Cross effects in mechanics of solid continua

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Abstract

In this paper we review the results obtained during last decade, concerning the various aspects of an important subject, with that mechanics and applied mathematics are dealing: the interaction of different physical fields with the deforming continuous solid media. Here are considered the influence of mechanical and geometrical factors on the behaviour of underground cavities, the thermomechanical models of underground cavities, and the influence of initial mechanical and electric fields on wave propagation in piezoelectric crystals.

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1 Stress concentration around circular and non-circular cavities

Underground cavities of cross sectional shapes circular, or non-circular are often used in mining and civil engineering. The elasto-viscoplastic constitutive equations became in last decades important tools of phenomenological description of the main physical phenomena encountered at geomaterials, like yield, failure, dilatancy, or compressibility of the volume. In the study of stress and displacements behaviour around underground cavities we could distinguish two time periods: the first one, in which the cavity is excavated, followed by the time interval in which the cavity is exploited. The first time period is usually much shorter than the second one. Thus, into the constitutive model we could emphasize two types of mechanical behaviour: one related to *instantaneous response* of the rock mass, which is related to elasticity, resp. an *evolution period* of the material that lasts over a long period of time, linked to viscoplastic deformation.

1.1 Basic equations

We consider the conformal mapping of the exterior of the unity circle γ onto the exterior of a square-like cavity, obtained *via* the well-known Schwarz-Christoffel formula:

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$$z = \omega(\zeta) = \zeta - \frac{1}{6\zeta^3} + \frac{1}{56\zeta^7} - \frac{1}{176\zeta^{11}} + \ldots, |\zeta| \geq 1. \tag{1.1}$$

Taking into account only three terms in the development (1.1), i.e.:

$$z = \omega(\zeta) = \zeta + \frac{m}{\zeta^3} + \frac{n}{\zeta^7}, |\zeta| \geq 1, \tag{1.2}$$

we obtain, if $m = -1/6$ and $n = 0$, a square-like cavity with a side $a = 5/3$ and a radius of curvature in the corner of 6% from a. If $m = -1/6$ and $n = 1/56$, our cavity approaches more the shape of a real square, having the side $a = 143/84$ and a radius of curvature in the corner of 2.5% from the side a. Finally, if $m = n = 0$, we obtain a circular cavity of diameter $a = 2$.

We suppose that the cavity is provided in an elastic, homogeneous and isotropic material and that we are in the hypothesis of plane strain problem with small deformations. At infinity we prescribe two far-field stresses σ_h (horizontal) and σ_v (vertical), generally distinct. The contour of the cavity is supposed to be free of stresses.

Using the method of complex potentials we have to find two holomorphic complex potentials φ and ψ satisfying the following integral equations:

$$\varphi(\zeta) + \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{\omega(\sigma)}{\overline{\omega'(\sigma)}} \frac{\overline{\varphi'(\sigma)}}{\sigma - \zeta} \mathrm{d}\sigma = \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{f(\sigma)}{\sigma - \zeta} \mathrm{d}\sigma \tag{1.3}$$

$$\psi(\zeta) = \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{\overline{f(\sigma)}}{\sigma - \zeta} \mathrm{d}\sigma - \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{\overline{\omega(\sigma)}}{\omega'(\sigma)} \frac{\varphi'(\sigma)}{\sigma - \zeta} \mathrm{d}\sigma \tag{1.4}$$

where $\sigma = \mathrm{e}^{\mathrm{i}\theta} \in \gamma$ and $f = 0$. Knowing the structure of the potentials φ and ψ in the case of an unending plane provided with a single cavity:

$$\varphi(\zeta) = \Gamma\omega(\zeta) + \varphi_0(\zeta) \tag{1.5}$$

$$\psi(\zeta) = \Gamma'\omega(\zeta) + \psi_0(\zeta) \tag{1.6}$$

with $\Gamma = \dfrac{\sigma_h + \sigma_v}{4}$, $\Gamma' = \dfrac{\sigma_v - \sigma_h}{2}$ and φ_0, ψ_0 holomorphic to infinity, we find the complex potentials φ and ψ in the following form, for a square-like cavity generated by a development (1.1) containing two terms:

$$\varphi(\zeta) = \frac{\sigma_h + \sigma_v}{4} \left[\zeta + \frac{1}{6\zeta^3}\right] - \frac{3}{7} \frac{\sigma_v - \sigma_h}{\zeta} \tag{1.7}$$

$$\psi(\zeta) = \frac{\sigma_v - \sigma_h}{2} \zeta - \frac{13(\sigma_v + \sigma_h)\zeta^3 + \frac{78}{7}(\sigma_v - \sigma_h)\zeta}{12(2\zeta^4 + 1)}, \tag{1.8}$$

respectively, for a square-like cavity generated by a development (1) containing three terms:

$$\begin{aligned} \varphi(\zeta) = {} & 0.25(\sigma_v + \sigma_h)\zeta + 0.426(\sigma_h - \sigma_v)\frac{1}{\zeta} + 0.046(\sigma_v + \sigma_h)\frac{1}{\zeta^3} + \\ & 0.008(\sigma_h - \sigma_v)\frac{1}{\zeta^5} + 0.004(\sigma_v + \sigma_h)\frac{1}{\zeta^7} \end{aligned} \tag{1.9}$$

$$\psi(\zeta) = 0.5(\sigma_v - \sigma_h)\zeta - (\sigma_h - \sigma_v)\zeta\, \frac{3.683\zeta^4 + 0.298}{1 - 4\zeta^4 - 8\zeta^8} - (\sigma_v + \sigma_h)\zeta^3\, \frac{-4.4\zeta^4 + 0.205}{1 - 4\zeta^4 - 8\zeta^8}. \tag{1.10}$$

The state of stress around the cavity is given, in polar coordinates, by the well-known Kolosov-Muskelisvili formulae:

$$\sigma_{rr} + \sigma_{\theta\theta} = 4\mathrm{Re}\left[\frac{\varphi'(\zeta)}{\omega'(\zeta)}\right] \tag{1.11}$$

$$\sigma_{\theta\theta} - \sigma_{rr} + 2i\sigma_{r\theta} = \frac{2\zeta^2}{|\zeta|^2\overline{\omega'(\zeta)}}\left[\overline{\omega(\zeta)}\left(\frac{\varphi'(\zeta)}{\omega'(\zeta)}\right)' + \psi'(\zeta)\right], \tag{1.12}$$

while

$$\sigma_{zz} = \sigma_h + \nu[(\sigma_{rr} - \sigma_{rr}^P) + (\sigma_{\theta\theta} - \sigma_{\theta\theta}^P)] \tag{1.13}$$

where $\sigma_{rr}^P = 0.5(\sigma_h + \sigma_v) - 0.5(\sigma_v - \sigma_h)\cos 2\theta$, $\sigma_{\theta\theta}^P = 0.5(\sigma_h + \sigma_v) + 0.5(\sigma_v - \sigma_h)\cos 2\theta$ are the cylindrical components of the far-field stresses.

1.2 Main results

In a series of papers (see [5]-[7] and [13]) we studied the evolution of viscoplastic zones, related to yield, failure and dilatancy/compressibility behaviour, which arise around a square-like cavity, or between two interacting circular, resp. square-like cavities. Our approach is semi-analytical, based on the complete elastic solution regarding the stresses taken as initial conditions, and on the numerical analysis performed programming with the code MATHEMATICA. The examples given in our paper are obtained using formal and numerical computations, the results showing quantitatively the effect of geometric and mechanical parameters, such as the form of the cavity contour, the distance between the cavities, or the ratio of the far-field stresses, on the location of the previous defined viscoplastic zones. The main difference observed, comparing the evolution period and the instantaneous one, was the appearance during the evolution time of the unloading zones, due to the stress relaxation between the cavities. We analyze the effect of this phenomenon on the viscoplastic zones evolution considering the constitutive equation of a rock salt.

In paper [4] was derived a numerical study of the evolution of a circular tunnel performed in a viscoplastic rock mass, sustained by an inelastic lining, while in paper [14] we made a comparative study between the analytical and numerical solutions in the problem of stress concentration between two square-like cavities. The previous results were presented and largely analyzed in monograph [8].

2 Thermomechanics of underground cavities

In recent years the problems of thermomechanical behaviour of underground structures have received considerable attention. In this frame, for example, the problem of a thick-walled tube subject to a mechanical, resp. to a thermal loading, and its equivalent homologue, the circular tunnel in an infinite medium, heated from the wall, become of great importance in engineering applications. Many practical problems can be treated by these types of modelling, such as underground storage of nuclear wastes, underground coal gasification, or the stability of deep petroleum borings. However, whereas the case of mechanical loading under isothermal conditions has been treated in a large number of publications, analytical solutions which take into account thermal loads are relatively rare.

2.1 Basic hypothesis

At the time $t = 0$, the medium is supposed to be in a state of hydrostatic stress $\boldsymbol{\sigma_0} = -P_\infty \mathbf{1}$ with zero displacement $\boldsymbol{u_0} = 0$ and zero strain $\boldsymbol{\varepsilon_0} = 0$ everywhere, and having a reference temperature T_0. We suppose that the material properties are temperature independent.

We are considering a quasi-static evolution, under the hypothesis of small axisymmetrical plane strains. Under the previous assumptions the displacement is purely radial:

$$\boldsymbol{u} = u(r,t)\boldsymbol{e}_r \tag{2.14}$$

and, consequently, the stress and strain tensors are diagonal:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \partial_r u(r,t) & & \\ & u(r,t)/r & \\ & & 0 \end{pmatrix}, \ \boldsymbol{\sigma} = \begin{pmatrix} \sigma_r(r,t) & & \\ & \sigma_\theta(r,t) & \\ & & \sigma_z(r,t) \end{pmatrix}. \tag{2.15}$$

We are imposing an internal pressure, which decreases monotonically from its initial value P_∞ to a final constant prescribed value P_i. We shall assume that the medium remains elastic during this stage of mechanical loading. Then, the boundary conditions take the form:

$$\sigma_r(a,t) = -P_i\ ,\ u(b,t) = 0. \tag{2.16}$$

Maintaining P_i at its previous prescribed value, the thick-walled tube is subjected to a heating process, resulting in an axisymmetrical temperature field $T(r,t)$. We shall restrict our attention to the case of a temperature field satisfying the following essential conditions:

$$\partial_r T(r,t) < 0\ ,\ \partial_t T(r,t) > 0. \tag{2.17}$$

Such is the case, for example, of a constant heat flux applied at the inner wall.

2.2 Fundamental equations

Under the previous assumptions, the constitutive equation takes the form:

$$\partial_t \boldsymbol{\varepsilon} \ = \frac{1+\nu}{E}\,\partial_t \boldsymbol{\sigma} \ - \frac{\nu}{E}\,tr(\partial_t \boldsymbol{\sigma}) + \alpha\,\partial_t T\,\mathbf{1} + \partial_t \boldsymbol{\varepsilon}^p \tag{2.18}$$

where ν is Poisson's ratio, E the Young's module, α the coefficient of linear thermal expansion and $T(r,t)$ the temperature field.

The form of the plastic strain rate tensor $\partial_t \boldsymbol{\varepsilon}^p$ depends on whether we are in the presence of face flow or corner flow:

$$\partial_t \boldsymbol{\varepsilon}^p = \left[\begin{array}{ll} \partial_t \lambda \, \dfrac{\partial F}{\partial \boldsymbol{\sigma}} & \text{if} \quad \sigma_i > \sigma_j > \sigma_k \\[2ex] \partial_t \lambda_{ij} \, \dfrac{\partial F_{ij}}{\partial \boldsymbol{\sigma}} + \partial_t \lambda_{ik} \, \dfrac{\partial F_{ik}}{\partial \boldsymbol{\sigma}} & \text{if} \quad \sigma_i > \sigma_j = \sigma_k \end{array} \right. \tag{2.19}$$

where $F = \sigma_i - \sigma_k - 2C$ is the Tresca's yield criterion (in the case of face flow), respectively $F_{lm} = |\sigma_l - \sigma_m| - 2C$ (in the case of corner flow). Here:

$$\partial_t \lambda = \left[\begin{array}{lllll} 0 & \text{if} & F < 0 & \text{or} & \partial_t \boldsymbol{\sigma} \cdot \dfrac{\partial F}{\partial \boldsymbol{\sigma}} < 0 \\ \\ > 0 & \text{if} & F = 0 & \text{and} & \partial_t \boldsymbol{\sigma} \cdot \dfrac{\partial F}{\partial \boldsymbol{\sigma}} = 0 \end{array} \right.$$

and the sign of $\partial_t \lambda_{lm}$ equals the sign of $\sigma_l - \sigma_m$.

The quantities $E, \sigma_r, \sigma_\theta, \sigma_z, P_\infty, P_i$ will be normalized with respect to the cohesion C and we shall denote by:

$$\theta(r,t) = \frac{E\alpha T(r,t)}{2C(1-\nu)} \qquad \textit{–dimensionless thermal loading}$$

$$\Delta P = \frac{P_\infty - P_i}{C} \qquad \textit{–dimensionless mechanical loading.}$$

Integrating the constitutive equation (18), with respect to time between $t = 0$ and any other instant $t > 0$ and taking into account the form (19) of $\partial_t \varepsilon^p$ we obtain the fundamental constitutive equations:

$$\begin{aligned} E\partial_r u &= \sigma_r - \nu(\sigma_\theta + \sigma_z) + 2(1-\nu)\theta + E\lambda + E\mu + (1-2\nu)P_\infty \\ E\, u/r &= \sigma_\theta - \nu(\sigma_z + \sigma_r) + 2(1-\nu)\theta - E\lambda + (1-2\nu)P_\infty \\ 0 &= \sigma_z - \nu(\sigma_r + \sigma_\theta) + 2(1-\nu)\theta - E\mu + (1-2\nu)P_\infty \end{aligned} \tag{2.20}$$

to which must be added the equilibrium equation:

$$\sigma_\theta - \sigma_r = r\, \partial_r \sigma_r \tag{2.21}$$

and the Tresca's yield condition:

$$PF\ (\textit{face flow}) \qquad \sigma_i - \sigma_k = 2 \qquad (\text{if } \sigma_i > \sigma_j > \sigma_k) \tag{2.22}$$

$$PC\ (\textit{corner flow}) \qquad \sigma_i - \sigma_k = 2\ ,\ \sigma_j = \sigma_k \qquad (\text{if } \sigma_i > \sigma_j = \sigma_k). \tag{2.23}$$

Note that in the system (2.20) the multipliers λ and μ are associated with the stress couples $\sigma_r - \sigma_\theta$, $\sigma_r - \sigma_z$ being non-zero only in the case of plastic flow.

To solve the system (2.20)-(2.23), a sequence of elastoplastic zones is assumed, for each phase encountered. The solution so established is verified, a posteriori, for consistency with respect to the following conditions:

- the boundary radii must be monotone increasing with time;
- the signs of $\partial_t \lambda$ and $\partial_t \mu$ must be the same as the corresponding differences $\sigma_r - \sigma_\theta$, respectively $\sigma_r - \sigma_z$ in each plastic zone, so that the plastic power is positive;

- the deviatoric stresses must stay below the yield limit in the elastic zone.

Using the uniqueness theorem of stresses in thermo-elasto-perfectly plastic problems we can state that the assumed solution is the real one.

2.3 Discussion of results

In papers [16]-[18], as well as in the monograph [8], we studied the thermoplastic behaviour of a circular tunnel, resp. of a thick-walled tube, subjected to thermo-mechanical loadings, supposing different plasticity conditions, like Tresca or Coulomb yield criteria and plastic potentials. We obtained analytical models of an elastoplastic circular tunnel, resp. of a thick-walled tube, subject to an internal pressure and to an axisymmetrical time dependent temperature field. The case of a cohesive-frictional material, with cohesion depending on the temperature, is also considered.

The subsequent thermal expansion generates plastic zones according to a precise predetermined order. Based on a set of simplifying, but realistic assumptions, we obtain a closed form solution expressed in terms of the main unknowns of the problem (i.e. the boundaries of the elastoplastic zones). These unknowns are simply the roots of a set of algebraic equations, and can easily be determined by simple numerical computations. Comparisons with two-dimensional numerical results are presented.

3 Wave propagation in piezoelectric crystals subject to initial fields

3.1 Basic equations

The basic equations of piezoelectric bodies for infinitesimal deformations and fields superposed on initial deformation and electric fields were given by Eringen and Maugin in their monograph [3]. An alternate derivation of this type of equations was obtained by Baesu, Fortuné and Soós in their paper [1].

We assume the material to be an elastic dielectric, which is nonmagnetizable and conducts neither heat, nor electricity. We shall use the quasi-electrostatic approximation of the equations of balance. Furthermore, we assume that the elastic dielectric is linear and homogeneous, that the initial homogeneous deformations are infinitesimal and that the initial homogeneous electric field has small intensity. To describe this situation we use three different configurations : the *reference configuration* B_R in which at time $t = 0$ the body is undeformed and free of all fields; the *initial configuration* $\overset{\circ}{B}$ in which the body is deformed statically and carries the initial fields; the *present (current) configuration* B_t obtained from $\overset{\circ}{B}$ by applying time dependent incremental deformations and fields. In what follows, all the fields related to the initial configuration $\overset{\circ}{B}$ will be denoted by a superposed "$\circ$".

In this case the field equations take the following form:

$$\overset{\circ}{\rho}\, \ddot{\boldsymbol{u}} = \operatorname{div} \boldsymbol{\Sigma} + \overset{\circ}{\rho}\, \bar{\boldsymbol{f}} + \bar{q}\, \overset{\circ}{\boldsymbol{E}}, \ \operatorname{div} \boldsymbol{\Delta} = \bar{q} \tag{3.24}$$

$$\operatorname{rot} \boldsymbol{e} = 0 \ \Leftrightarrow \ \boldsymbol{e} = -\operatorname{grad} \varphi$$

where $\overset{\circ}{\rho}$ is the mass density, $\overset{\circ}{\boldsymbol{E}}$ is the initial applied electric field, $\boldsymbol{u}$ is the incremental displacement from $\overset{\circ}{B}$ to B_t, $\boldsymbol{\Sigma}$ is the incremental mechanical nominal stress tensor, $\bar{\boldsymbol{f}}$ is the incremental body force density, $\bar{q}$ is the incremental volumetric charge density, $\boldsymbol{\Delta}$ is the incremental electric displacement vector, $\boldsymbol{e}$ is the incremental electric field and φ is the incremental electric potential. All incremental fields involved into the above equations depend on the spatial variable $\boldsymbol{x}$ and on the time t. We suppose a homogeneous process ($\bar{\boldsymbol{f}} = \boldsymbol{0}$, $\bar{q} = 0$).

We have the following incremental constitutive equations:

$$\Sigma_{kl} = \overset{\circ}{\Omega}_{klmn}\, u_{m,n} - \overset{\circ}{\Lambda}_{mkl}\, e_m = \overset{\circ}{\Omega}_{klmn}\, u_{m,n} + \overset{\circ}{\Lambda}_{mkl}\, \varphi_{,\,m} \tag{3.25}$$
$$\Delta_k = \overset{\circ}{\Lambda}_{kmn}\, u_{n,m} + \overset{\circ}{\epsilon}_{kl}\, e_l = \overset{\circ}{\Lambda}_{kmn}\, u_{n,m} - \overset{\circ}{\epsilon}_{kl}\, \varphi_{,\,l}.$$

In these equations $\overset{\circ}{\Omega}_{klmn}$ are the components of the instantaneous elasticity tensor, $\overset{\circ}{\Lambda}_{kmn}$ are the components of the instantaneous coupling tensor and $\overset{\circ}{\epsilon}_{kl}$ are the components of the instantaneous dielectric tensor. These coefficients can be expressed in terms of the classical moduli of the material and on the initial applied fields as follows:

$$\overset{\circ}{\Omega}_{klmn} = \overset{\circ}{\Omega}_{nmlk} = c_{klmn} + \overset{\circ}{S}_{kn}\, \delta_{lm} - e_{kmn}\, \overset{\circ}{E}_l - e_{nkl}\, \overset{\circ}{E}_m - \eta_{kn}\, \overset{\circ}{E}_l \overset{\circ}{E}_m \tag{3.26}$$
$$\overset{\circ}{\Lambda}_{mkl} = e_{mkl} + \eta_{mk}\, \overset{\circ}{E}_l, \quad \overset{\circ}{\epsilon}_{kl} = \overset{\circ}{\epsilon}_{lk} = \epsilon_{kl} = \delta_{kl} + \eta_{kl}$$

where c_{klmn} are the components of the constant elasticity tensor, e_{kmn} are the components of the constant piezoelectric tensor, ϵ_{kl} are the components of the constant dielectric tensor, $\overset{\circ}{E}_i$ are the components of the initial applied electric field and $\overset{\circ}{S}_{kn}$ are the components of the initial applied symmetric (Cauchy) stress tensor.

It is important to observe that the previous material moduli have the following symmetry properties:

$$c_{klmn} = c_{lkmn} = c_{klnm} = c_{mnkl},\ e_{mkl} = e_{mlk},\ \epsilon_{kl} = \epsilon_{lk}.$$

Hence, in general there are 21 independent elastic coefficients c_{klmn}, 18 independent piezoelectric coefficients e_{klm} and 6 independent dielectric coefficients ϵ_{kl}. From the relations (3) we see that $\overset{\circ}{\Omega}_{klmn}$ is not symmetric in indices (k, l) and (m, n) and $\overset{\circ}{\Lambda}_{mkl}$ is not symmetric in indices (k, l). It follows that, generally, there are 45 independent instantaneous elastic moduli $\overset{\circ}{\Omega}_{klmn}$, 27 independent instantaneous coupling moduli $\overset{\circ}{\Lambda}_{mkl}$ and 6 independent instantaneous dielectric moduli $\overset{\circ}{\epsilon}_{kl}$.

Our main goal is to study the conditions of propagation for incremental progressive plane waves in an unbounded three dimensional material described by the previous constitutive equations. Therefore, we suppose that the displacement vector and the electric potential will have the following form:

$$\boldsymbol{u} = \boldsymbol{a} \exp[\mathrm{i}(\boldsymbol{p} \cdot \boldsymbol{x} - \omega t)],\ \varphi = a \exp[\mathrm{i}(\boldsymbol{p} \cdot \boldsymbol{x} - \omega t)]. \tag{3.27}$$

Here $\boldsymbol{a}$ and a are constants, characterizing the *amplitude* of the wave, $\boldsymbol{p} = p\,\boldsymbol{n}$ (with $\boldsymbol{n}^2 = 1$) is a constant vector, p representing the *wave number* and $\boldsymbol{n}$ denoting the *direction of propagation* of the wave, ω being the *frequency* of the wave.

Introducing these forms of $\boldsymbol{u}$ and φ into the field equations (3.24) and taking into account the constitutive equations (3.25), (3.26) we obtain the *condition of propagation* of progressive waves:

$$\overset{\circ}{\boldsymbol{Q}}\ \boldsymbol{a} = \overset{\circ}{\rho}\ \omega^2 \boldsymbol{a} \tag{3.28}$$

with

$$\overset{\circ}{Q}_{lm} = \overset{\circ}{A}_{lm} + \frac{\overset{\circ}{\Gamma}_l \overset{\circ}{\Gamma}_m}{\Gamma}, \quad \overset{\circ}{A}_{lm} = \overset{\circ}{\Omega}_{klmn}\ p_k p_n = p^2\ \overset{\circ}{\Omega}_{klmn}\ n_k n_n \tag{3.29}$$
$$\overset{\circ}{\Gamma}_l = \overset{\circ}{\Lambda}_{mkl}\ p_m p_k = p^2\ \overset{\circ}{\Lambda}_{mkl}\ n_m n_k, \ \Gamma = \epsilon_{kl} p_k p_l = p^2 \epsilon_{kl} n_k n_l.$$

Since the *acoustic tensor* $\overset{\circ}{\boldsymbol{Q}}$ is symmetric, the eigenvalues $\overset{\circ}{\rho}\ \omega^2$ are real numbers. Moreover, if we assume the positive definiteness of the instantaneous moduli tensors $\overset{\circ}{\boldsymbol{\Omega}}$ and $\boldsymbol{\epsilon}$ (i.e. if the initial configuration $\overset{\circ}{B}$ is locally stable), it follows from the definition of the acoustic tensor $\overset{\circ}{\boldsymbol{Q}}$ that it is positive definite. Consequently, the eigenvalues $\overset{\circ}{\rho}\ \omega^2$ are positive quantities for any $\boldsymbol{p}$. Thus, if $\overset{\circ}{\boldsymbol{\Omega}}$ and $\boldsymbol{\epsilon}$ satisfy the given conditions, in a prestressed and prepolarized piezoelectric material, then incremental progressive waves can propagate in any direction, the direction of propagation $\boldsymbol{n}$, the wave number p and the frequency ω being connected by the *dispersion equation*:

$$\det\,(\overset{\circ}{\boldsymbol{Q}} - \overset{\circ}{\rho}\ \omega^2 \mathbf{1}) = 0. \tag{3.30}$$

The *velocity of propagation* of the wave is defined by $v = \omega/p$.

3.2 Main results

In papers [9]-[12] and [15] we studied the wave propagation conditions in piezoelectric crystals of second order, i. e. piezoelectric crystals subjected to infinitesimal deformations and electric fields superposed on initial fields. In the case of a 6mm-type crystal, using realistic values of the initially applied fields, we obtain that the progressive waves can propagate along the symmetry elements of the crystal, i.e. the symmetry axis, the meridian plane and in the plane normal to the symmetry axis. We determine the velocities of propagation and the amplitude vectors in closed forms. The polarization of the waves is influenced only by the initial electric field components, and not by the components of the initial stress field. On the other hand, the velocities of propagation are influenced by both initial fields. Sections of the *slowness surfaces* in the mentioned planes of symmetry are obtained and the respective *coupling coefficients* are analyzed, for several crystals of 6mm-type and for various initial fields. Our results generalize the wave propagation in dielectric crystals without initial fields problem and are compatible with the previous results. The cases of an isotropic material, as well as of a cubic crystal, are also analyzed.

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