Non-Existence of Real Lightlike Hypersurfaces of an Indefinite Complex Space Form

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Dedicated to Prof.Dr. Constantin UDRIŞTE
on the occasion of his sixtieth birthday

Abstract

The purpose of this paper is to give some characterizations of non-existence of real lightlike hypersurfaces in an indefinite complex space form

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1 Introduction

The general theory of lightlike (or, null) hyperurfaces is one of the most important topics of differential geometry. Many authors have studied lightlike (null) hypersurfaces (or submanifolds) of semi-Riemannian manifold(or indefinite Kaehler manifold) [1], [2], [3], [4], and others. In [1], the authors have constructed the vector bundles related to a degenerate submanifold in a semi-Riemann manifold and obtained many properties about this submanifolds. On the other hand, in [3], the authors initiated to study CR-lightlike submanifolds of indefinite Kaehlerian manifolds. It is well known that real lightlike hypersurface of an indefinite Kaehler manifold is a CR-lightlike submanifold [3].

In the present paper, we consider real lightlike hypersurfaces of an indefinite complex space form. The main purpose of this paper is to investigate non-existence of real lightlike hypersurfaces of an indefinite complex space form.

2 Preliminaries

Firstly, we note that the notation and fundamental formulas used in this study are the same as [3]. Let $\overline{M}$ be a $(m+2)-$ dimensional semi-Riemannian manifold with index $q \in \{1, ..., m+1\}$. Let M be a hypersurface of $\overline{M}$. Denote by g the induced tensor field by $\overline{g}$ on M. M is called a *lightlike hypersurface* if g is of constant rank m. Consider the vector bundle $TM^{\perp}$whose fibres are defined by

$$T_xM^{\perp} = \left\{Y_x \in T_x\overline{M} \mid \overline{g}_x\left(Y_x, X_x\right) = 0, \quad \forall X_x \in T_xM\right\}$$

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for any $x \in M$. Thus, a hypersurface M of $\overline{M}$ is lightlike if and only if $TM^{\perp}$is a distribution of rank 1 on M.

If M is a lightlike hypersurface, then we consider the complementary distribution $S(TM)$ of $TM^{\perp}$in TM which is called a screen distribution. From [1], we know that it is non-degenerate. Thus we have direct orthogonal sum $TM = S(TM) \perp TM^{\perp}$. Since $S(TM)$ is non-degenerate with respect to $\overline{g}$ we have

$$T\overline{M} = S(TM) \perp S(TM)^{\perp}$$

where $S(TM)^{\perp}$ is the orthogonal complementary vector bundle to $S(TM)$ in $T\overline{M} \mid_M$.

Theorem 2.1 *[2] Let $(M, g, S(TM))$ be a lightlike hypersurface of $\overline{M}$. Then, there exists a unique vector bundle $tr(TM)$ of rank 1 over M such that for any non-zero section ξ of $TM^{\perp}$ on a coordinate neighborhood $U \subset M$, there exist a unique section N of $tr\,(TM)$ on U satisfying*

$$g(N, \xi) = 1$$

and

$$\overline{g}\,(N, N) = \overline{g}\,(W, W) = 0, \forall W \in \Gamma\,(S(TM)\mid_U)$$

From Theorem 2.1, we have

$$T\overline{M}\mid_M = S(TM) \perp \left(TM^{\perp} \oplus tr(TM)\right) = TM \oplus tr(TM).$$

$tr(TM)$ is called the *null* transversal vector bundle of M with respect to $S(TM)$.

Let $\overline{\nabla}$ be levi-Civita connection on $\overline{M}$. We have

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), X, Y \in \Gamma\,(TM) \tag{1}$$

and

$$\overline{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, X \in \Gamma\,(TM)\,, V \in \Gamma\,(tr(TM))\,, \tag{2}$$

where $\nabla_X Y, A_V X \in \Gamma\,(TM)$ and $h(X,Y), \nabla^{\perp}_X V \in \Gamma\,(tr(TM)).\nabla$ is a symmetric linear connection on M called an induced linear connection, $\nabla^{\perp}$ is a linear connection on the vector bundle $tr(TM)$, h is a $\Gamma\,(tr(TM))$-valued simetric bilinear form and A_V is the shape operator of Mconcerning V.

Locally, suppose $\{\xi, N\}$ is a pair of sections on $U \subset M$ in Theorem 2.1. Then define a symmetric $F(U)$-bilinear form B and a 1-form τ on U by

$$B(X, Y) = \overline{g}(h(X, Y), \xi), \forall X, Y \in \Gamma\,(TM\mid_U)$$

and

$$\tau\,(X) = \overline{g}\left(\nabla^{\perp}_X N, \xi\right)$$

Thus (1) and (2) locally become

$$\overline{\nabla}_X Y = \nabla_X Y + B(X, Y)N \tag{3}$$

and

$$\overline{\nabla}_X N = -A_N X + \tau\,(X)\,N \tag{4}$$

respectively.

Let denote P as the projection of TM on $S(TM)$. We consider decomposition

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi \tag{5}$$

and

$$\nabla_X \xi = -A^*_\xi X + \epsilon(X)\xi \tag{6}$$

where $\nabla^*_X PY, A^*_\xi X$ belong to $S(TM)$ and C is a 1-form on U. From (4) and (6) it is easy to check that $\epsilon = -\tau$. Thus

$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi \tag{7}$$

Note that ∇ is not metric connection [3]. We have the following equations [3]

$$g(A_N X, PY) = C(X, PY), \overline{g}(A_N X, N) = 0 \tag{8}$$

$$g(A^*_\xi X, PY) = B(X, PY), \overline{g}(A^*_\xi X, N) = 0 \tag{9}$$

for any $X, Y \in \Gamma(TM)$.

Let $(\overline{M}, \overline{g}, \overline{J})$ be an indefinite almost Hermitian manifold, then $\overline{M}$ is called an indefinite Kaehler manifold if $\overline{J}$ is parallel with respect to $\overline{\nabla}$, i.e.,

$$(\nabla_X \overline{J})\, Y = 0, \forall X, Y \in \Gamma(T\overline{M}).$$

An indefinite complex space form is a connected indefinite Kaehler manifold with constant holomorphic sectional curvature c and it is denoted by $\overline{M}(c)$. The curvature tensor field of $\overline{M}(c)$ is given by

$$\begin{aligned} \overline{R(X,Y)Z} &= \frac{c}{4}\{\overline{g}(Y,Z)X - \overline{g}(X,Z)Y + \overline{g}(\overline{J}Y,Z)\overline{J}X - \\ &- \overline{g}(\overline{J}X,Z)\overline{J}Y + 2\overline{g}(X,\overline{J}Y)\overline{J}Z\}, \end{aligned} \tag{10}$$

for any $X, Y, Z \in \Gamma(T\overline{M})$.

Let $(\overline{M}, \overline{g}, \overline{J})$ be a real $2m$-dimensional, $m > 1$ indefinite almost Hermitian manifold, where $\overline{g}$ is a semi-Riemannian metric of index $\nu = 2q, 0 < q < m$. Suppose that (M, g) is lightlike hypersurface of $\overline{M}$, where g is the degenarate induced metric of M. Then there exists a non-degenarate and almost complex distribution on M such that

$$S(TM) = \{\overline{J}TM^\perp \oplus \overline{J}tr(TM)\} \perp D_0. \tag{11}$$

Thus

$$TM = \{\overline{J}TM^\perp \oplus \overline{J}tr(TM)\} \perp D_0 \perp TM^\perp$$

and

$$T\overline{M} = \{\overline{J}TM^\perp \oplus \overline{J}tr(TM)\} \perp D_0 \perp \{TM^\perp \oplus tr(TM)\}.$$

Now, we consider the local lightlike vector fields $U = -\overline{J}N$ and $V = -\overline{J}\xi$, then any vector field on$M$ is expressed as follows

$$X = SX + u(X)U \tag{12}$$

where S is the projection on $D = TM^{\perp} \perp \overline{J}TM^{\perp} \perp D_0$ and

$$u(X) = g(X, V). \tag{13}$$

Hence

$$\overline{J}X = FX + u(X)N, \tag{14}$$

where F is a tensor field of type (1,1) globally defined on M by

$$FX = \overline{J}SX. \tag{15}$$

By using (12), (13), (14) and taking into account that $\left(D, \overline{J}\right)$ is an almost complex distribution, we derive

$$F^2X = -X + u(X)U; u(U) = 1. \tag{16}$$

3 Real lightlike hypersurfaces of an indefinite complex space form

We start with the following preparatory results.

Lemma 3.1 *Let $\overline{M}(c)$ be an indefinite complex space form and M be a real lightlike hypersurface of $\overline{M}(c)$. Then*

$$\begin{aligned} R(X,Y)Z = {} & \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(\overline{J}Y,Z)FX \\ & -g(\overline{J}X,Z)FY + 2g(X,\overline{J}Y)FZ\} + B(Y,Z)A_NX - B(X,Z)A_NY, \end{aligned} \tag{17}$$

and

$$\begin{aligned} (\nabla_Y h)(X,Z) - (\nabla_X h)(Y,Z) &= \frac{c}{4}\{-g(\overline{J}Y,Z)u(X) + g(\overline{J}X,Z)u(Y) \\ &- g(X,\overline{J}Y)u(Z)\}N, \end{aligned} \tag{18}$$

for any $X, Y, Z \in \Gamma(TM)$.

Proof. Since $\overline{M}$ be an indefinite complex space form, we obtain

$$R(X,Y)Z = \frac{c}{4}\{g(Y,Z))X - g(X,Z)Y + g(\overline{J}Y,Z)\overline{J}X - g(\overline{J}X,Z)\overline{J}Y$$
$$+2g(X,\overline{J}Y)\overline{J}Z\} + A_{h(Y,Z)}X - A_{h(X,Z)}Y + (\nabla_Y h)(X,Z) - (\nabla_X h)(Y,Z).$$

By using (3) and (13) we find

$$R(X,Y)Z = \frac{c}{4}\{g(Y,Z))X - g(X,Z)Y + g(\overline{J}Y,Z)(FX + u(X)N) - g(\overline{J}X,Z)$$
$$(FY + u(Y)N) + 2g(X,\overline{J}Y)(FZ + u(Z)N)\} + B(Y,Z)A_NX - B(X,Z)A_NY$$
$$+ (\nabla_Y h)(X,Z) - (\nabla_X h)(Y,Z).$$

Comparing the tangential and transversal vector bundle parts of the both sides of this equation, we have (17) and (18).

By using the lemma 3.1, we may state the following result

Lemma 3.2 *Let M be a real lightlike hypersurface of $\overline{M}(c)$. Then,*

$$g\left(R(X,\overline{J}\xi)Y,\overline{J}N\right) = \tfrac{c}{4}\left\{g(\overline{J}\xi,Y)g(X,\overline{J}N) - g(X,Y)\right\} -B(X,Y)C(\overline{J}\xi,\overline{J}N) + B(\overline{J}\xi,Y)C(X,\overline{J}N) \tag{19}$$

$$g\left(R(X,\overline{J}N)Y,\overline{J}\xi\right) = \tfrac{c}{4}\left\{g(\overline{J}N,Y)g(X,\overline{J}\xi) - g(X,Y)\right\} -B(X,Y)C(\overline{J}N,\overline{J}\xi) + B(\overline{J}N,Y)C(X,\overline{J}\xi) \tag{20}$$

and

$$g\left(R(X,\xi)Y,N\right) = \frac{c}{4}\{-g(X,Y) + g\left(\overline{J}\xi,Y\right)g\left(\overline{J}X,N\right) + 2g\left(X,\overline{J}\xi\right)g\left(\overline{J}Y,N\right), \tag{21}$$

for any $X,Y \in \Gamma(TM)$, $\xi \in \Gamma\left(TM^{\perp}\right)$ and $N \in \Gamma(tr(TM))$.

Now, consider the local frame $\{z_i,\xi,\overline{J}\xi,\overline{J}N\}$on $U \subset M$, where $\{z_i\}$, $(i = 1,...,2m-4)$ is an orthonormal basis of $\Gamma(D_0)$, $\xi \in \Gamma\left(TM^{\perp}\right)$, $\overline{J}\xi \in \Gamma\left(TM^{\perp}\right)$ and $\overline{J}N \in \Gamma(tr(TM))$. Then by the definition of real lightlike hypersurface and by using theorem 2.1. We obtain

$$\begin{aligned} Ric(X,Y) &= \sum_{i=1}^{2m-4}\epsilon_i g\left(R(X,z_i)Y,z_i\right) + g\left(R(X,\xi)Y,N\right) + g\left(R(X,\overline{J}\xi)Y,\overline{J}N\right) \\ &+ g\left(R(X,\overline{J}N)Y,\overline{J}\xi.\right) \end{aligned}$$

From lemma 3.1 and lemma 3.2, we get

$$\begin{aligned} &Ric(X,Y) = \frac{c}{4}\{\epsilon_i g(z_i,Y)g(z_i,X) - (2m-1)g(X,Y) \\ &-\epsilon_i g(z_i,\overline{J}Y)g(z_i,\overline{J}X) - g(X,\overline{J}\xi)g(Y,\overline{J}N)\} + B(\overline{J}N,Y)C(X,\overline{J}\xi) \\ &+\epsilon_i B(z_i,Y)C(X,z_i) + B\left(\overline{J}\xi,Y\right)C(X,\overline{J}N) \\ &-B(X,Y)\left\{\epsilon_i C(z_i,z_i) + C(\overline{J}N,\overline{J}\xi) + C(\overline{J}\xi,\overline{J}N)\right\}. \end{aligned} \tag{22}$$

Lemma 3.3 *Let $\overline{M}$ be an indefinite Kaehler manifold and M be a real lightlike hypersurface of $\overline{M}$. Then*

$$B(\overline{J}N,Y) = C(Y,\overline{J}\xi)$$

for any $X,Y \in \Gamma(TM)$.

Proof. By the definition of B and using Kaehler structure of $\overline{M}$, we have

$$\begin{aligned} B(\overline{J}N,Y) &= \overline{g}(h(\overline{J}N,Y),\xi) \\ &= \overline{g}(\overline{\nabla}_Y\overline{J}N,\xi) \\ &= -\overline{g}(\overline{\nabla}_Y N,\overline{J}\xi). \end{aligned}$$

From (4) we find

$$B(\overline{J}N,Y) = g\left(A_N Y,\overline{J}\xi\right).$$

From (6) we obtain

$$B(\overline{J}N, Y) = C\left(Y, \overline{J}\xi\right)$$

which proves assertion.

It is known that B is a degenerate bilinear form [3]. Combining this fact with (22) and lemma 3.3, we have the following theorem.

Theorem 3.4 *There are no real lightlike hypersurfaces in $\overline{M}(c)$ with positive or negative Ricci curvature.*

Theorem 3.5 *There are no real lightlike hypersurfaces of indefinite complex space form $\overline{M}\,(c)$ $(c \neq 0)$ with parallel second fundemental form.*

Proof. Suppose c is not zero and second fundemental form is parallel. Then, if we take $Y = \xi, Z = \overline{J}N$ in (18), we obtain

$$\frac{c}{4}\{-g(\overline{J}\xi, \overline{J}N)u(X) + g(\overline{J}X, \overline{J}N)u(\xi) + 2g(X, \overline{J}\xi)u(\overline{J}N)\} = 0$$

$$\frac{c}{4}\{u(X) - 2g(X, \overline{J}\xi)\} = 0.$$

From (13), we have

$$\frac{c}{4}\{-g(X, \overline{J}\xi) - 2g(X, \overline{J}\xi)\} = 0.$$

If we take $X = \overline{J}N$ in this equation, we deduce

$$-\frac{3}{4}c = 0,$$

which is a contradiction.

Theorem 3.6 . *There are no real lightlike hypersurfaces of indefinite complex space form $\overline{M}$ $(c \neq 0)$ with parallel screen distribution.*

Proof. Suppose M be a real lightlike hypersurface with integrable screen distribution in an indefinite complex space form with $c \neq 0$. Then $C = 0$ (See, [2]), thus

$$\overline{g}\left(\overline{R}(\xi, \overline{J}N)\overline{J}\xi, N)\right) = 0.$$

Since $\overline{M}$ is an indefinite complex space form, we have

$$\begin{aligned} 0 \;\; = \;\; & \frac{c}{4}\{g\left(\overline{J}N, \overline{J}\xi\right)\overline{g}\left(\xi, N\right) - \overline{g}\left(\overline{J}N, N)\right)\overline{g}\left(\xi, \overline{J}\xi\right) + \overline{g}\left(\overline{JJ}N, \overline{J}\xi\right) \\ & \overline{g}\left(N, \overline{J}\xi\right) - g\left(\overline{J}\xi, \overline{J}\xi\right)\overline{g}\left(\overline{JJ}N, N\right) + 2\overline{g}\left(\xi, \overline{JJ}N\right)\overline{g}\left(N, \overline{JJ}\xi\right) \end{aligned}$$

or

$$\frac{3}{4}c = 0 \Rightarrow c = 0.$$

This is a contradiction.

From theorem 3.6, we have the following

Corollary 3.7 *Let $\overline{M}$ be positively or negatively curved indefinite complex space form. Then, there are no real lightlike hypersurfaces of $\overline{M}$ such that $\nabla C = 0$ and $C(V, V) = 0$.*

Theorem 3.8 *There are no real lightlike hypersurfaces of indefinite complex space form* $\overline{M}$ $(c \neq 0)$ *such that* $\overline{g}\left(R\left(\overline{J}\xi,\xi\right)\overline{J}N,N\right)=0$.

Proof. Suppose M be a real lightlike hypersurface of positively or negatively curved an indefinite complex space form such that

$$g(R(\overline{J}\xi,\xi)\overline{J}N,N)=0.$$

If we take $X=\overline{J}\xi, Y=\overline{J}N$ in (21). Then we get

$$\begin{aligned} 0 &= \overline{g}(R\left(\overline{J}\xi,\xi\right)\overline{J}N,N) \\ &= \frac{c}{4}\{-g\left(\overline{J}\xi,\overline{J}N\right)+g\left(\overline{J}\xi,\overline{J}N\right)\overline{g}\left(\overline{J}\overline{J}\xi,N\right)) \\ &\quad +2g\left(\overline{J}\xi,\overline{J}\xi\right)\overline{g}\left(\overline{J}\overline{J}N,N\right)\} \end{aligned}$$

or

$$c=0.$$

From theorem 3.8, we have the following corollary

Corollary 3.9 . *There are no real lightlike hypersurfaces of* $\overline{M}(c), c \neq 0$ *such that* $\overline{g}(R(\overline{J}N,\xi)\overline{J}\xi,N)=0$.

Lemma 3.10 *Let* $\overline{M}$ *be an indefinite Kaehler manifold and* M *be a real lightlike hypersurface of* $\overline{M}$. *If* V *is a principle vector field, then*

$$B(V,U)=0$$

and

$$C(V,V)=0.$$

Proof. From the Kaehler structure of $\overline{M}$, we have

$$\overline{\nabla}_X U=-\overline{J\nabla}_X N.$$

From (3) and (4), we get

$$\nabla_X U+B(X,U)N=-\overline{J}\left(-A_N X+\tau\left(X\right)N\right)$$

$$\nabla_X U+B(X,U)N=\overline{J}A_N X-\tau\left(X\right)\overline{J}N.$$

Thus using (14), we obtain

$$\nabla_X U+B(X,U)N=FA_N X+u(A_N X)N+\tau\left(X\right)U$$

taking the transversal vector bundle parts, we have

$$B(X,U)=u(A_N X),$$

which proves assertion.

Lemma 3.11 *Let $\overline{M}(c)$ be a positively or negatively curved indefinite complex space form and M be a real lightlike hypersurface of $\overline{M}(c)$. Then the equation of Codazzi is given by*

$$(\nabla_Y A)\,X - (\nabla_X A)\,Y = \frac{c}{4}\{\overline{g}(Y,N)X - \overline{g}(X,N)Y + g(\overline{J}Y,N)FX - g(\overline{J}X,N)FY$$
$$+2\overline{g}(X,\overline{J}Y)\overline{J}N\} + \tau\,(Y)\,A_N X - \tau\,(X)\,A_N Y$$

Proof. By straight forward calculations using (3) and (4) we obtain equation.

Now, we consider an orthonormal basis $\{z_1, z_2, ..., z_{2m-4}\}$ of D_0 such that $\{z_1, z_2, ..., z_p\}$ and $\{z_{p+1}, z_{p+2}, ..., z_{p+q}\}$, $p+q = 2m-4, p \neq q$ are unit spacelike and timelike vector fields, respectively.

Lemma 3.12 *Let $\overline{M}$ be an indefinite Kaehler manifold and M be a real lightlike hypersurface of $\overline{M}$. Then*

$$A_N U = \sum_{i=1}^{2m-4} \frac{C(U,z_i)}{\epsilon_i} z_i + C(U,U)V + C(U,V)U \tag{23}$$

and

$$A_N \xi = \sum_{i=1}^{2m-4} \frac{C(\xi,z_i)}{\epsilon i} z_i + C(\xi,U)V. \tag{24}$$

Proof. Firstly, from definition of real lightlike hypersurface of an indefinite Kaehler manifold, we have

$$A_N U = \sum_{i=1}^{2m-4} \lambda_i z_i + \beta_1 \xi + \beta_2 \overline{J}\xi + \beta_3 \overline{J}N. \tag{25}$$

From (4) and (7) we obtain

$$\begin{aligned} g(z_i,z_i)\lambda_i &= g(A_N U, z_i) \\ &= \frac{1}{\epsilon_i} C(U,z_i) \end{aligned}$$

and $\beta_1 = 0, \beta_2 = C(U,U), \beta_3 = C(U,V)$. Thus we derive, equation (23). Similarly we obtain (24).

Theorem 3.13 *There are no real lightlike hypersurfaces of indefinite complex space form $\overline{M}$ $(c \neq 0)$ such that $g\left((\nabla_U A)\,\xi, V\right) = g\left((\nabla_\xi A)\,U, V\right)$ and $B(U,U) = 0$.*

Proof. If we take $Y = U$ and $X = \xi$ in lemma 3.5. Then we have

$$(\nabla_U A)\,\xi - (\nabla_\xi A)\,U = \frac{c}{4}\{\overline{g}(U,N)\xi - \overline{g}(\xi,N)U + g(\overline{J}U,N)F\xi - g(\overline{J}\xi,N)FU$$
$$+2\overline{g}(\xi,\overline{J}U)\overline{J}N\} + \tau\,(U)\,A_N\xi - \tau\,(\xi)\,A_N U$$

or

$$(\nabla_U A)\,\xi - (\nabla_\xi A)\,U = \frac{c}{4}\{-U - 2U\} + \tau\,(U)\,A_N\xi - \tau\,(\xi)\,A_N U$$

From equations of (23) and (24) we find

$$\begin{aligned}(\nabla_U A)\,\xi - (\nabla_\xi A)\,U &= -\frac{3}{4}cU \\ &\quad -\tau\,(\xi)\left\{\sum_{i=1}^{2m-4} C(U,z_i)z_i + C(U,U)V + C(U,V)U\right\} \\ &\quad +\tau\,(U)\left\{\sum_{i=1}^{2m-4} C(\xi,z_i)z_i + C(\xi,U)V\right\}.\end{aligned}$$

Thus we obtain

$$g\,((\nabla_U A)\,\xi - (\nabla_\xi A)\,U, V)\,. = -\frac{3}{4}c + \tau\,(\xi)\,B(U,U).$$

Hence, the proof is complete.

Theorem 3.14 *There are no real lightlike hypersurfaces with positively or negatively null curved in* $\overline{M}\,(c)$.

Proof. By the definition of nullsectional curvature of M at $x \in M$ with respect to ξ_x, we have

$$K_{\xi_x}(M) = \frac{g(R(Z_x,\xi_x)\xi_x, Z_x)}{g(Z_x,Z_x)} \tag{26}$$

where Z_x is an arbitrary non-null vector in T_xM. Now, we take $Z = Z_0 \in \Gamma\,(D_0)$ in (26), then

$$\begin{aligned}K_\xi(M) &= \frac{1}{g(Z_0,Z_0)}\{\frac{c}{4}\{g\,(\xi,\xi)\,g\,(Z_0,Z_0) - g(Z_0,\xi)^2 \\ &\quad +g(\overline{J}\xi,\xi)\,(FZ_0,Z_0) - g(\overline{J}Z_0,\xi)g(F\xi,Z_0) \\ &\quad +2g(Z_0,\overline{J}\xi)g(F\xi,Z_0)\} + B\,(\xi,\xi)\,g(A_N Z_0,Z_0) \\ &\quad -B\,(Z_0,\xi)\,g(A_N\xi,Z_0)\}\end{aligned}$$

or

$$K_\xi(M) = 0.$$

Thus, the proof is complete.

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