

On the stability of some fixed point iteration procedures with errors

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Abstract. Several results have been obtained on the stability of Mann, Ishikawa and Kirk iteration procedures (without errors) when dealing with different classes of quasi-contractive operators. In this paper, we consider the stability of Mann, Ishikawa and Kirk iteration procedures *with errors* and show that they are *almost stable* with respect to some classes of quasi-contractive maps. The results obviously generalise the results of several authors.

Resumen. Varios resultados se han obtenido sobre la estabilidad del proceso de iteración de Mann, Ishikawa y Kirk (sin errores) cuando se trata con clases diferentes de operadores cuasi-contractivos. En este trabajo, consideramos la estabilidad del proceso de la iteración de Mann, Kirk Ishikawa con errores y se prueba que son semi estable con respecto a algunas clases de mapas cuasi-contractivo. Los resultados, obviamente, generalizar los resultados de varios autores.

1. Introduction

Suppose X is a normed linear space and T is a self map of X . Suppose $x_0 \in X$ and $x_{n+1} = f(T, x_n)$ defines an iteration procedure which yields a sequence of points $\{x_n\}$ in X . Suppose $F(T) = \{x \in X | Tx = x\} \neq \phi$ and that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $p \in F(T)$. Suppose $\{y_n\}_{n=0}^{\infty}$ is a sequence in X and $\{\epsilon_n\}$ is a sequence in $[0, \infty)$ given by $\epsilon_n = \|y_{n+1} - f(T, y_n)\|$. If $\lim_{n \rightarrow \infty} \epsilon_n = 0$ implies that $\lim_{n \rightarrow \infty} y_n = p$, then the iteration procedure defined by $x_{n+1} = f(T, x_n)$ is said to be T -stable or *stable* with respect to T . If $\sum_{n=0}^{\infty} \epsilon_n < \infty$ implies $y_n \rightarrow p$, then the iteration procedure is said to be *almost T-stable*. Clearly any T -stable iteration procedure is almost T -stable, but an almost T -stable iteration procedure may fail to be T -stable.

The stability of several iteration procedures for certain contractive operators has been studied by several authors. Harder and Ricks [2] proved the stability of Picard and Mann iteration procedures for the quasi-contractive operators, called Zamfirescu operators, satisfying the following condition:

Let (X, d) be a complete metric space and $T : X \rightarrow X$ a map for which there exists the real numbers a, b and c satisfying $0 < a < 1$, $0 < b, c < 1/2$ such that for each pair $x, y \in X$, at least one of the following is true:

- (i) $d(Tx, Ty) \leq ad(x, y)$;
- (ii) $d(Tx, Ty) \leq b[d(x, Tx) + d(y, Ty)]$;
- (iii) $d(Tx, Ty) \leq c[d(x, Ty) + d(y, Tx)]$.

In [13], Rhoades considered the stability of some iteration procedures for the contractive operators satisfying the following definition:

there exists a constant $c \in [0, 1)$ such that for each $x, y \in X$,

$$d(Tx, Ty) \leq c \max\{d(x, y), \frac{1}{2}[d(x, Tx) + d(y, Ty)], d(x, Ty), d(y, Tx)\}. \quad (0.1)$$

The Zamfirescu operators are independent of the operators satisfying (0.1). However, it was shown by Rhoades, B. E. 1976 [Comments on fixed point iteration methods. J. Math. Anal. Appl., 56:741-750], that if T satisfies (0.1) or if T is a Zamfirescu operator, then for $\delta \in [0, 1)$

$$d(Tx, Ty) \leq \frac{\delta}{1-\delta}d(x, Tx) + \delta d(x, y). \quad (0.2)$$

Osilike [11] considered a more general contractive definition: there exists $L \geq 0$, $a \in [0, 1)$ such that, for each $x, y \in X$

$$d(Tx, Ty) \leq Ld(x, Tx) + ad(x, y). \quad (0.3)$$

Recently, Imoru and Olatinwo [3] proved the stability of the Picard and the Mann iteration process for the following operator which is more general than the one introduced by Osilike [11]. The operator satisfies the following contractive definition: there exist a constant $q \in [0, 1)$ and a monotone increasing function $\Phi : R_+ \rightarrow R_+$ with $\Phi(0) = 0$ such that for each $x, y \in X$,

$$\|Tx - Ty\| \leq \Phi(\|x - Tx\|) + q\|x - y\|. \quad (0.4)$$

Olaleru [9] also considered the stability of Ishikawa and Kirk iteration procedures when the operator satisfies (0.4).

Osilike [10] proved that the Ishikawa iteration is *almost T-stable* when the operator is a Lipschitz strongly pseudocontractive operator. It is the purpose of this paper to generalise previous results on the stability of iteration procedures in literature by studying the stability of those iteration procedures *with errors* when the operators satisfy (0.4).

2. Preliminaries

The idea of considering fixed point iteration procedures *with errors* comes from practical numerical computations.

Definition 1 [1]. Let K be a nonempty convex subset of a Banach space X and $T : K \rightarrow X$ a mapping. The sequence $\{x_n\}_{n=0}^{\infty}$ defined iteratively by

$$x_o \in K, \quad (0.5)$$

$$x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \quad (0.6)$$

$$y_n = a'_n x_n + b'_n T x_n + c'_n v_n, \quad n \geq 0 \quad (0.7)$$

where $\{u_n\}$, $\{v_n\}$ are bounded sequences in K and $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$ and $\{c'_n\}$ are sequences in $[0, 1]$ such that

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = 1, \quad n \geq 0 \quad (0.8)$$

is called the *Ishikawa iteration sequence with errors*.

Remark 1. If u_n and v_n are null sequences, then we have the usual Ishikawa iteration [4]. If $b'_n = c'_n = 0$, $n \geq 0$, then the sequence $\{x_n\}$ will be called *Mann iteration with errors*. For recent results on Ishikawa iteration with errors, see [6] and [14]. If in addition, u_n is a null sequence, then we have the usual Mann iteration procedure [7-8].

Definition 2. The Kirk iteration *with errors* is defined as the sequence $\{x_n\}_{n=1}^{\infty}$ given iteratively by

$$x_{n+1} = a_o x_n + a_1 T x_n + a_2 T^2 x_n + \dots + a_k T^k x_n + w_n u_n, \quad n \geq 0 \quad (0.9)$$

where $\{u_n\}$ is a bounded sequence in X , $\{a_n\}$ and $\{w_n\}$ are sequences in $[0, 1]$, $k \geq 1$ an integer, $i = 0, 1, \dots, k$ such that $a_1 > 0$ and $\sum_{i=0}^k a_i + w_n = 1$ for each n .

Remark 2. If u_n is a null sequence, then we have the usual Kirk iteration ([1], [11]).

In the sequel we shall require the following Lemma.

Lemma [1]. Let $\{r_n\}$, $\{s_n\}$, and $\{k_n\}$ be sequences of nonnegative numbers satisfying

$$r_{n+1} \leq (1 - t_n)r_n + s_n + k_n$$

for all $n \geq 0$, where $\{t_n\}_{n=0}^{\infty} \subset [0, 1]$. If $\sum_{n=0}^{\infty} t_n = \infty$, $s_n = o(t_n)$ and $\sum_{n=0}^{\infty} k_n < \infty$ hold, then $\lim_{n \rightarrow \infty} r_n = 0$.

3. Main Results

Theorem 1. Let X be a normed linear space, $T : X \rightarrow X$ a self map of X satisfying (0.4) and $\{y_n\}_{n=0}^{\infty} \subset X$. For arbitrary $x_o \in X$, define sequence $\{x_n\}_{n=0}^{\infty}$ iteratively by

$$x_{n+1} = f(T, x_n) = a_n x_n + b_n T x_n + c_n u_n, \quad n \geq 0$$

where $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are sequences in $[0,1]$ such that $a_n + b_n + c_n = 1$ and $\{u_n\}$ is a bounded sequence in X . Suppose $\epsilon_n = \|y_{n+1} - (a_n y_n + b_n T y_n + c_n u_n)\|$ and suppose T has a fixed point p . Assume that $M = \sup_{n \geq 0} \|u_n - p\|$. If $\lim_{n \rightarrow \infty} c_n = 0$, then

(i) $\sum_{n=0}^{\infty} \epsilon_n < \infty$ implies $\lim_{n \rightarrow \infty} y_n = p$, i.e., the Mann iteration with errors is almost T -stable;

(ii) $\lim_{n \rightarrow \infty} y_n = p$ implies $\lim_{n \rightarrow \infty} \epsilon_n = 0$.

Proof. (i) Let $\sum_{n=0}^{\infty} \epsilon_n < \infty$. We shall establish that $\lim_{n \rightarrow \infty} y_n = p$. In view of (0.4) and the fact that $\|T y_n - p\| = \|T y_n - T p\|$,

$$\begin{aligned} \|y_{n+1} - p\| &\leq \|y_{n+1} - (a_n y_n + b_n T y_n + c_n u_n)\| \\ &\quad + \|(a_n y_n + b_n T y_n + c_n u_n) - (a_n + b_n + c_n)p\| \\ &\leq \epsilon_n + a_n \|y_n - p\| + b_n \|T y_n - p\| + c_n \|u_n - p\| \\ &\leq \epsilon_n + a_n \|y_n - p\| + b_n [\Phi \|p - T p\| + q \|p - y_n\|] \\ &\quad + c_n \|u_n - p\| \\ &= \epsilon_n + a_n \|y_n - p\| + b_n q \|y_n - p\| + c_n \|u_n - p\| \\ &= \epsilon_n + (a_n + b_n q) \|y_n - p\| + M c_n. \end{aligned}$$

Thus

$$\|y_{n+1} - p\| \leq (a_n + b_n q) \|y_n - p\| + M c_n + \epsilon_n. \quad (0.10)$$

In view of the Lemma and considering the fact that $0 \leq a_n + b_n q < 1$, (0.10) yields $\lim_{n \rightarrow \infty} \|y_n - p\| = 0$. Consequently, $\lim_{n \rightarrow \infty} y_n = p$.

(ii) Suppose $\lim_{n \rightarrow \infty} y_n = p$. Following the same method as above, it is easy to see that,

$$\begin{aligned} \epsilon_n &= \|y_{n+1} - (a_n y_n + b_n T y_n + c_n u_n)\| \\ &= \|y_{n+1} - (a_n + b_n + c_n)p + (a_n + b_n + c_n)p \\ &\quad - (a_n y_n + b_n T y_n + c_n u_n)\| \\ &\leq \|y_{n+1} - p\| + a_n \|y_n - p\| + b_n \|T y_n - p\| + c_n \|u_n - p\| \\ &\leq \|y_{n+1} - p\| + a_n \|y_n - p\| + b_n [\Phi \|p - T p\| + q \|p - y_n\|] \\ &\quad + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + a_n \|y_n - p\| + b_n q \|y_n - p\| + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + (a_n + b_n q) \|y_n - p\| + M c_n. \end{aligned}$$

If $n \rightarrow \infty$, it is clear that $\epsilon_n \rightarrow 0$ and this completes the proof.

Remark 3. If $c_n \equiv 0$ in Theorem 1(ii), we obtain a corresponding stability result for the Mann iteration procedure in [3] and the corresponding stability result in [11] if specifically, $\Phi(d(x, Tx))$ is $Ld(x, Tx)$, $L \geq 0$.

Example 1. Let X and T be defined as in Theorem 1. Suppose $\|u_n\| = \frac{1}{n+7}$, $a_n = \frac{1}{n+3}$, $b_n = \frac{n}{n+3}$, $c_n = \frac{2}{n+3}$, $n \geq 0$. Then, Theorem 1 is satisfied.

Theorem 2. Let X be a normed linear space, $T : X \rightarrow X$ a self map of X satisfying (0.4). Suppose T has a fixed point p . For arbitrary $x_0 \in X$, define

sequence $\{x_n\}_{n=0}^\infty$ iteratively by

$$z_n = a'_n x_n + b'_n T x_n + c'_n v_n$$

$$x_{n+1} = f(T, x_n) = a_n x_n + b_n T z_n + c_n u_n, \quad n \geq 0$$

where $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{a'_n\}$, $\{b'_n\}$ and $\{c'_n\}$ are sequences in $[0,1]$ such that $a_n + b_n + c_n = 1$, $a'_n + b'_n + c'_n = 1$ and $\{u_n\}$, $\{v_n\}$ are bounded sequences in X . Let $\{y_n\}$ be any sequence in X and define

$$s_n = a'_n y_n + b'_n T y_n + c'_n v_n$$

and

$$\epsilon_n = \|y_{n+1} - a_n y_n + b_n T s_n + c_n u_n\|, \quad n \geq 0$$

Assume that $M = \sup_{n \geq 0} \|u_n - p\|$ and $N = \sup_{n \geq 0} \|v_n - p\|$. If $\forall n$, $c_n c'_n$ and $c'_n \rightarrow 0$, then

(i) $\sum_{n=0}^\infty \epsilon_n < \infty$ implies $\lim_{n \rightarrow \infty} y_n = p$, i.e. Ishikawa iteration with errors is almost T -stable;

(ii) $\lim_{n \rightarrow \infty} y_n = p$ implies $\lim_{n \rightarrow \infty} \epsilon_n = 0$.

Proof. (i) Suppose $\sum_{n=0}^\infty \epsilon_n < \infty$, we shall establish that $\lim_{n \rightarrow \infty} y_n = p$. In view of (0.4), it follows that

$$\begin{aligned} \|y_{n+1} - p\| &= \|y_{n+1} - (a_n y_n + b_n T s_n + c_n u_n) \\ &\quad + (a_n y_n + b_n T s_n + c_n u_n) - (a_n + b_n + c_n)p\| \\ &\leq \epsilon_n + a_n \|y_n - p\| + b_n \|T s_n - p\| + c_n \|u_n - p\| \\ &\leq \epsilon_n + a_n \|y_n - p\| + b_n [\Phi \|p - T p\| + q \|p - s_n\|] \\ &\quad + c_n \|u_n - p\| \\ &= \epsilon_n + a_n \|y_n - p\| + b_n q \|p - s_n\| + c_n \|u_n - p\| \\ &= \epsilon_n + a_n \|p - y_n\| \\ &\quad + b_n q \|a'_n y_n + b'_n T y_n + c'_n v_n - (a'_n + b'_n + c'_n)p\| + c_n \|u_n - p\| \end{aligned}$$

i.e.

$$\begin{aligned} \|y_{n+1} - p\| &\leq \epsilon_n + a_n \|p - y_n\| + b_n q \|(a'_n (y_n - p) + b'_n (T y_n - p) \\ &\quad + c'_n (v_n - p))\| + c_n \|u_n - p\| \\ &\leq \epsilon_n + a_n \|p - y_n\| + b_n q a'_n \|y_n - p\| + b_n q b'_n \|T y_n - p\| \\ &\quad + b_n q c'_n \|v_n - p\| + c_n \|u_n - p\| \\ &\leq \epsilon_n + a_n \|p - y_n\| + b_n q a'_n \|y_n - p\| \\ &\quad + b_n q b'_n [\Phi \|p - T p\| + q \|p - y_n\|] + b_n q c'_n \|v_n - p\| + c_n \|u_n - p\| \\ &= \epsilon_n + a_n \|p - y_n\| + b_n q a'_n \|y_n - p\| \\ &\quad + b_n q^2 b'_n \|p - y_n\| + b_n q c'_n \|v_n - p\| + c_n \|u_n - p\| \\ &= \epsilon_n + (a_n + b_n q a'_n + b_n q^2 b'_n) \|p - y_n\| + b_n q c'_n \|v_n - p\| \\ &\quad + c_n \|u_n - p\| \\ &\leq (a_n + b_n q) \|y_n - p\| + (M c_n + b_n q c'_n N) + \epsilon_n. \end{aligned}$$

Hence

$$\|y_{n+1} - p\| \leq (a_n + b_n q)\|y_n - p\| + (Mc_n + b_n qc'_n N) + \epsilon_n. \quad (0.11)$$

In view of the Lemma, (0.11) yields $\lim_{n \rightarrow \infty} y_n = p$.

(ii) Suppose $\lim_{n \rightarrow \infty} y_n = p$.

$$\begin{aligned} \epsilon_n &= \|y_{n+1} - (a_n y_n + b_n T s_n + c_n u_n)\| \\ &= \|y_{n+1} - (a_n + b_n + c_n)p + (a_n + b_n + c_n)p \\ &\quad - (a_n y_n + b_n T s_n + c_n u_n)\| \\ &\leq \|y_{n+1} - p\| + a_n \|y_n - p\| + b_n \|T s_n - p\| + c_n \|u_n - p\| \\ &\leq \|y_{n+1} - p\| + a_n \|y_n - p\| + b_n [\Phi \|p - T p\| + q \|p - s_n\|] \\ &\quad + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + a_n \|y_n - p\| + b_n q \|p - s_n\| + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + a_n \|p - y_n\| \\ &\quad + b_n q \|(a'_n y_n + b'_n T y_n + c'_n v_n - (a'_n + b'_n + c'_n)p) + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + a_n \|p - y_n\| \\ &\quad + b_n q \|(a'_n (y_n - p) + b'_n (T y_n - p) + c'_n (v_n - p))\| + c_n \|u_n - p\| \\ &\leq \|y_{n+1} - p\| + a_n \|p - y_n\| + b_n q a'_n \|y_n - p\| \\ &\quad + b_n q b'_n \|T y_n - p\| + b_n q c'_n \|v_n - p\| + c_n \|u_n - p\| \\ &\leq \|y_{n+1} - p\| + a_n \|p - y_n\| + b_n q a'_n \|y_n - p\| \\ &\quad + b_n q b'_n [\Phi (p - T p) + q \|p - y_n\|] + b_n q c'_n \|v_n - p\| + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + a_n \|p - y_n\| + b_n q a'_n \|y_n - p\| \\ &\quad + b_n q^2 b'_n \|p - y_n\| + b_n q c'_n \|v_n - p\| + c_n \|u_n - p\| \\ &= \|y_{n+1} - p\| + (a_n + b_n q a'_n + b_n q^2 b'_n) \|p - y_n\| + b_n q c'_n \|v_n - p\| \\ &\quad + c_n \|u_n - p\| \\ &\leq \|y_{n+1} - p\| + (a_n + b_n q)\|y_n - p\| + Mc_n + b_n q c'_n N. \end{aligned}$$

Thus $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$.

Remark 4. If $\Phi(d(x, Tx))$ is $Ld(x, Tx)$, $L \geq 0$ and $\forall n$, $c_n = c'_n = 0$, then Theorem 2(ii) gives the corresponding result of [11].

Example 2. Let X and T be defined as in Theorem 2. Suppose $\|u_n\| = \frac{1}{n+7}$, $\|v_n\| = \frac{1}{n+1}$, $a_n = \frac{1}{n+3}$, $a'_n = \frac{n+2}{2n+3}$, $b_n = \frac{n}{n+3}$, $b'_n = \frac{n}{2n+3}$, $c_n = \frac{2}{n+3}$, $c'_n = \frac{1}{2n+3}$, $n \geq 0$. Then, Theorem 2 is satisfied.

Theorem 3. Let X be a normed linear space and let $T : X \rightarrow X$ be a selfmap of X satisfying (0.4). Suppose T has a fixed point p . Let $x_o \in X$ and suppose that

$$x_{n+1} = f(T, x_n) = a_o x_n + a_1 T x_n + a_2 T^2 x_n + \dots + a_k T^k x_n + w_n u_n, \quad n \geq 0,$$

$k \geq 1$ an integer, where $\{u_n\}$ is a bounded sequence in X , $\{a_n\}$ and $\{w_n\}$ are sequences in $[0, 1]$ such that $\lim_{n \rightarrow \infty} w_n = 0$, $\sum w_n = \infty$, $a_1 > 0$ and $a_o + a_1 + \dots + a_k + w_n = 1$ for any $n \geq 0$. Let $\{y_n\}$ be any sequences in X and suppose $\epsilon_n = \|y_{n+1} - \sum_{i=0}^k a_i T^i y_n + w_n u_n\|$. If $w_n \rightarrow 0$, then

(i) $\sum_{n=0}^{\infty} \epsilon_n < \infty$ implies $\lim_{n \rightarrow \infty} y_n = p$ i.e the Kirk iteration with errors is almost T-stable;

(ii) $\lim_{n \rightarrow \infty} y_n = p$ implies $\lim_{n \rightarrow \infty} \epsilon_n = 0$.

Proof. (i) Let $\sum_{n=0}^{\infty} \epsilon_n < \infty$. We shall establish that $\lim_{n \rightarrow \infty} y_n = p$. It is clear that $\|Ty_n - p\| = \|Tp - Ty_n\| \leq q\|y_n - p\|$ and hence for $i = 1, 2, \dots, k$, $\|T^i y_n - p\| \leq q^i \|y_n - p\|$. Thus

$$\begin{aligned} \|y_{n+1} - p\| &\leq \|y_{n+1} - (\sum_{i=0}^k a_i T^i y_n + w_n u_n)\| \\ &\quad + \|\sum_{i=0}^k a_i T^i y_n + w_n u_n - p(\sum_{i=0}^k a_i + w_n)\| \\ &\leq \epsilon_n + (\sum_{i=0}^k a_i) \|T^i y_n - T^i p\| + w_n \|u_n - p\| \\ &\leq (\sum_{i=0}^k a_i q^i) \|y_n - p\| + w_n \|u_n - p\| + \epsilon_n. \end{aligned}$$

Since $\sum_{i=0}^k a_i q^i < 1$, in view of the Lemma, it follows that $\lim_{n \rightarrow \infty} y_n = p$.

(ii) If $\lim_{n \rightarrow \infty} y_n = p$, it follows easily that

$$\epsilon_n = \|y_{n+1} - (\sum_{i=0}^k a_i T^i y_n + w_n u_n)\| \leq \|y_{n+1} - p\| + (\sum_{i=0}^k a_i q^i) \|y_n - p\| + w_n \|u_n\|$$

and $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$. Thus the proof of the Theorem is complete.

Remark 5. If $\Phi(d(x, Tx))$ is $Ld(x, Tx)$, $L \geq 0$ and $w_n \equiv 0$, then Theorem 2(ii) is the same as Theorem 7 of [11].

It is still an open problem to show that our Theorems can be proved for T-stability instead of almost T-stability for the iteration processes with errors.

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