi(e_2) = e_1$ and $\phi(e_3) = 0$. Let the contravariant vector field $\xi = e_3$.
Then $\eta(\xi) = 1$, $\phi^2(X) = -X + \eta(X)\xi$, $g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y)$, where $X, Y \in \chi(M)$.
Also $\eta(\phi(e_i)) = 0$, for all $i = 1,2,3$, $\phi\xi = 0$. Thus (ϕ, ξ, η, g) is an almost contact metric structure on M. Also we obtain
$[e_1, e_2] = 0$, $[e_2, e_3] = -e_2$ and $[e_1, e_3] = -e_1$.
By Koszul's formula we get
$\nabla_{e_1} e_1 = e_3$, $\nabla_{e_2} e_1 = 0$, $\nabla_{e_3} e_1 = 0$,
$\nabla_{e_1} e_2 = 0$, $\nabla_{e_2} e_2 = e_3$, $\nabla_{e_3} e_2 = 0$,
$\nabla_{e_1} e_3 = -e_1$, $\nabla_{e_2} e_3 = -e_2$, $\nabla_{e_3} e_3 = 0$.
It can be shown that M is a trans-Sasakian manifold of type $(0,-1)$ [2].

Now using (16) we find D, the quarter-symmetric connection on M
$D_{e_1} e_1 = e_3$, $D_{e_2} e_1 = 0$, $D_{e_3} e_1 = 0$,
$D_{e_1} e_2 = 0$, $D_{e_2} e_2 = e_3$, $D_{e_3} e_2 = 0$,
$D_{e_1} e_3 = -e_1 - e_2$, $D_{e_2} e_3 = e_1 - e_2$, $D_{e_3} e_3 = 0$.
Using (17), the torsion tensor $\bar{T}$, with respect to quarter-symmetric connection D is given by $\bar{T}(e_i, e_i) = 0$, for all $i = 1,2,3$, $\bar{T}(e_1, e_2) = 0$, $\bar{T}(e_2, e_3) = -e_1$, $\bar{T}(e_3, e_1) = -e_2$.

Now using (18), we calculate the metric g with respect to the quarter-symmetric connection D as follows:
$(D_{e_1}g)(e_2, e_3) = -\{\eta(e_2)g(\phi e_1, e_3) + \eta(e_3)g(\phi e_1, e_2)\} = 1,$
$(D_{e_2}g)(e_3, e_1) = -\{\eta(e_3)g(\phi e_2, e_1) + \eta(e_1)g(\phi e_2, e_3)\} = -1,$
$(D_{e_3}g)(e_1, e_2) = -\{\eta(e_1)g(\phi e_3, e_2) + \eta(e_2)g(\phi e_3, e_1)\} = 0.$
From these we can conclude that $(D_X g)(Y, Z) \neq 0$, where X, Y, Z are any vector field in $\chi(M)$.
Hence D is a quarter-symmetric non-metric connection on a trans-Sasakian manifold M.

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