

ESTIMATION OF REINSURANCE PHT PREMIUM FOR AR(1) PROCESS WITH INFINITE VARIANCE

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ABSTRACT. The estimation of the price of an insurance risk is a very important actuarial problem. This price has to reflect the property of the distribution of the random variable describing the corresponding loss. If the loss variable has a heavy-tailed distribution (i.e. distribution with an infinite variance) then, the risk measure (as a measure of the risk premium) should be higher. In this paper, we extend estimate of PHT premium developed by Necir et al [14] to autoregressive processes with infinite variance.

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1. INTRODUCTION

Quantifying the risk associated with a random financial outcome is an important actuarial problem. Based on various systems of axioms, a number of risk measures have been proposed in the literature, and their properties have been investigated. On the subject, we refer to, for example, Wang [21, 23, 24] and Wirth and Hardy [26]. Artzner et al. [1] proposed a set of four axioms for a coherent risk measure. For loss variables X and Y a coherent measure μ is a real functional defined on a space of random variables, satisfying the following axioms:

- *Bounded above by the maximum loss:* $\mu(X) \leq \max(X)$.
- *Bounded below by the mean loss:* $\mu(X) \geq \mathbf{E}(X)$.
- *Scaler additive and multiplicative:* $\mu(cX + d) = c\mu(X) + d$, for $c \geq 0, d \in \mathbf{R}$.
- *Subadditivity:* $\mu(X + Y) \leq \mu(X) + \mu(Y)$.

The first use of risk measures in actuarial science was the development of premium principles. These were applied to a loss distribution to determine an appropriate premium to charge for the risk. Some traditional premium principle examples include: expected value, variance, standard deviation, modified variance, value at risk, etc. (see for instance Rolski et al. [18]).

The class of the distortion risk measures are closely related to coherent measures. They were introduced by Denneberg [6] and Wang [22] and have been applied to a wide variety of insurance problems, most particularly to the determination of insurance premiums. For example, if $X \geq 0$ represents an insurance loss with distribution function F , the distortion risk premium is defined by

$$\mu_g(X) = \int_0^\infty g(1 - F_X(x))dx. \tag{1}$$

Here g , the distortion function, is an increasing function defined on $[0, 1]$ with $g(0) = 0$ and $g(1) = 1$. If g is concave the distortion risk measure further satisfies the subadditivity and becomes coherent; see, e.g., Wirch and Hardy [26] and Dhaene et al. [7].

Families of distortion risk measures

- *Conditional tail expectation (CTE) (Hürlimann [9]):* For $0 \leq \nu < 1$, the distortion function is

$$g(s) = \begin{cases} s/(1 - \nu) & , \quad 0 \leq s < 1 - \nu, \\ 1 & , \quad 1 - \nu < s \leq 1, \end{cases}$$

and the risk measure becomes

$$CTE_\nu(X) = \frac{1}{\nu} \int_{1-\nu}^1 F_X^{-1}(t)dt.$$

- *Wang Transform (WT) (Wang [23]):* For the WT measure the distortion function $g(s) = \phi(\phi^{-1}(s) + \varrho)$, $0 \leq \varrho < \infty$, the measure can be defined by

$$WT_\varrho(X) = \int_0^\infty (\phi(\phi^{-1}(1 - F_X(u)) + \varrho))du$$

where $\phi(\cdot)$ and $\phi^{-1}(\cdot)$, respectively, denote the cdf and the inverse of the standard normal distribution, and parameter ϱ reflects the level of systematic risk and is called the market price of risk.

- *Proportional hazards transform(PHT) (Wang [21]):* If $g(s) = s^{1/\rho}$, $\rho \geq 1$, the distortion function is the power-law transformation and the associated risk measure is

$$\Pi_{\rho}(X) = \int_0^{\infty} (1 - F_X(x))^{1/\rho} dx,$$

where ρ is called distortion parameter. The interpretation of the distortion above is the following: The initial survival function $S(x) = 1 - F(x)$ is replaced by the transformed survival function $S_{\rho}^*(x) = (S(x))^{1/\rho}$. Therefore, we have:

$$\Pi_{\rho}(X) = \int_0^{\infty} S_{\rho}^*(x) dx.$$

The relationship between the initial and transformed survival functions can also be written : $\log S_{\rho}^*(x) = \frac{1}{\rho} \log S(x)$, which implies

$$\frac{-d \log S_{\rho}^*(x)}{dx} = \frac{1}{\rho} \left(\frac{-d \log S(x)}{dx} \right).$$

Thus, the hazard functions associated with both distributions are proportional, which explains the name of the risk measure.

Reinsurance PHT premium

Insurance companies often seek reinsurance to protect themselves against catastrophic losses. Reinsurance is the transfer of risk from a direct insurer (the cedent), to a second insurance carrier. The reinsurance PHT premium with retention level $R > 0$, is defined as follows

$$\Pi_{\rho,R}(X) = \int_R^{\infty} (1 - F_X(x))^{1/\rho} dx \quad (2)$$

For high-excess loss layers ($R \rightarrow \infty$) Necir and Boukhetala [13], Vandewalle and Beirlant [20] and Necir et al. [14] have proposed different asymptotically normal estimators for $\Pi_{\rho,R}$ based on samples of claim amounts of reinsurance covers of heavy tailed i.i.d. risks.

Most applications in statistics need time dependence. To illustrate some results on $\Pi_{\rho,R}(X)$ estimation, we consider some models of ergodic processes, particularly MA and AR processes driven by regularly varying tail innovation with infinite variance. For more information on this kind of processes see, for example, Mikosch and Samorodnitsky [12], Samorodnitsky and Taqqu [19]. This paper is organized as follows: in Section 2 we introduce linear processes with infinite variance. In Section 3, we construct a reinsurance PHT premium estimation for an AR(1) process which is the main result. In Section 4, we provide the proof of our result.

2. LINEAR PROCESSES WITH INFINITE VARIANCE

We consider the moving average process of order infinity, written $MA(\infty)$ of the form

$$X_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j}, \quad t \in \mathbf{Z}, \quad (3)$$

where the i.i.d. innovations ε_t , $t \in \mathbf{Z}$ are non-negative random variables having distribution F for which $S = 1 - F$ is regularly varying at infinity with index $-\alpha$, that is:

$$\lim_{v \rightarrow \infty} \frac{S(vx)}{S(v)} = x^{-\alpha}, \quad \text{for any } x > 0 \text{ and } 1 < \alpha < 2. \quad (4)$$

We define the quantile function associated to the df F as $F^{-1}(s) = \inf\{x \in \mathbf{R} : F(x) \geq s\}$, $0 < s < 1$. Note that the condition (4) is equivalent to

$$\lim_{t \rightarrow \infty} \frac{U(tx)}{U(t)} = x^{1/\alpha}, \quad \text{for any } x > 0, \quad (5)$$

where $U(t) = F^{-1}(1/t)$, $t \geq 1$. To get asymptotic normality of estimators of parameters of extreme events, it is usual to assume the following extra second regular variation condition, that involves a second order parameter $\eta < 0$:

$$\lim_{t \rightarrow \infty} (A(t))^{-1} \left(\frac{U(tx)}{U(t)} - x^{1/\alpha} \right) = x^{1/\alpha} \frac{x^\eta - 1}{\eta}, \quad \text{for any } x > 0, \quad (6)$$

where A is a suitably chosen function of constant sign near infinity. Our concern is with non-negative time series and we will assume that the coefficients c_j are non-negative satisfying $\sum_{j=0}^{\infty} c_j < \infty$.

The moving average (3) has the same tail behavior as the innovations ε_t , $t \in \mathbf{Z}$. More precisely Datta and McCormick [3] proved that

$$\lim_{x \rightarrow \infty} \frac{P(X_t > x)}{P(\varepsilon_t > x)} = \sum_{j=0}^{\infty} c_j^\alpha \quad (7)$$

Examples of linear processes with infinite variance include finite-order autoregressive AR, moving-average MA and autoregressive moving-average ARMA processes.

3. ESTIMATING $\Pi_{\rho,R}(X_t)$ FOR AR(1) PROCESS WITH INFINITE VARIANCE

Let us consider a finite sequence X_0, X_1, \dots, X_n of random variables which we suppose verifying the autoregressive AR(1) process given by :

$$X_t = a_1 X_{t-1} + \varepsilon_t, \quad 0 \leq t \leq n \quad (8)$$

with $0 < a_1 < 1$, and ε_t be an i.i.d. innovations with common distribution F satisfying (4), (6). There are two possible ways to estimate α :

1. Apply the Hill estimator [8] directly to X_t , i.e

$$1/\hat{\alpha}_X = \frac{1}{k} \sum_{i=1}^k \log(X_{n-i+2,n+1}) - \log(X_{n-k+1,n+1}),$$

where $X_{j,n+1}$ is the j th largest order statistic of X_t

2. Estimate autoregressive coefficient a_1 with the consistent estimator

$$\hat{a}_1 = \frac{\sum_{t=1}^n (X_t - \bar{X})(X_{t+1} - \bar{X})}{\sum_{t=1}^n (X_t - \bar{X})^2},$$

where $\bar{X} = n^{-1} \sum_{t=1}^n X_t$ (see, Davis and Resnik [4, 5]), then estimate the residuals

$$\hat{\varepsilon}_t = X_t - \hat{a}_1 X_{t-1}, \quad 1 \leq t \leq n,$$

and apply Hill's estimator to residuals, we get:

$$1/\hat{\alpha}_{\hat{\varepsilon}} = \frac{1}{k} \sum_{i=1}^k \log(\hat{\varepsilon}_{n-i+1,n}) - \log(\hat{\varepsilon}_{n-k,n}),$$

where $\hat{\varepsilon}_{j,n}$ is the j th largest order statistic of $\hat{\varepsilon}_t$

Resnick and Stărică [17] demonstrated that the Hill estimator performs better in the second approach. A similar result was proved by Ling and Peng [10]. The autoregressive process X_t in (8) can be written as an $MA(\infty)$ like in (3) with $c_j = a_1^j$, for estimate the right extreme quantile $F_{X_t}^{-1}(1-u)$, $0 < u < 1$ relation (7) can be written

$$\lim_{x \rightarrow \infty} \frac{1 - F_{X_t}(x)}{1 - F_{\varepsilon_t}(x)} = 1/(1 - a_1^\alpha),$$

using the regular variation of $1 - F_{\varepsilon_t}$, we obtain the following relationship between the corresponding right quantile functions:

$$\lim_{u \downarrow 0} \frac{F_{X_t}^{-1}(1-u)}{F_{\varepsilon_t}^{-1}(1 - (1 - a_1^\alpha)u)} = 1.$$

Then we approximate $F_{X_t}^{-1}(1-u)$ by $F_{\hat{\varepsilon}_t}^{-1}(1 - (1 - \hat{a}_1^\alpha)u) \sim F_{\hat{\varepsilon}_t}^{-1}(1-u) (1 - \hat{a}_1^\alpha)^{-1/\alpha}$ and estimate the latter by the Weissman estimator [25]

$$\widehat{\varepsilon}_{n-k,n} \left(\frac{n(1 - \widehat{a}_1^{\widehat{\alpha}_\varepsilon})u}{k} \right)^{-1/\widehat{\alpha}_\varepsilon} \quad (9)$$

Defining the estimator and main results

To estimate the risk measure $\Pi_{\rho,R}(X_t)$, given in (2), when X_t is an AR(1) like in (8), and $R = F_{X_t}^{-1}(1 - k/n)$. Let $k = k_n$ be sequence of integer satisfying $1 < k < n, k \rightarrow \infty, k/n \rightarrow 0$. We present now our risk measure $\Pi_{\rho,R}(X_t)$ as

$$\Pi_{\rho,R}(X_t) = - \int_0^{k/n} s^{1/\rho} dF_{X_t}^{-1}(1 - s) \quad (10)$$

To estimate $\Pi_{\rho,R}$ we use derivation in (9). After an integration, we obtain the following estimator

$$\widehat{\Pi}_{\rho,R}(X_t) = \frac{\rho(k/n)^{1/\rho} \left(1 - \widehat{a}_1^{\widehat{\alpha}_\varepsilon}\right)^{-1/\widehat{\alpha}_\varepsilon}}{\widehat{\alpha}_\varepsilon - \rho} \widehat{\varepsilon}_{n-k,n}, \quad (11)$$

Theorem 1. *Let X_t an AR(1) process satisfying (8), and assume that (6) holds with $t^{-1/\rho} F_\varepsilon^{-1}(1 - 1/t) \rightarrow 0$ as $t \rightarrow \infty$, and $k = k_n$ be such that $k \rightarrow \infty, k/n \rightarrow 0$. If $\sqrt{n}A(k/n) \rightarrow 0$ as $n \rightarrow \infty$ and if the distortion parameter $\rho \in [1, \alpha]$, then*

$$\frac{(k/n)^{-1/\rho} k^{1/2}}{\widehat{\varepsilon}_{n-k,n}} [\widehat{\Pi}_{\rho,R}(X_t) - \Pi_{\rho,R}(X_t)] \xrightarrow{D} \mathcal{N}(0, \sigma^2(\rho, \alpha, a_1)), \quad \text{as } n \rightarrow \infty,$$

$$\sigma^2(\rho, \alpha, a_1) = (1 - a_1^\alpha)^{-2/\alpha} \frac{\rho\alpha^2 - 2\rho^2\alpha + \rho^3 + \rho\alpha^4}{\alpha^3(\alpha - \rho)^2}.$$

Proof. Denoting

$$\begin{aligned} H_1 &= \rho(k/n)^{1/\rho} \left(1 - \widehat{a}_1^{\widehat{\alpha}_\varepsilon}\right)^{-1/\widehat{\alpha}_\varepsilon} \widehat{\varepsilon}_{n-k,n} \left\{ \frac{1}{\widehat{\alpha}_\varepsilon - \rho} - \frac{1}{\alpha - \rho} \right\} \\ H_2 &= \frac{\rho(k/n)^{1/\rho} \left(1 - \widehat{a}_1^{\widehat{\alpha}_\varepsilon}\right)^{-1/\widehat{\alpha}_\varepsilon} F_{\widehat{\varepsilon}}^{-1}(1 - k/n)}{\alpha - \rho} \left\{ \frac{\widehat{\varepsilon}_{n-k,n}}{F_{\widehat{\varepsilon}}^{-1}(1 - k/n)} - 1 \right\}, \\ H_3 &= \frac{\rho(k/n)^{1/\rho} \left(1 - \widehat{a}_1^{\widehat{\alpha}_\varepsilon}\right)^{-1/\widehat{\alpha}_\varepsilon} F_{\widehat{\varepsilon}}^{-1}(1 - k/n)}{\alpha - \rho} - \int_{F_{X_t}^{-1}(1 - k/n)}^{\infty} (S_X(x))^{1/\rho} dx. \end{aligned}$$

Then, we can verifies easily that

$$\widehat{\Pi}_{\rho,R}(X_t) - \Pi_{\rho,R}(X_t) = H_1 + H_2 + H_3.$$

H_1 can be written also

$$H_1 = \frac{\rho \widehat{\alpha}_{\widehat{\varepsilon}} \alpha \left(1 - \widehat{a}_1^{\widehat{\alpha}_{\widehat{\varepsilon}}}\right)^{-1/\widehat{\alpha}_{\widehat{\varepsilon}}} (k/n)^{1/\rho \widehat{\varepsilon}_{n-k,n}} \left\{ \frac{1}{\widehat{\alpha}_{\widehat{\varepsilon}}} - \frac{1}{\alpha} \right\}}{(\widehat{\alpha}_{\widehat{\varepsilon}} - \rho)(\alpha - \rho)}$$

Since $\widehat{\alpha}$ and \widehat{a}_1 are consistent estimators of α and a_1 respectively, then for all large n

$$H_1 = (1 + o_P(1)) \frac{\rho \alpha^2 (1 - a_1^\alpha)^{-1/\alpha} (k/n)^{1/\rho \widehat{\varepsilon}_{n-k,n}} \left\{ \frac{1}{\widehat{\alpha}_{\widehat{\varepsilon}}} - \frac{1}{\alpha} \right\}}{(\alpha - \rho)^2}$$

and

$$H_2 = (1 + o_P(1)) \frac{\rho (k/n)^{1/\rho} (1 - a_1^\alpha)^{-1/\alpha} F_{\widehat{\varepsilon}_t}^{-1}(1 - k/n)}{\alpha - \rho} \left\{ \frac{\widehat{\varepsilon}_{n-k,n}}{F_{\widehat{\varepsilon}_t}^{-1}(1 - k/n)} - 1 \right\}$$

In view of Theorems 2.3 and 2.4 of Csörgő and Mason [2], Peng [16], and Necir et al. [14] has been shown that under the second-order condition (6) and for all large n

$$\sqrt{k} \alpha \left(\frac{1}{\widehat{\alpha}_{\widehat{\varepsilon}}} - \frac{1}{\alpha} \right) = \sqrt{\frac{n}{k}} B_n \left(1 - \frac{k}{n} \right) - \sqrt{\frac{n}{k}} \int_{1-k/n}^1 \frac{B_n(s)}{1-s} ds + o_P(1),$$

$$\sqrt{k} \left(\frac{\widehat{\varepsilon}_{n-k,n}}{F_{\widehat{\varepsilon}_t}^{-1}(1 - k/n)} - 1 \right) = -\alpha^{-1} \sqrt{\frac{n}{k}} B_n \left(1 - \frac{k}{n} \right) + o_P(1),$$

and

$$\frac{\widehat{\varepsilon}_{n-k,n}}{F_{\widehat{\varepsilon}_t}^{-1}(1 - k/n)} = 1 + o_P(1),$$

where $\{B_n(s), 0 \leq s \leq 1, n = 1, 2, \dots\}$ is the sequence of Brownian bridges. This implies that for all large n

$$H_1 = (1 + o_P(1)) \frac{\rho \alpha (1 - a_1^\alpha)^{-1/\alpha} (k/n)^{1/\rho} F_{\widehat{\varepsilon}_t}^{-1}(1 - k/n)}{k^{1/2} (\alpha - \rho)^2} \left(\sqrt{\frac{n}{k}} B_n \left(1 - \frac{k}{n} \right) - \sqrt{\frac{n}{k}} \int_{1-k/n}^1 \frac{B_n(s)}{1-s} ds + o_P(1) \right)$$

$$H_2 = (1 + o_P(1)) \frac{\rho (1 - |a_1|^\alpha)^{-1/\alpha} (k/n)^{1/\rho} F_{\widehat{\varepsilon}_t}^{-1}(1 - k/n)}{k^{1/2} \alpha (\alpha - \rho)} \left(-\sqrt{\frac{n}{k}} B_n \left(1 - \frac{k}{n} \right) + o_P(1) \right).$$

We have from Necir et al [14] and Necir et al [15]

$$\frac{(k/n)^{-1/\rho} k^{1/2}}{F_{\hat{\varepsilon}}^{-1}(1-k/n)}(H_3) = o(1) \quad n \rightarrow \infty,$$

and

$$\frac{(k/n)^{-1/\rho} k^{1/2}}{F_{\hat{\varepsilon}}^{-1}(1-k/n)}(H_1 + H_2) = \Delta_n + o_P(1),$$

with

$$\begin{aligned} \Delta_n = & (1 - a_1^\alpha)^{-1/\alpha} \left[\frac{\rho\alpha}{(\alpha - \rho)^2} \left(\frac{\rho}{\alpha^2} - \frac{1}{\alpha} + 1 \right) (n/k)^{1/2} B_n(1 - k/n) \right. \\ & \left. - \frac{\rho\alpha}{(\alpha - \rho)^2} (n/k)^{1/2} \int_{1-k/n}^1 \frac{B_n(s)}{1-s} ds \right], \end{aligned}$$

then the asymptotic variance of $\frac{(k/n)^{-1/\rho} k^{1/2}}{F_{\hat{\varepsilon}}^{-1}(1-k/n)}(\hat{\Pi}_{\rho,R}(X_t) - \Pi_{\rho,R}(X_t))$ will be computed by

$$\sigma^2(\rho, \theta, \alpha) = \lim_{n \rightarrow \infty} E(\Delta_n)^2 = (1 - a_1^\alpha)^{-2/\alpha} \frac{\rho\alpha^2 - 2\rho^2\alpha + \rho^3 + \rho\alpha^4}{\alpha^3(\alpha - \rho)^2}.$$

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