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NUMERICAL SIMULATION FOR THE SOLUTION OF NONLINEAR JAULENT-MIODEK COUPLED EQUATIONS USING QUARTIC B-SPLINE

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Abstract. In this paper, an efficient algorithm which combines quartic B-spline for the space discretization and classical finite difference methods such as Crank-Nicolson, for the time discretization to solve nonlinear Jaulent-Miodek coupled equations has been studied. Von-Neumann stability analysis shows that the numerical scheme is unconditionally stable. The numerical results show that the proposed method is a successful numerical technique for solving these problems. Also, comparing numerical solution with other methods demonstrates that our method is accurate and readily implemented.

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1. Introduction

Multi-scale problems in physics and engineering are often described in nonlinear partial differential equations [1]. These equations arise in many areas of science such as condense matter physics, fluid mechanics, plasma physics and optics [2]. The nonlinear Jaulent-Miodek equation (JM equation) is considered in [3] in the following form:

$$u_t + u_{xxx} + A(v)v_{xxx} + f(u, v, u_x, v_x, v_{xx}) = 0 \quad (1)$$

$$v_t + v_{xxx} + g(u, v, u_x, v_x) = 0 \quad (2)$$

with the initial and boundary conditions as

$$u(x,0) = g_1(x), v(x,0) = g_2(x),$$

$$u(a,t) = g_3(t), u_x(a,t) = g_4(t), u(b,t) = g_5(t),$$
$$v(a,t) = g_6(t), v_x(a,t) = g_7(t), v(b,t) = g_8(t), \quad (3)$$

where $(x,t) \in [a,b] \times [0,T]$.

Both equations (1) and (2) obey the oscillations related to energy-dependent Schrdinger potential as in [4]-[6]. These equations have many interesting characteristics, such as finite-band solution [7], Solitary solution and compacton-like solution [8]. The numerical analysis literature on JM equation contains little on the numerical solution of the problem (1-3). For example, D. Kaya adapted the Adomians Decomposition method to find the solitary solution of the JM equation [9]. Also, A.M. Wazwaz used the tanh-coth and sech methods to find a form of the traveling wave solution of the equation [10]. Raslan and Abusheer, discussed the Differential transform method to solve Jaulent-Miodek and the Hirota-Satsuma equations [11]. Other methods have been applied to find a compact form for the solution of JM equations such as Homotopy perturbation method [12]-[15] and the Exp-function method [16]. A recent study illustrated the variational iteration method and the Homotopy perturbation method for solving such kind of equations, [13],[17]. The aim of this paper is to adapt quartic B-spline method for the space discretization and Crank-Nicolson, for the time discretization for solving nonlinear JM equation (1-3). This algorithm has its advantages in handling nonlinear partial differential equations.

The paper is organized as follows: Section 2 contains a description of the quartic B-spline. The implementation of B-spline method for the solution of coupled Jaulent-Miodek equation is discussed in details in Section 3. While, in section 4, the stability analysis of the proposed scheme is presented. Finally, section 5 is devoted to conclude the results of the suggested method.

2. Quartic B-spline functions

To construct numerical solutions, consider nodal points (x_i, t_n) defined in the region $[a,b] \times [0,T]$ where

$$a = x_0 < x_1 < \ldots < x_{N-1} < x_N = b, \qquad x_{i+1} - x_i = h,$$

we can define

$$x_i = a + ih, \qquad i = 0, 1, 2, \ldots, N.$$

The quartic B-spline basis functions at the knots are given by

$$B_0(x) = \begin{cases} (x-x_{i-2})^4, & [x_{i-2}, x_{i-1}] \\ (x-x_{i-2})^4 - 5(x-x_{i-1})^4, & [x_{i-1}, x_i] \\ (x-x_{i-2})^4 - 5(x-x_{i-1})^4 + 10(x-x_i)^4, & [x_i, x_{i+1}] \\ (x-x_{i-2})^4 - 5(x-x_{i-1})^4, & [x_{i+1}, x_{i+2}] \\ (x-x_{i-2})^4, & [x_{i+2}, x_{i+3}] \\ 0 & otherwise \end{cases} \tag{4}$$

where

$$B_{i-1}(x) = B_0(x-(i-1)h), \qquad i = 2, 3, \ldots. \tag{5}$$

Using quartic B-spline functions in equation (5), the values of $B_i(x)$ and its derivatives at the nodal points can be calculated, which are tabulated in Table 1.

Table 1: Coefficients of B_i and its derivatives

x	x_{i-1}	x_i	x_{i+1}	x_{i+2}
B_i	1	11	11	1
hB'_i	4	12	-12	-4
$h^2B''_i$	12	-12	-12	12
$h^3B'''_i$	-24	72	-72	24

3. Solution of coupled JaulentMiodek equations

To apply the proposed method, we discretize the time derivative in the usual finite difference way and applying Crank-Nicolson method. Let $\{0 = t_0 < t_1 < t_2 < \ldots < t_n = T\}$ be a partition on $[0, T]$, and let $\triangle t$ be the step size defined as

$$t_n = n\triangle t, \quad \triangle t = \frac{T}{M}, \quad n = 0, 1, 2, \ldots, M,$$

where M is the number of the time steps. We define the usual finite difference and Crank-Nicolson method as follows

$$t_{n+\frac{1}{2}} = (n+\frac{1}{2})\triangle t, \quad U^n = u(x, t_n), \quad V^n = v(x, t_n),$$

$$U_t^n = \frac{(U^{n+1} - U^n)}{\triangle t}, \quad V_t^n = \frac{(V^{n+1} - V^n)}{\triangle t}, \quad U_x^{n+\frac{1}{2}} = \frac{(U_x^{n+1} + U_x^n)}{2},$$

$$V_x^{n+\frac{1}{2}} = \frac{(V_x^{n+1} + V_x^n)}{2},$$

$$U_{xx}^n = \frac{(U_{xx}^{n+1} - U_{xx}^n)}{2}, \quad V_{xx}^n = \frac{(V_{xx}^{n+1} - V_{xx}^n)}{2}, \quad U_{xxx}^{n+\frac{1}{2}} = \frac{(U_{xxx}^{n+1} + U_{xxx}^n)}{2},$$

$$V_{xxx}^{n+\frac{1}{2}} = \frac{(V_{xxx}^{n+1} + V_{xxx}^n)}{2}, \tag{6}$$

applying to both equations (1) and (2) then we get

$$\frac{U^{n+1} - U^n}{\triangle t} + U_{xxx}^{n+\frac{1}{2}} + A(V^{n+\frac{1}{2}})V_{xxx}^{n+\frac{1}{2}} + f(u,v,u_x,v_x,v_{xx})^{n+\frac{1}{2}} = 0, \tag{7}$$

$$\frac{V^{n+1} - V^n}{\triangle t} + V_{xxx}^{n+\frac{1}{2}} + g(u,v,u_x,v_x)^{n+\frac{1}{2}} = 0. \tag{8}$$

Then equations (7) and (8) can be rewritten in the following form after simplification

$$\frac{2U^{n+1}}{\triangle t} + U_{xxx}^{n+1} + A(V^{n+1})V_{xxx}^{n+1} + f(u,v,u_x,v_x,v_{xx})^{n+1}$$
$$= \frac{2U^n}{\triangle t} - U_{xxx}^n - A(V^n)V_{xxx}^n + f(u,v,u_x,v_x,v_{xx})^n, \tag{9}$$

$$\frac{2V^{n+1}}{\triangle t} + V_{xxx}^{n+1} + g(u,v,u_x,v_x)^{n+1} = \frac{2V^n}{\triangle t} - V_{xxx}^n - g(u,v,u_x,v_x)^n. \tag{10}$$

If we define the terms

$$A(V) = \frac{3}{2}V, \quad f(u,v,u_x,v_x,v_{xx}) = \frac{9}{2}VV_x - 6UU_x - 6UVV_x - \frac{3}{2}U_xV^2 \quad and$$

$$g(u,v,u_x,v_x) = -6U_xV - \frac{15}{2}V_xV^2,$$

then equations (9) and (10) become

$$\frac{2}{\triangle t}U^{n+1} + U_{xxx}^{n+1} + \frac{3}{2}V^{n+1}V_{xxx}^{n+1} + f^{n+1} = F^n, \tag{11}$$

$$\frac{2}{\triangle t}V^{n+1} + V_{xxx}^{n+1} + g^{n+1} = G^n, \tag{12}$$

where

$$f^{n+1} = \frac{9}{2}V_x^{n+1}V_{xx}^{n+1} - 6U^{n+1}U_x^{n+1} - 6(UV_xV)^{n+1} - \frac{3}{2}(U)_x^{n+1}(V^2)^{n+1},$$

$$F^n = \frac{2}{\triangle t}U^n - U^n_{xxx} - (VV_{xxx})^n - \frac{9}{2}V^n_x V^n_{xx} + (6U)^n U^n_x + 6(UV_xV)^n + (\frac{3}{2}U)^n_x (V^2)^n,$$

$$g^{n+1} = -6(U_xV)^{n+1} - 6(UV_x)^{n+1} - \frac{15}{2}(V_xV^2)^{n+1},$$

$$G^n = \frac{2}{\triangle t}V^n - V^n_{xxx} + 6(U_xV)^n + 6(UV_x)^n + \frac{15}{2}(V_xV^2)^n.$$

In the Crank-Nicolson scheme, the time stepping process is half explicit and half implicit. So the method is better than simple finite difference method. The nonlinear terms in both equations (11) and (12) are linearized using the form given by Rubin and Graves [18]

$$(VV_{xxx})^{n+1} = V^{n+1}V^n_{xxx} + V^nV^{n+1}_{xxx} - (VV_{xxx})^n$$

$$(V_xV_{xx})^{n+1} = (V_x)^{n+1}V^n_{xx} + (V_x)^nV^{n+1}_{xx} - (V_xV_{xx})^n,$$

$$(UU_x)^{n+1} = U^{n+1}U^n_x + U^nU^{n+1}_x - (UU_x)^n,$$

$$(U^2U_x)^{n+1} = (U^2)^nU^{n+1}_x + 2U^nU^n_xU^{n+1} - 2(U^2)^nU^n_x$$

$$(V^2V_x)^{n+1} = (V^2)^nV^{n+1}_x + 2V^nV^n_xV^{n+1} - 2(V^2)^nV^n_x,$$

$$(UV_xV)^{n+1} = [(U^{n+1}V^n_x + U^nV^{n+1}_x - (UV_x)^n]V^n + (UV_x)^nV^{n+1} - (UV_xV)^n.$$

Expressing $U(x,t)$ and $V(x,t)$ by using quartic B-spline functions $B_0(x)$ and the time dependent parameters $\delta_m(t)$ and $\sigma_m(t)$ for $u(x,t)$ and $v(x,t)$ respectively, the approximate solutions can be written as:

$$U_m(x,t) = \sum_{m=-2}^{N+1} \delta_m(t)B_m(x), \quad V_m(x,t) = \sum_{m=-2}^{N+1} \sigma m(t)B_m(x), \tag{13}$$

then by using the approximate solution in equation (13) and quartic B-spline functions in equation (4), the approximate values of u and v denoted by U, V and their derivatives up to third order are determined in terms of the time parameters $\delta_m(t)$ and $\sigma_m(t)$, respectively as

$$U_m = \delta_{m-2} + 11\delta_{m-1} + 11\delta_m + \delta_{m+1}, \quad hU'_m = 4\delta_{m-2} + 12\delta_{m-1} - 12\delta_m - 4\delta_{m+1},$$

$$h^2U''_m = 12\delta_{m-2} - 12\delta_{m-1} - 12\delta_m + 12\delta_{m+1},$$

$$h^3U'''_m = -24\delta_{m-2} + 72\delta_{m-1} - 72\delta_m + 24\delta_{m+1},$$

$$V_m = \sigma_{m-2} + 11\sigma_{m-1} + 11\sigma_m + \sigma_{m+1}, \quad hV'_m = 4\sigma_{m-2} + 12\sigma_{m-1} - 12\sigma_m - 4\sigma_{m+1},$$

$$h^2V''_m = 12\sigma_{m-2} - 12\sigma_{m-1} - 12\sigma_m + 12\sigma_{m+1},$$

$$h^3V'''_m = -24\sigma_{m-2} + 72\sigma_{m-1} - 72\sigma_m + 24\sigma_{m+1}.$$

On substituting the approximate solution of U and V and thier derivatives and the nonlinear terms yields the following difference equation with the variables δ_m and σ_m.

$$\alpha_1\delta_{m-2}^{n+1}+\alpha_2\delta_{m-1}^{n+1}+\alpha_3\delta_{m}^{n+1}+\alpha_4\delta_{m+1}^{n+1}+\beta_1\sigma_{m-2}^{n+1}+\beta_2\sigma_{m-1}^{n+1}+\beta_3\sigma_{m}^{n+1}+\beta_4\sigma_{m+1}^{n+1}=\eta_m \quad (14)$$

$$\alpha_5\delta_{m-2}^{n+1}+\alpha_6\delta_{m-1}^{n+1}+\alpha_7\delta_{m}^{n+1}+\alpha_8\delta_{m+1}^{n+1}+\beta_5\sigma_{m-2}^{n+1}+\beta_6\sigma_{m-1}^{n+1}+\beta_7\sigma_{m}^{n+1}+\beta_8\sigma_{m+1}^{n+1}=\mu_m \quad (15)$$

where $m=0,1,\ldots,N$ and

$$\alpha_1 = 1-X_1-Y_1-Z_1-M_1-G_1-F_1,$$

$$\alpha_2 = 11+3X_1-11Y_1-3Z_1-11M_1-11G_1-3F_1,$$

$$\alpha_3 = 11-3X_1-11Y_1+3Z_1-11M_1-11G_1+3F_1, \quad \alpha_4 = 1+X_1-Y_1+Z_1-M_1-G_1+F_1$$

$$\alpha_5=\frac{-V^n}{h^2}X_1-\frac{h^2}{3}Y_2, \quad \alpha_6=\frac{-3V^n}{h^2}X_1-11\frac{h^2}{3}Y_2, \quad \alpha_7=\frac{3V^n}{h^2},X_1-11h\frac{h^2}{3}Y_2,$$

$$\alpha_8=\frac{V^n}{h^2}X_1-\frac{h^2}{3}Y_2,$$

$$\beta_1=X_2+Y_2-Z_1, \quad \beta_2=11X_2-Y_2-3Z_1, \quad \beta_3=11X_2-Y_2+3Z_1, \quad \beta_4=X_2+Y_2+Z_1$$

$$\beta_5=1-X_1-Y_1-Z_1-5F_1, \quad \beta_6=11+3X_1-11Y_1-3Z_1-15F_1$$

$$\beta_7=11-3X_1-11Y_1+3Z_1+15F_1, \quad \beta_8=1+X_1-Y_1+Z_1+5F_1$$

and

$$X_1=\frac{12\triangle t}{h^3}, \quad X_2=\frac{3\triangle t}{h}V_{xxx}^n, \quad Y_1=3\triangle t U_x^n, \quad Y_2=\frac{9\triangle t}{h^2}V_x^n, \quad Z_1=\frac{12\triangle t}{h}U^n,$$

$$M_1=3\triangle t V^n V_x^n, \quad G_1=3\triangle t(UV_x)^n, \quad F_1=\frac{3\triangle t}{h}(V^2)^n,$$

$$\eta_m=U^n-\frac{\triangle t}{2}U_{xxx}^n-3\triangle t V^n(UV_x)^n-d_m^2U_x^n,$$

$$\mu_m=V^n-\frac{\triangle t}{2}V_{xxx}^n+\frac{15}{4}(v^2u_x)^n+15\frac{15\triangle t}{4}d_m^2V_x^n,$$

$$d_m^2=(V^2)^{n+1}=(\sigma_{m-1}^{n+1}+11\sigma_m^{n+1}+11\sigma_{m+1}^{n+1}+\sigma_{m+2}^{n+1})^2.$$

The above system given by equations (14) and (15) can be represented in matrix form as follows

$$\mathbf{A}\delta^*=\mathbf{C}, \quad (16)$$

where

$$A=\begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix}, \quad \delta^*=\begin{pmatrix}\delta\\ \sigma\end{pmatrix}, \quad C=\begin{pmatrix}C_1\\ C_2\end{pmatrix}$$

and the matrices A_1, A_2, B_1 and B_2 are defined as

$$A_1 = \begin{pmatrix} 1 & 11 & 11 & 1 & 0 & 0 & 0 \\ \frac{4}{h} & \frac{12}{h} & \frac{-12}{h} & \frac{-4}{h} & 0 & 0 & 0 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 1 & 11 & 11 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 11 & 11 & 1 & 0 & 0 & 0 \\ \frac{4}{h} & \frac{12}{h} & \frac{-12}{h} & \frac{-4}{h} & 0 & 0 & 0 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & 0 & 0 & 0 \\ 0 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & 0 & 0 \\ 0 & 0 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 1 & 11 & 11 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 1 & 11 & 11 & 1 & 0 & 0 & 0 \\ \frac{4}{h} & \frac{12}{h} & \frac{-12}{h} & \frac{-4}{h} & 0 & 0 & 0 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 & 0 & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 1 & 11 & 11 & 1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 11 & 11 & 1 & 0 & 0 & 0 \\ \frac{4}{h} & \frac{12}{h} & \frac{-12}{h} & \frac{-4}{h} & 0 & 0 & 0 \\ \beta_5 & \beta_6 & \beta_7 & \beta_8 & 0 & 0 & 0 \\ 0 & \beta_5 & \beta_6 & \beta_7 & \beta_8 & 0 & 0 \\ 0 & 0 & \beta_5 & \beta_6 & \beta_7 & \beta_8 & 0 \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 1 & 11 & 11 & 1 \end{pmatrix}$$

$$\delta = \begin{pmatrix} \delta_-2 \\ \delta_-1 \\ \delta_0 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \delta_N \\ \delta_{n+1} \end{pmatrix}, \quad \sigma = \begin{pmatrix} \sigma_-2 \\ \sigma_-1 \\ \sigma_0 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \sigma_N \\ \sigma_{n+1} \end{pmatrix}, \quad C_1 = \begin{pmatrix} g_3(t) \\ g_4(t) \\ \eta(x_0,t,u,u_x) \\ \eta(x_1,t,u,u_x) \\ \eta(x_2,t,u,u_x) \\ \cdots \\ \cdots \\ \cdots \\ g_5(t) \end{pmatrix}, \quad C_2 = \begin{pmatrix} g_6(t) \\ g_7(t) \\ \mu(x_0,t,u,u_x) \\ \mu(x_1,t,u,u_x) \\ \mu(x_2,t,u,u_x) \\ \cdots \\ \cdots \\ \cdots \\ g_8(t) \end{pmatrix}.$$

The above system that obtained from equation (16) consists of $(2N+2)$ equations in $(2N+8)$ unknowns, so six additional constraints are required for a unique solution. These equations are obtained by imposing the boundary conditions given by equation (3). Thus the system can be solved as in [19].

4. Stability analysis of the method

In this section, we will investigate the stability of the proposed method by applying von-Neumann method. To apply this method, we have linearized the nonlinear terms into equations (14) and (15) by considering U and V as a local constants γ_1 and γ_2 respectively. Substituting the approximate solution for U and V and their derivatives at these knots in the modified equations after discretizing the time derivatives in the usual finite difference way and applying Crank-Nicolson scheme yields a difference equations with the variables δ_m and σ_m given as

$$w_1\delta_{m-1}^{n+1}+w_2\delta_m^{n+1}+w_3\delta_{m+1}^{n+1}+w_4\delta_{m+2}^{n+1}+w(\sigma_{m-1}^{n+1}+3\sigma_m^{n+1}-3\sigma_{m+1}^{n+1}-\sigma_{m+2}^{n+1})=$$
$$(1-\frac{w}{h^2})\delta_{m-1}^n+(11+\frac{3w}{h^2})\delta_m^n+(11-\frac{3w}{h^2})\delta_{m+1}^n+(1+\frac{w}{h^2})\delta_{m+2}^n, \tag{17}$$

$$w_5\sigma_{m-1}^{n+1}+w_6\sigma_m^{n+1}+w_7\sigma_{m+1}^{n+1}+w_8\sigma_{m+2}^{n+1}+w(\delta_{m-1}^{n+1}+3\delta_m^{n+1}-3\delta_{m+1}^{n+1}-\delta_{m+2}^{n+1})=$$
$$(1-\frac{w}{h^2})\sigma_{m-1}^n+(11+\frac{3w}{h^2})\sigma_m^n+(11-\frac{3w}{h^2})\sigma_{m+1}^n+(1+\frac{w}{h^2})\sigma_{m+2}^n, \tag{18}$$

where

$$w_1=1-\frac{12\triangle t}{h^3}-\frac{3\triangle t}{h}(\gamma_2)^2-\frac{12\triangle t}{h}\gamma_1,\quad w_2=11+\frac{36\triangle t}{h^3}-\frac{9\triangle t}{h}(\gamma_2)^2-\frac{36\triangle t}{h}\gamma_1,$$

$$w_3=11-\frac{36\triangle t}{h^3}+\frac{9\triangle t}{h}(\gamma_2)^2+\frac{36\triangle t}{h}\gamma_1,\quad w_4=1+\frac{12\triangle t}{h^3}+\frac{3\triangle t}{h}(\gamma_2)^2+\frac{12\triangle t}{h}\gamma_1,$$

$$w_5=1-\frac{12\triangle t}{h^3}-\frac{15\triangle t}{h}(\gamma_2)^2-\frac{12\triangle t}{h}\gamma_1,\quad w_6=11+\frac{36\triangle t}{h^3}-\frac{45\triangle t}{h}(\gamma_2)^2-\frac{36\triangle t}{h}\gamma_1,$$

$$w_7=11-\frac{36\triangle t}{h^3}+\frac{45\triangle t}{h}(\gamma_2)^2+\frac{36\triangle t}{h}\gamma_1,\quad w_8=1+\frac{12\triangle t}{h^3}+\frac{15\triangle t}{h}(\gamma_2)^2+\frac{12\triangle t}{h}\gamma_1,$$

$$w=-\frac{12\triangle t}{h}$$

Now substituting $\delta_m^n=A\xi^n exp(im\phi h)$ and $\sigma_m^n=B\xi^n exp(im\phi h)$ into equations (17) and (18) and simplifying, where A and B are the harmonic amplitude, is the mode number, h is the element size and $i=\sqrt{-1}$, we obtain

$$\xi_1=\frac{(X_1+iY_1)}{(X_2+iY_2)},\quad \xi_2=\frac{(X_+iY_3)}{(X_4+iY_4)}$$

where

$$X_1=A((12+4w_1')\cos\phi h+(1-w_1')\cos 2\phi h+11-3w_1'),$$
$$X_2=(A(w_1+w_3)+B)\cos\phi h+(w_4-3)\cos 2\phi h+w_2,$$

$$X_3 = B((12+4w)\cos\phi h + (1-w)\cos 2\phi h + 11 - 3w),$$

$$X_4 = (-2Aw_1' + B(w_5+w_7))\cos\phi h + (Aw_1' + Bw_8)\cos 2\phi h + 3Aw_1' + Bw_6,$$

$$X_4 = (-2Aw_1' + B(w_5+w_7))\cos\phi h + (Aw_1' + Bw_8)\cos 2\phi h + 3Aw_1' + Bw_6,$$

$$Y_2 = (A(-w_1+w_3) - 4B)\sin\phi h + (Aw_4 - B)\sin 2\phi h,$$

$$Y_3 = B((10+2w)\sin\phi h + (1-w)\sin 2\phi h),$$

$$Y_4 = (-4Aw_1' + B(w_5 - w_7))\sin\phi h + (Aw_1' + Bw_8)\sin 2\phi h$$

We note that $|\xi_1| = \sqrt{\xi_1\bar{\xi_1}} = \sqrt{\frac{X_1^2+Y_1^2}{X_2^2+Y_2^2}} \leq 1$ and $|\xi_2| = \sqrt{\xi_2\bar{\xi_2}} = \sqrt{\frac{X_3^2+Y_3^2}{X_4^2+Y_4^2}} \leq 1$which implies that the scheme is unconditionally stable for both equations.

5. Numerical results and discussion

To gain insight into the performance of the suggested method, we use L_∞ errors for both u and v obtained by formula given by

$$L_U\infty = \max_j |u_j - U_j|, \quad L_V\infty = \max_j |v_j - V_j|.$$

Consider nonlinear Jaulent-Miodek coupled equations given by equations (1) and (2) with the following initial and boundary conditions
Initial conditions

$$u(x,0) = s - \frac{1}{2}b_0 ksech(kx) - \frac{3}{4}c_2 sech(kx)^2, \quad v(x,0) = b_0 + ksech(kx),$$

and boundary conditions:

$$u(a,t) = s - \frac{1}{2}b_0 ksech(k(Rt+a)) - \frac{3}{4}c_2 sech^2(k(Rt+a)),$$

$$v(a,t) = b_0 + ksech(k(Rt+a)),$$

$$u_x(a,t) = \frac{b_0 k^2 sinh(k(Rt+a))}{2cosh^2(kRt)} + \frac{3c_2 ksinh(k(Rt+a))}{2cosh^3(kRt)},$$

$$v_x(a,t) = \frac{-(k^2 sinh(k(Rt+a)))}{cosh^2(k(Rt+a))},$$

$$u(b,t) = s - \frac{b_0}{2ksech(k(Rt+b))} - \frac{3c_2}{4} sech^2(k(Rt+b)),$$

$$v(b,t) = b_0 + ksech(k(Rt+b)),$$

with the exact solution as given in [9]

$$u(x,t) = s - \frac{b_0}{2} k sech(k(Rt+x)) - \frac{3c_2}{4} sech^2(k(Rt+x)),$$

$$v(x,t) = b_0 + k sech(k(Rt+x)),$$

where $k = \sqrt{c_2}, \quad R = \frac{1}{2}(b_0^2 + c_2^2), \quad s = \frac{1}{4}(c_2 - b_0^2)$

To demonstrate the accuracy of the method, we assign $b_0 = c_2 = 0.01$ with the maximum error for both $u(x,t)$ and $v(x,t)$ are shown in the Tables (2-7) for different time and space levels. Also for the sake of comparison, Table 8, shows that our method has the advantage of providing better results than the results given in [9]. In Fig. (1), we illustrate the numerical and exact solutions for both $u(x,t)$ and $v(x,t)$ and the error for both of them. Also, in Fig. (2) and Fig. (3) results are depicted graphically for both $u(x,t)$ and $v(x,t)$ for different time levels and space levels. These figures reveal that the numerical and exact solutions are in good agreement with each other.

Table 2: Absolute maximum error for $u(x,t)$ for $a = 0 < x < 1 = b, \quad 0 < t < 1$

(x_i, t_n)	0.2	0.4	0.6	0.8	1
0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.1	7.1909E-11	3.5976E-11	3.4800E-11	3.4882E-11	3.5004E-11
0.2	2.8149E-10	1.3934E-10	1.3464E-10	1.3491E-10	1.3535E-10
0.3	6.0828E-10	2.9688E-10	2.8648E-10	2.8699E-10	2.8784E-10
0.4	1.0128E-09	4.8679E-10	4.6910E-10	4.6980E-10	4.7110E-10
0.5	1.4323E-09	6.7858E-10	6.5305E-10	6.5389E-10	6.5557E-10
0.6	1.7790E-09	8.3303E-10	8.0079E-10	8.0166E-10	8.0360E-10
0.7	1.9413E-09	9.0232E-10	8.6674E-10	8.6754E-10	8.6951E-10
0.8	1.7890E-09	8.3035E-10	7.9740E-10	7.9803E-10	7.9974E-10
0.9	1.1846E-09	5.5323E-10	5.3147E-10	5.3184E-10	5.3292E-10
1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Table 3: Absolute maximum error for $v(x,t)$ for $a = 0 < x < 1 = b$, $0 < t < 1$

(x_i, t_n)	0.2	0.4	0.6	0.8	1
0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.1	1.6105E-10	1.0982E-10	1.0895E-10	1.0930E-10	1.0969E-10
0.2	6.2775E-10	4.2510E-10	4.2150E-10	4.2275E-10	4.2413E-10
0.3	1.3488E-09	9.0507E-10	8.9689E-10	8.9933E-10	9.0203E-10
0.4	2.2318E-09	1.4828E-09	1.4686E-09	1.4723E-09	1.4764E-09
0.5	3.1376E-09	2.0654E-09	2.0445E-09	2.0492E-09	2.0546E-09
0.6	3.8782E-09	2.5338E-09	2.5071E-09	2.5125E-09	2.5187E-09
0.7	4.2181E-09	2.7435E-09	2.7138E-09	2.7192E-09	2.7255E-09
0.8	3.8832E-09	2.5245E-09	2.4968E-09	2.5015E-09	2.5070E-09
0.9	2.5762E-09	1.6826E-09	1.6643E-09	1.6672E-09	1.6707E-09
1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Table 4: Absolute maximum error for $u(x,t)$ for $a = 0 < x < 1 = b$, $0 < t < 10$

(x_i, t_n)	1.0	3.0	5.0	7.0	9.0
0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.1	1.6105E-10	1.0982E-10	1.0895E-10	1.0930E-10	1.0969E-10
0.2	6.2775E-10	4.2510E-10	4.2150E-10	4.2275E-10	4.2413E-10
0.3	1.3488E-09	9.0507E-10	8.9689E-10	8.9933E-10	9.0203E-10
0.4	2.2318E-09	1.4828E-09	1.4686E-09	1.4723E-09	1.4764E-09
0.5	3.1376E-09	2.0654E-09	2.0445E-09	2.0492E-09	2.0546E-09
0.6	3.8782E-09	2.5338E-09	2.5071E-09	2.5125E-09	2.5187E-09
0.7	4.2181E-09	2.7435E-09	2.7138E-09	2.7192E-09	2.7255E-09
0.8	3.8832E-09	2.5245E-09	2.4968E-09	2.5015E-09	2.5070E-09
0.9	2.5762E-09	1.6826E-09	1.6643E-09	1.6672E-09	1.6707E-09
1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Table 5: Absolute maximum error for $v(x,t)$ for $a = 0 < x < 1 = b, \quad 0 < t < 10$

(x_i, t_n)	1.0	3.0	5.0	7.0	9.0
0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.1	1.0076E-10	1.3081E-10	1.3469E-10	1.3856E-10	1.4243E-10
0.2	3.8205E-10	4.9923E-10	5.1301E-10	5.2676E-10	5.4051E-10
0.3	7.9674E-10	1.0498E-09	1.0770E-09	1.1040E-09	1.1311E-09
0.4	1.2798E-09	1.7015E-09	1.7428E-09	1.7840E-09	1.8252E-09
0.5	1.7508E-09	2.3475E-09	2.4013E-09	2.4549E-09	2.5085E-09
0.6	2.1154E-09	2.8558E-09	2.9177E-09	2.9794E-09	3.0411E-09
0.7	2.2641E-09	3.0692E-09	3.1323E-09	3.1952E-09	3.2582E-09
0.8	2.0690E-09	2.8059E-09	2.8608E-09	2.9155E-09	2.9702E-09
0.9	1.3770E-09	1.8596E-09	1.8942E-09	1.9288E-09	1.9634E-09
1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Table 6: Absolute maximum error for $u(x,t)$ for $a = -1 < x < 1 = b, \quad 0 < t < 1$

(x_i, t_n)	0.2	0.4	0.6	0.8	1.0
-1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
-0.8	1.0076E-10	1.3081E-10	1.3469E-10	1.3856E-10	1.4243E-10
-0.6	3.8205E-10	4.9923E-10	5.1301E-10	5.2676E-10	5.4051E-10
-0.4	7.9674E-10	1.0498E-09	1.0770E-09	1.1040E-09	1.1311E-09
-0.2	1.2798E-09	1.7015E-09	1.7428E-09	1.7840E-09	1.8252E-09
0.0	1.7508E-09	2.3475E-09	2.4013E-09	2.4549E-09	2.5085E-09
0.2	2.1154E-09	2.8558E-09	2.9177E-09	2.9794E-09	3.0411E-09
0.4	2.2641E-09	3.0692E-09	3.1323E-09	3.1952E-09	3.2582E-09
0.6	2.0690E-09	2.8059E-09	2.8608E-09	2.9155E-09	2.9702E-09
0.8	1.3770E-09	1.8596E-09	1.8942E-09	1.9288E-09	1.9634E-09
1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Table 7: Absolute maximum error for $v(x,t)$ for $a = -1 < x < 1 = b, \quad 0 < t < 1$

(x_i, t_n)	0.2	0.4	0.6	0.8	1.0
-1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
-0.8	4.46E-08	3.37E-08	4.97E-08	6.89E-08	1.06E-07
-0.6	1.53E-07	1.34E-07	1.95E-07	2.72E-07	4.18E-07
-0.4	2.90E-07	2.92E-07	4.21E-07	5.94E-07	9.05E-07
-0.2	4.25E-07	4.89E-07	7.03E-07	9.97E-07	1.51E-06
0.0	5.29E-07	6.92E-07	9.93E-07	1.42E-06	2.13E-06
0.2	5.78E-07	8.56E-07	1.23E-06	1.76E-06	2.64E-06
0.4	5.52E-07	9.24E-07	1.33E-06	1.91E-06	2.85E-06
0.6	4.43E-07	8.37E-07	1.21E-06	1.74E-06	2.59E-06
0.8	2.53E-07	5.41E-07	7.82E-07	1.13E-06	1.67E-06
1.0	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

Table 8: Comparison between errors of different methods for both $u(x,t)$ and $v(x,t)$, $0 < x < 1, \quad 0 < t < 1$

(x_i, t_n)	Adomians method [19]	Our method
$\lvert u_j - U_j \rvert$		
(0.1,0.1)	$1.0329E-06$	$1.471E-10$
(0.2,0.2)	$4.1263E-06$	$2.814E-10$
(0.3,0.3)	$9.2645E-06$	$2.221E-10$
(0.4,0.4)	$1.6421E-05$	$4.868E-10$
(0.5,0.5)	$2.5561E-05$	$6.462E-10$
$\lvert v_j - V_j \rvert$		
(0.1,0.1)	$1.8383E-06$	$2.355E-10$
(0.2,0.2)	$7.3454E-06$	$6.277E-10$
(0.3,0.3)	$1.6497E-05$	$8.212E-10$
(0.4,0.4)	$2.9256E-05$	$1.4828E-09$
(0.5,0.5)	$4.5567E-05$	$2.0369E-09$

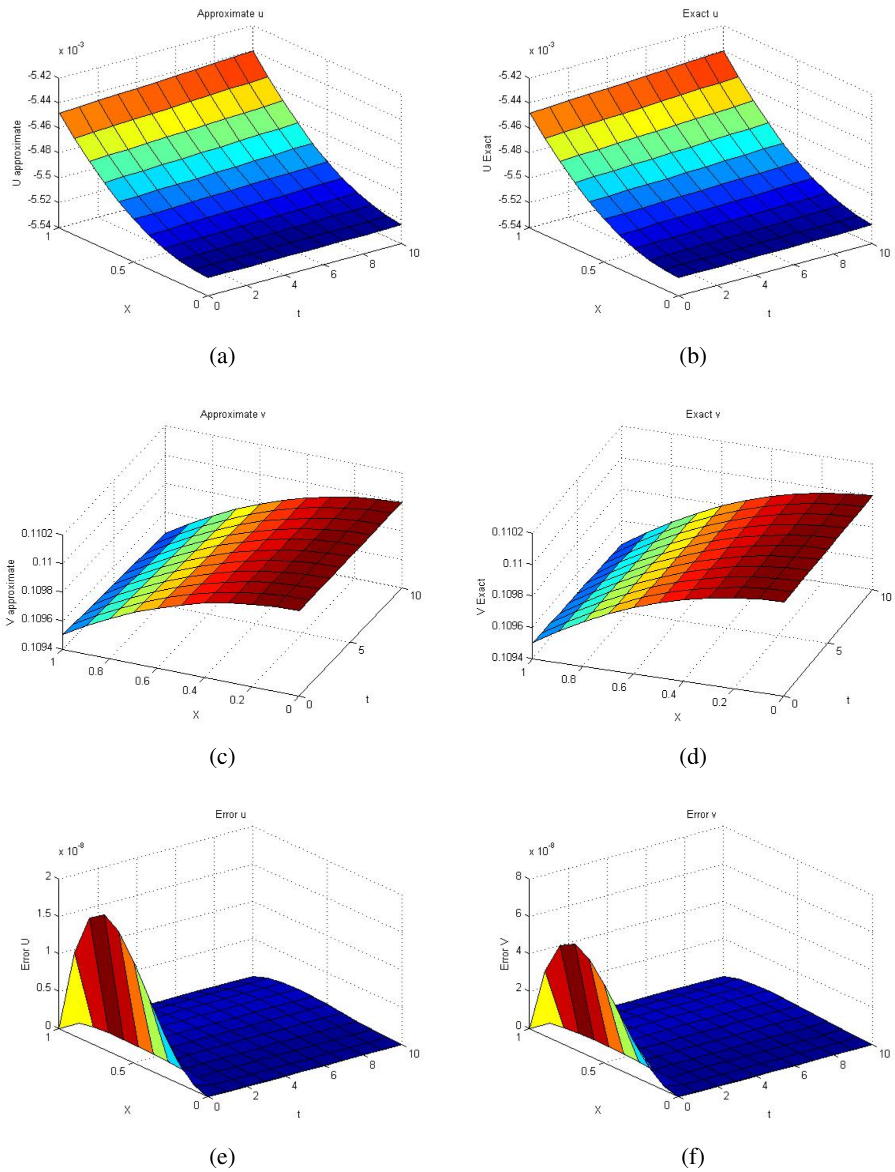


Figure 1: (a) Approximate solution for $u(x,t)$, (b) Approximate solution for $v(x,t)$, (c) Exact solution for $u(x,t)$, (d) Exact solution for $v(x,t)$, (e) Absolute error for $u(x,t)$, (f) Absolute error for $v(x,t)$ for $0 < x < 1$ and $0 < t < 10$.

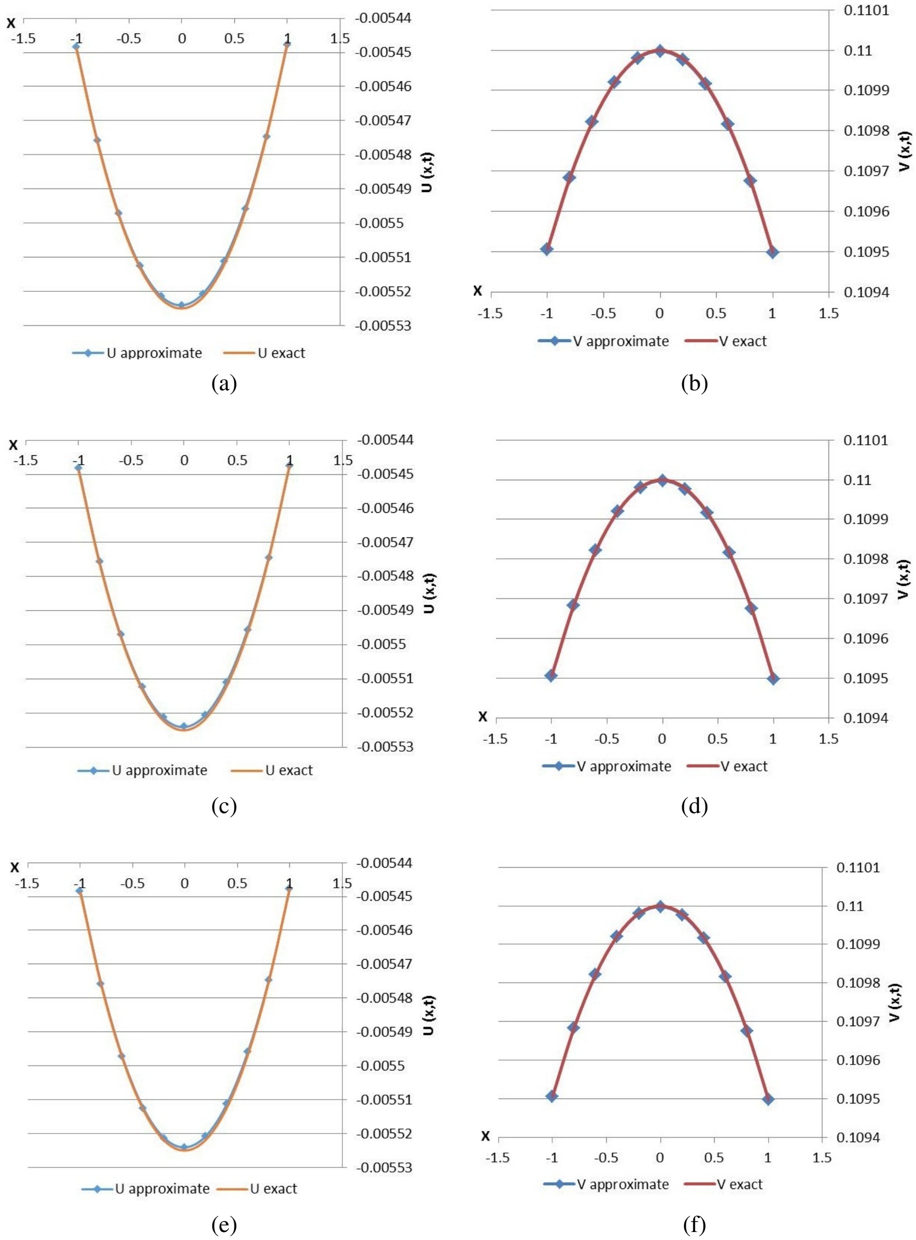
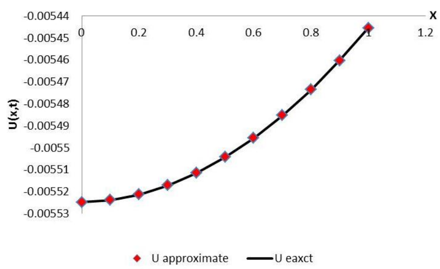
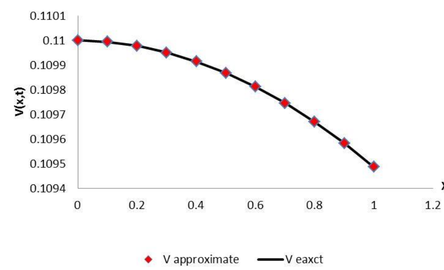


Figure 2: Approximate versus Exact solution for both $u(x,t)$ and $v(x,t)$ solutions for $-1 < x < 1$ and (g), (h) at $t = 2$, (i), (j) at $t = 6$, (k), (l) at $t = 8$.



(a)



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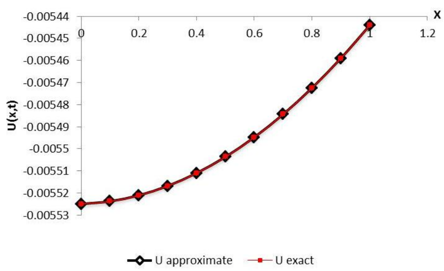

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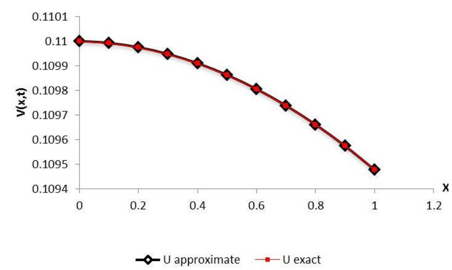

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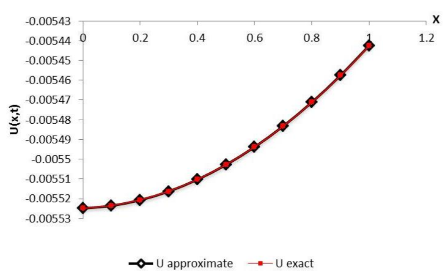
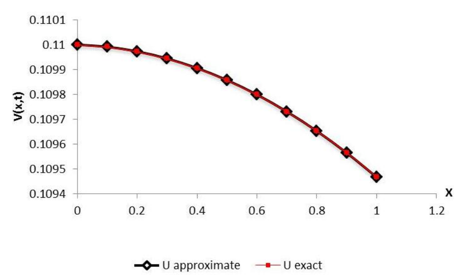


(e)



(f)

Figure 3: Approximate versus Exact solution for both $u(x,t)$ and $v(x,t)$ solutions for $0 < x < 1$ and (m), (n) at $t = 2$, (o), (p) at $t = 5$, (q), (s) at $t = 7$.

6. Conclusion

In this paper, a numerical technique for the nonlinear Jaulent Miodek equation system is presented using Quartic B-spline method. The stability of the method has been investigated using Von-Neumann stability analysis showing that the system is unconditionally stable. The results reported in this paper prove that this simulation provides superior results than the results given in [9]. The authors believe that this method is an efficient technique for solving nonlinear partial differential systems.

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